

Fault detection and diagnosis for three-tank system using robust residual generator

A. Asokan and D. Sivakumar

Dept of E&I, Annamalai University, Annamalai Nagar-608 002, Chidambaram, India

asokan_me@yahoo.co.in

Abstract: Fault detection and diagnosis (FDD) is a task to deduce from observed variable of the system if any component is faulty, to locate the faults and also to estimate the fault magnitude present in the system. The main goal when synthesizing robust residual generators, for diagnosis and supervision, is to attenuate influence from model uncertainty on the residuals while keeping fault detection performance. In this paper, a design procedure for robust residual generators is developed with two key elements. One is the use of a reference model that represents desired performance. The other is an optimization criterion, based on robust H_∞ filtering, used to synthesize the residual generator.

Keywords: Fault detection, robust residual generation, structured residual approach, H_∞ filtering.

Introduction

Diagnosis and supervision are important in many applications. Different approaches for fault detection using mathematical models have been developed in the last 20 years. The task consists of the detection of faults in the processes, actuators and sensors by using the dependencies between different measurable signals. They are based either on the model of the system (Gertler, 1998; Gertler *et al.*, 2002) or on the knowledge about the system (Wu *et al.*, 2005; Gao *et al.*, 2007). One way to detect faults in a system is by means of analytical redundancy (Gertler, 1988). This consists of comparing the behaviour of the real system with that of its model. In an ideal case, the system and the model behave exactly the same and when a fault is detected the behaviours are different. But usually there are differences between the behaviours of the system and the model.

The implementation procedure of the proposed FDI scheme is illustrated in Fig. 1. The controller in the system is used to maintain the process variable at its set point. When there is a fault in the process, its output differs with model output. In the presence of modeling errors output differs with model output even though there is no fault in the system. This difference is termed as residual. In this paper fault isolation scheme is studied under modeling uncertainties and also considers the isolation of multiple faults that may occur at same time.

Model based diagnosis uses a model to obtain residuals, which are signals that are zero in the fault free case and non-zero otherwise, to perform the diagnosis. Since available models of real processes always are uncertain, there is naturally a need for robust methods minimizing the sensitivity to the model uncertainties. This paper addresses the problem of synthesizing and analyzing robust residual generators in the presence of parametric uncertainties that influence the process. Here, focus is on designing robust residual generators, dealing with model uncertainty, to fit in a structured residuals framework. It is

advantageous to introduce a reference model that describes desired behavior of the residuals with respect to faults.

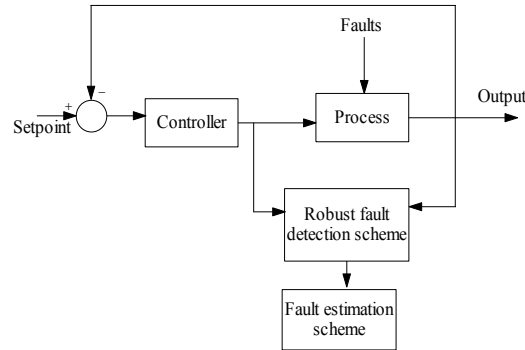


Fig. 1. Block diagram representation of proposed FDI scheme

Problem formulation

The system under consideration is assumed to be on the form: $Y = G^\Delta(s)u + L^\Delta(s)f$ (1)

Where y is the measurement vector, u the control signal, and f is the fault vector, matrices $G^\Delta(s)$, and $L^\Delta(s)$ are all rational transfer matrices. The superscripts Δ indicates that the model is subject to bounded parametric uncertainties. The residual generator is a finite dimensional linear filter $Q(s)$ that uses available known signals, that is Y and u , to form a vector of residuals r that can be used to detect and isolate the different faults f

$$r = Q(s) \begin{pmatrix} Y \\ u \end{pmatrix} \quad (2)$$

The basic requirement on $Q(s)$, besides being RH_∞ , is that the residuals, r , should also be insensitive to control actions u , but it should be sensitive to faults f . The technique of fault isolation considered here is structured residual approach where faults are decoupled in each residual. By generating a set of residuals where different subsets of faults are decoupled in each residual, fault isolation is possible.

Inserting (1) into (2) gives

$$r = Q(s) \begin{bmatrix} G^\Delta(s) \\ 0 \end{bmatrix} u + Q(s) \begin{bmatrix} L^\Delta(s) \\ 0 \end{bmatrix} f \quad (3)$$

Nominally, to achieve decoupling of u the first term of (3) must be zero while the second term must be $\neq 0$. However, with uncertain models it is in most cases impossible to get the first term = 0 for all Δ i.e. for all possible instances of uncertainties, without losing some or all of the desired fault sensitivity. Generally some tradeoff between sensitivity to faults and uncertainty attenuation is required. The problem studied here is how to find the filter $Q(s)$ such that a proper trade off between fault sensitivity and robustness towards model uncertainty

is achieved. The solution is based on a performance specification and a synthesis procedure based on robust H_∞ -filtering.

