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Node numbering in a topological structure of interconnection network

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Abstract: The topological design of a computer communication network assigns the links and link capacities for connecting network nodes within several constraints, with a goal to achieve a specified performance at a minimal cost. A reasonable approach is to generate a potential network topology. This paper presents a spanning tree based method for systematic numbering of nodes in any communication network. When the nodes are numbered in a systematic manner the potential network topology generated will have lesser amount of perturbation before an acceptable network is found.

Keywords: Topological design, Computer, Link deficit algorithm, Wireless network, Minimum spanning tree.

Introduction

The topological design of a network assigns the links and link capacities for connecting the network nodes. This is a critical phase of network synthesis, partly because the routing, flow control and other behavioral design algorithms rest largely on the given network topology. The topological design has also several performance and economic implications. The node locations, link connections and link speeds directly determine the transit time through the network. For reliability or security considerations, some networks may be required to provide more than one distinct path for each node pair, thereby resulting in a minimum degree of connectivity between the nodes (Kamlesh & Srivatsa, 2007).

The goal of the topological design of a computer communication network is to achieve a specified performance at minimal cost (Andrew S. Tanenbaum *et al*., 1987). A reasonable approach is to generate a potential network topology (starting network) and see if it satisfies the connectivity and delay constraints. If not, the starting network topology is subjected to a small modification ("perturbation") yielding a slightly different network, which is now checked to see if it is better. If a better network is found, it is used as the base for more perturbations. If the network resulting from the perturbation is not better, the original network is perturbed in some other way. This process is repeated till the computer budget is used up (Gerla *et al.,* 1974; Lavia & Manning, 1975, Andrew S. Tanenbaum*,* 1987).

One of the many heuristics for generating a potential network topology is the link deficit algorithm (Steiglitz *et al.,*1969). This heuristic begins by numbering the nodes at random. We present a systematic method for numbering the nodes. When the nodes are numbered in a systematic fashion, the starting network will need relatively lesser amount of perturbation in order to satisfy constraints. A brief review of the existing methods for numbering the nodes can be found in (Kamalesh & Srivatsa, 2007)

One of the most useful measure of a communication network performance is the transmission delay (or time delay) encountered by a message in traveling through the network from its source to its destination. In a store-forward network a message may have to be stored and forwarded by several intermediate processors before reaching its destination. The transmission delay or signal degradation is approximately proportional to the number of edges a message must travel. Thus, minimizing this number obviously leads to more efficient communication networks. Furthermore, the cost of the interconnection among the processors increases with the number of physical lines between two processors in the networks. The distance, eccentricity and diameter of a graph play significant roles in analyzing efficiency of interconnection networks. They provide efficient parameters to measure the transmission delay in the network (Latha & Srivatsa, 2007; Kamalesh & Srivatsa, 2009a,b).

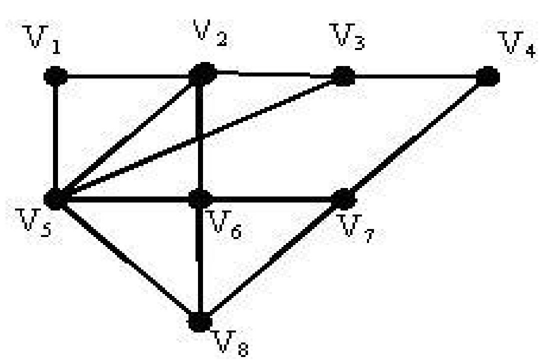
Definitions

Distance between two nodes

Let N = (V, E) be a communication network. Let v and w be any two nodes of N. The length of the shortest path (Douglas B. West *et al*., 2001) connecting v and w is called the distance between v and w and is denoted by d (v, w). The following examples depict the concept of distance between two nodes.

Example 1:

Fig. 1. A Network to calculate distance between nodes

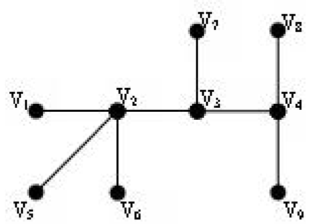


$d(v_1, v_7) = 3$, $d(v_2, v_8) = 2$, $d(v_4, v_5) = 2$, the cost associated with each link is assumed to be unity.

Indian Journal of Science and Technology Vol.2 No. 11 (Nov. 2009) ISSN: 0974- 6846

Example 2:

Fig. 2. A network graph to calculate distance between any two nodes



$d(v_1, v_9) = 4$, $d(v_5, v_7) = 3$, $d(v_2, v_6) = 1$ etc.
Further, the distance function d(v, w) is a metric because it satisfies following properties.

- $d(v,w) \geq 0$
- $d(v,w) = 0$ if $u = w$
- $d(v,w) \leq d(v,x) + d(x,w)$ for all nodes $x \in N$

Eccentricity of a node

Let N= (V, E) be a communication network. Let v be a node in N. Suppose we determine the distance between v and every other node of N. The largest of these distances is called the eccentricity of the node v (Douglas B. West, 2001) and is denoted by E (v).

$$E(v) = \max \{d(v, x) \mid \forall x \in N\}$$

That is, eccentricity of a node v in a communication network N is the distance between the node v and farthest node from v in N.

The node with minimum eccentricity is called the **center**.

Spanning tree of a graph

Let G be a connected Graph. A subgraph T of G is a spanning tree of G, if T is a tree and T contains all nodes of G. (Douglas B. West, 2001)

Example:

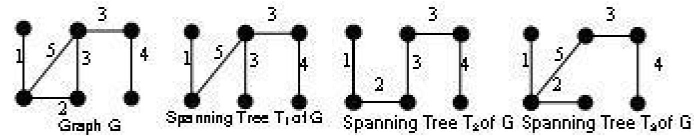
Fig. 3. Connected graph & its corresponding spanning tree

Minimum spanning Tree of a graph

Let G be a weighted connected graph. A spanning tree whose total weight is the least is called a minimum spanning tree of G (Douglas B. West, 2001).

Example:

Fig.4. A network graph and its corresponding spanning trees



The total weights of the three spanning trees T_1, T_2 and T_3 are 16, 13, and 15 respectively. Of these trees T_2 has the minimum weight and hence T_2 is the minimum spanning tree of G.

Theorem

Statement: Every tree topology has either one or two centers.

Proof: Let T(V, E) be a tree topology. For any node $v \in T(V)$, d (v, w) is a maximum if w is a pendent node. Delete all the pendent nodes from T (V, E). The resultant network topology T′ (V′, E′) is again a tree topology with node having eccentricities reduced by one. Further centre of T continues to be centre for T′.

Delete all the pendent nodes from T′ (V′, E′) and get a new tree topology T″ (V″, E″). The centre of T′ continues to be the centre of T″. Proceeding in this way finally we end up with a single node or link with two nodes. Thus a tree topology has either one or two adjacent centers.

Proposed method

This section presents the proposed method for systematic numbering of nodes for any communication (wired / wireless) networks, keeping in view of transmission delay. Consider the given communication network, N(V, E). To start with, the nodes are labeled using some symbols. Find the minimum spanning tree topology T(V, E) of the given communication network N(V, E) for every node $v \in T$ the eccentricity E(v) is calculated. A table displaying the node and the corresponding eccentricity values is constructed. The nodes are sorted based on eccentricity values, subsequently the index value of the sorted list is assigned as the representative number for the respective nodes.

The major steps of the proposed algorithm are as follows:

Algorithm: Systematic numbering of nodes in a communication network.
Input : Unlabeled network N= (V, E), where $|V| = n$
Output : n labels for n distinct nodes of the network N.

Indian Journal of Science and Technology Vol.2 No. 11 (Nov. 2009) ISSN: 0974- 6846

Method :

1. Consider a given communication network N(V, E)
2. Name the nodes using some symbols (V_1, V_2, …,V_n)
3. Find the minimum spanning tree topology T of the given communication network.
4. For every node V ∈ T(V), Find the eccentricity E(V)
5. Sort the vertices based on Eccentricity value subsequently assign the index value of the sorted list as the representative number for the nodes.

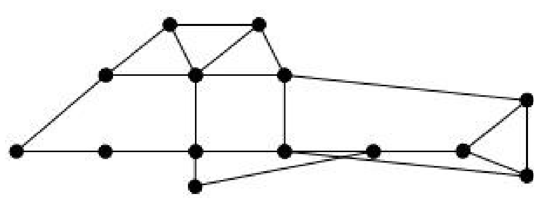
Algorithm end

Illustration

Stage by stage illustration of the proposed method is presented in this section.