Reference model

When synthesizing a robust residual generator, it is desired that the design freedom available should be used to achieve both robustness and detection performance. It is chosen that a reference model, $R(s)$, to describe the desired behavior of the residual vector r , with respect to faults f . Define desired residual behavior $r_0(s)$, of the residual, via the reference model as

$$r_0(s) = R(s) f(s)$$

The matrix $R(s)$ is an arbitrary RH_∞ transfer matrix of appropriate dimensions. It is of course necessary that the reference model, $R(s)$, contains the necessary structure for $Q(s)$ to be a residual generator. This includes decoupling properties of faults, i.e. zeros at proper positions in $R(s)$ corresponding to the desired residual structure

Robust residual generator synthesis

The main idea with robust residual generation is to minimize the unwanted influences on the residuals while maintaining the fault sensitivity in the residuals. This trade-off, fault sensitivity vs. disturbance attenuation, is normally formulated as an optimization problem in different formulations and norms. A common choice is to utilize H_∞ theory to perform the synthesis. Powerful synthesis tools are important, but also worst-case analysis tools are important to aid example robust threshold selection or robustness evaluation.

Robustness criterion

The optimization criterion used here is formulated as a robust H_∞ filtering problem (Frank & Ding, 1997).

The criterion is given by

$$J = \sup_{v \in L} \frac{\|r_0 - r\|_2}{\|v\|_2} \tag{4}$$

where $v = [u^T \ f \ d^T]^T$. The optimization criterion J is thus the worst case distance between the residual r and the idealized residual r_0 , defined by transfer matrix $R(s)$, normed by the size of the inputs. The optimal residual generator $Q(s)$ is the filter that minimizes J for all $\|\Delta\|$ inside some bounded ball.

The optimization criterion J can be rewritten as

$$J = \sup_{v \in L} \frac{\|r_0 - r\|_2}{\|v\|_2} = \sup_{v \in L} \frac{\|T_{zv}(s)v\|_2}{\|v\|_2} = \|T_{zv}(s)\|_\infty$$

Where $z(t) = r_0(t) - r(t)$, and

$$T_{zv} = [-G_{ru}^\Delta(s)(R(s) - G_{rf}^\Delta(s))] \tag{5}$$

is the transfer matrix from $v(t)$ to $z(t)$. The transfer matrices from u to r , $G_{ru}(s)$, and from f to r , $G_{rf}(s)$ all depend on the residual generator $Q(s)$. Minimizing J , i.e. minimizing the 1-norm of expression (5), has a simple interpretation, the first makes sure that the influence from

u on the residual are attenuated. The next term keeps fault sensitivity, and also shapes the fault to residual transfer function $G_{rf}(s)$ by minimizing the distance to the reference model $R(s)$.

The optimization performance index minimizes the absolute difference between $R(s)$ and $G_{rf}(s)$. A reasonable assumption is that it is the relative difference that needs to be minimized, otherwise in high-gain models even very small relative errors will dominate the loss function and therefore move away optimization focus from robustness to fault sensitivity in an unwanted manner. Therefore it is important to normalize and weigh the model appropriately to avoid such effects.

Implementation

The residual generation optimization problem can be described by an upper and lower LFT, including the structured parametric uncertainty, as in Fig. 1, where $P(s)$ is an augmented system description including a description on how the parametric uncertainties Δ influence the system, fault models, disturbance models, dynamic weighting matrices and also the reference model. With this problem setup, there exist algorithms minimizing J with respect to $Q(s)$ by example μ synthesis. The algorithm used in this work is basic DK-iterations.

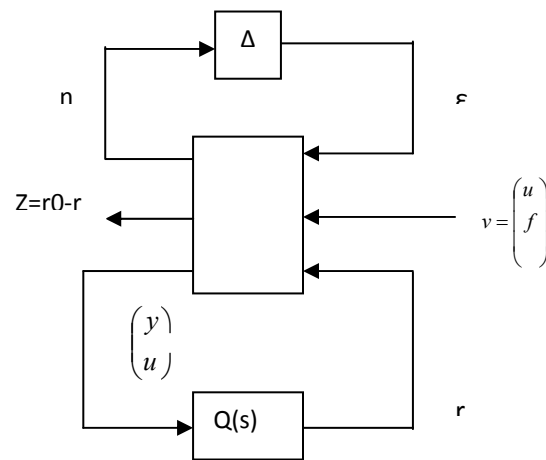


Fig.2. LFT formulation of residual generation problem

Forming the reference model

Reference model with unrealistic performance properties, can result in a residual generator with unnecessary poor robustness properties. The main idea is thus to use a nominal design of the residual generator to shape the reference model when synthesizing the robust residual generator, thus assuring attainable reference models. This is to avoid specifying an unrealistic performance criterion and thereby inflicting unnecessary poor robustness properties on the residual generator. The formation of the criterion for the robust design is straightforward, given that a nominal residual generator, i.e. a $Q_{nom}(s)$, has been derived that nominally fulfills all demands. The reference model $R(s)$ is then selected as