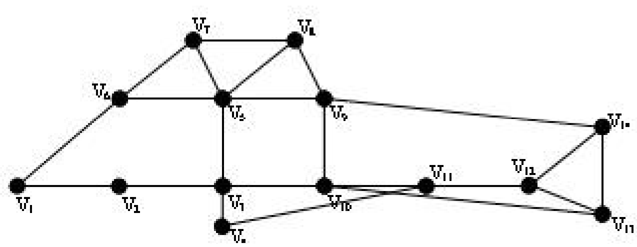
Let us consider the following network, with cost associated with each link is unity

Fig.5. Unlabelled network graph



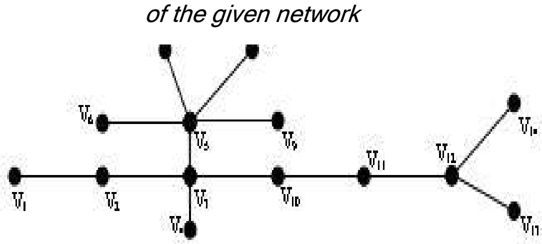
Name the nodes using some symbols.

Fig. 6. Network graph whose nodes are named using symbols



Find the minimum spanning tree topology of the given network.

Fig.7. Minimum spanning tree topology of the given network



Distance between various nodes of the tree topology is computed and the same is tabulated (Table 1).

Table 1. Distance between various nodes of the tree topology

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
V_1	0	1	2	3	3	4	4	4	4	3	4	5	6	6
V_2	1	0	1	2	2	3	3	3	3	3	3	4	5	5
V_3	2	1	0	1	1	2	2	2	2	1	2	3	4	4
V_4	3	2	1	0	2	3	3	3	3	2	3	4	5	5
V_5	3	2	1	2	0	1	1	1	1	2	3	4	5	5
V_6	4	3	2	3	1	0	2	2	2	3	4	5	6	6
V_7	4	3	2	3	1	2	2	0	2	3	4	5	6	6
V_8	4	3	2	3	1	2	2	0	2	3	4	5	6	6
V_9	4	3	2	3	1	2	2	2	0	3	4	5	6	6
V_{10}	3	2	1	2	2	3	3	3	3	0	1	2	3	3
V_{11}	4	3	2	3	3	4	4	4	4	1	0	1	2	2
V_{12}	5	4	3	4	4	5	5	5	5	2	1	0	1	1
V_{13}	6	5	4	5	5	6	6	6	6	3	2	1	0	2
V_{14}	6	5	4	5	5	6	6	6	6	3	2	1	2	0

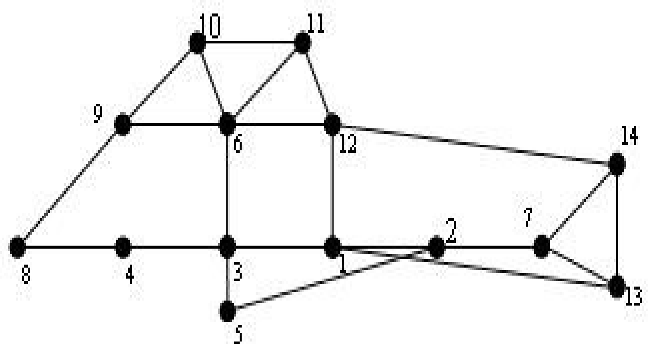
The eccentricity is computed for every node of the tree topology as shown in column 2 of Table 2.

Table 2: Eccentricity and node numbering

Vertex	E(V)	Node Number
V_1	6	8
V_2	5	4
V_3	4	3
V_4	5	5
V_5	5	6
V_6	6	9
V_7	6	10
V_8	6	11
V_9	6	12
V_{10}	3	1
V_{11}	4	2
V_{12}	5	7
V_{13}	6	13
V_{14}	6	14

The vertices are sorted based on the eccentricity value. Subsequently, the index value of the sorted list is assigned as representative number of the nodes as shown in column 3 of Table 2. The network after numbering the nodes is as shown below:

Fig.8. A network graph whose nodes are numbered systematically



Conclusion

This paper presents a generic method for systematic numbering of nodes of any communication network. Most of the existing methods rely heavily on heuristics and lack sound mathematical background. The proposed method is based on graph theoretical approach. Also the method works equally well irrespective of the network connectivity number. Where as the methods suggested earlier (Srivatsa & Seshaiah, 1995; Latha & Srivatsa, 2007) works for k-connected network. The proposed method is much more efficient than the previous ones (Henzinger *et al.,* 2000; Kamalesh & Srivatsa, 2007; 2009a,b). The method suggested earlier by Kamalesh & Srivatsa (2007) requires the consideration of multiple paths to compute the distance between pair of nodes. This demands computational effort, whereas in the proposed method the distance between the pair of nodes is computed from the minimum spanning tree topology of the given communication network.

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