$$R(s) = Q_{nom}(s) \begin{bmatrix} L(s) \\ 0 \end{bmatrix} \quad (6)$$

Since this is the nominal fault to residual transfer function, compare with Equation (3). Of course, if no design based on a nominal model is available that meets the requirements of the application, then no feasible design with an uncertain model is available either.

System descriptions

The three-tank system considered for study (Wu *et al.*, 2005) is shown in Fig.2. The controlled variables are the level of the tank1 (h1) and level of the tank3 (h3). In flow of tank1 (fin₁) and in flow of tank3 (fin₃) are chosen as manipulated variables to control the level of the tank1 and tank3. The unmeasured outflow of that is leak of tank1, tank2 and tank3 have been considered as fault variables (L₁,L₂ and L₃).In this paper it is assumed that the leaks are independent of level of the tank.

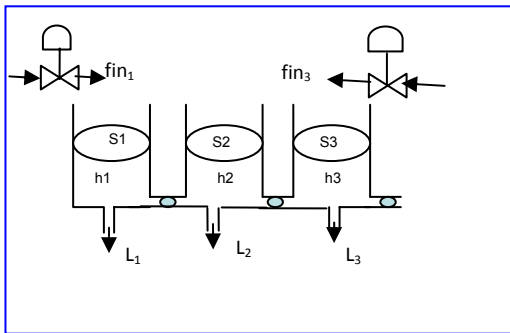


Fig. 3. Three tank systems

The steady state operating data of the Three-tank system is given in Table1. The state space model of the three tank system around the operating point given in the table is

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Where

$$A = \begin{bmatrix} -0.00723996 & 0.00723996 & 0 \\ 0.00723996 & -0.01707086 & 0.0098303 \\ 0 & 0.0098309 & -0.012540553 \end{bmatrix}$$

$$B = \begin{bmatrix} 64.935 & 0 & 0 \\ 0 & 64.935 & 0 \\ 0 & 0 & 64.935 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

When fault (leak) occurs the state space model is given

$$\dot{x} = Ax + Bu + B_f f$$

by

$$y = Cx + Du$$

$$B_f = \begin{bmatrix} 64.935 & 0 & 0 \\ 0 & 64.935 & 0 \\ 0 & 0 & 64.935 \end{bmatrix}$$

Design of structured residual generator

To transform raw residual R(s) into structured form R_t(s), multiply R(s) with weighting matrix W(s)

$$R_t(s) = W(s)R(s) \quad (7)$$

Weighting matrix is chosen as to contain inverse of Identified fault model to cancel the effects of fault transfer function present in residual.

$$w = z(s) Q_{nom}^{-1}(s) \quad (8)$$

Where Z(s) is user defined matrix

Substituting W(s) in expression (7)

$$R_t = w Q_{nom}(s) L(s)$$

Substituting R_t(s) in the above expression

$$R_t = z(s) Q_{nom}^{-1}(s) Q_{nom}(s) L(s) \quad (9)$$

Where I is the identity matrix.

$$R_t(s) = Z(s)L(s) \quad (10)$$

Z(s) is chosen as diagonal matrix so as to fault isolation and estimation

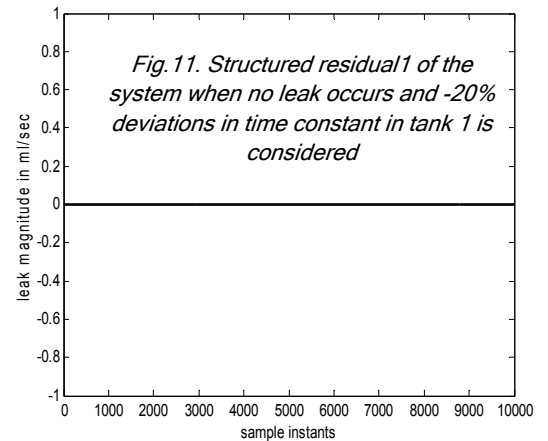
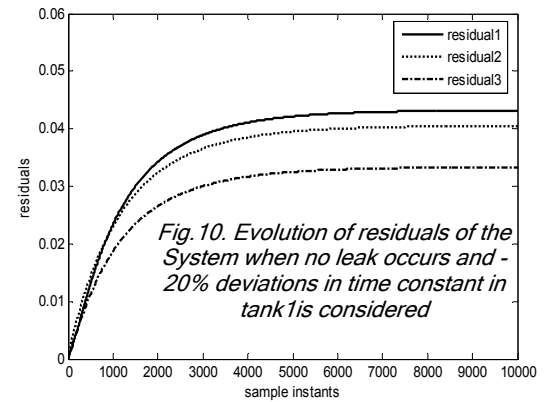
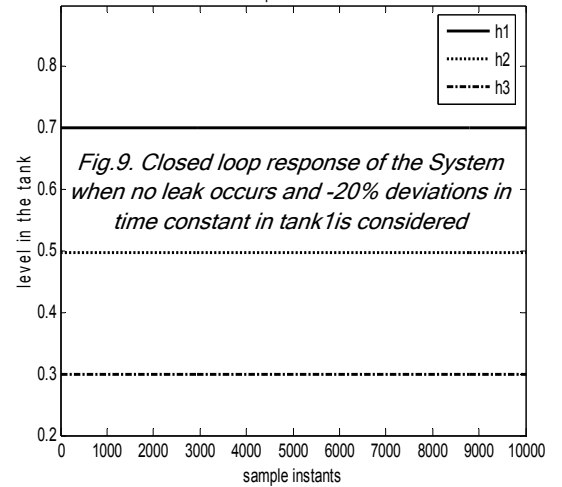
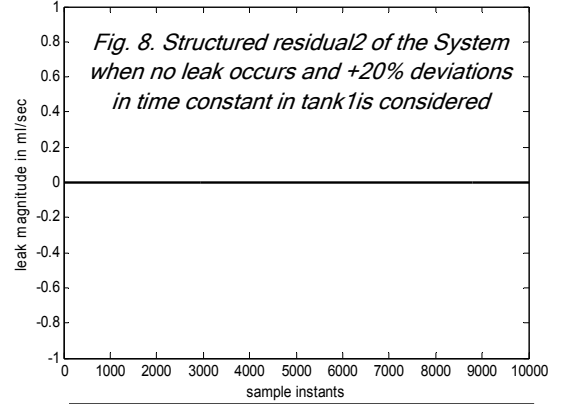
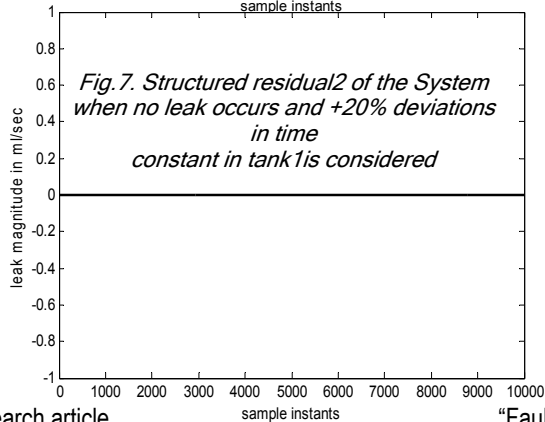
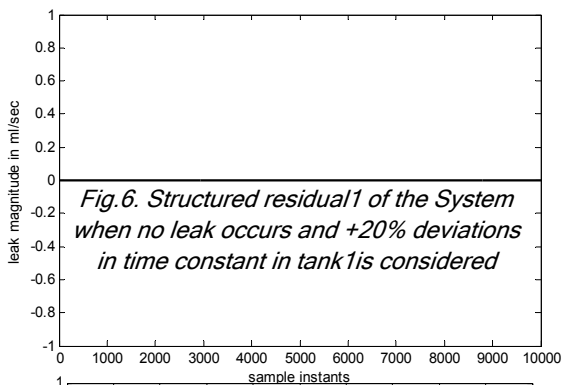
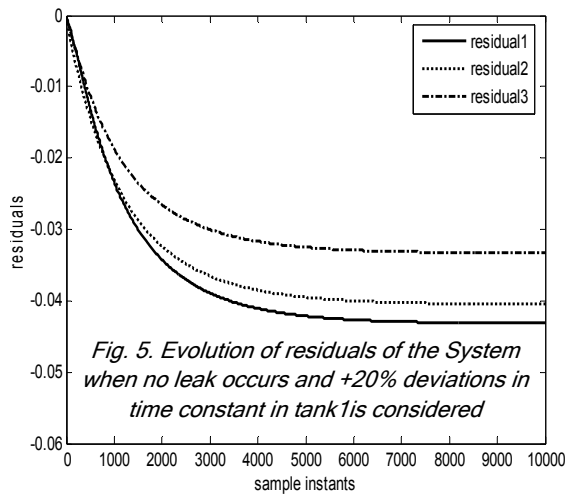
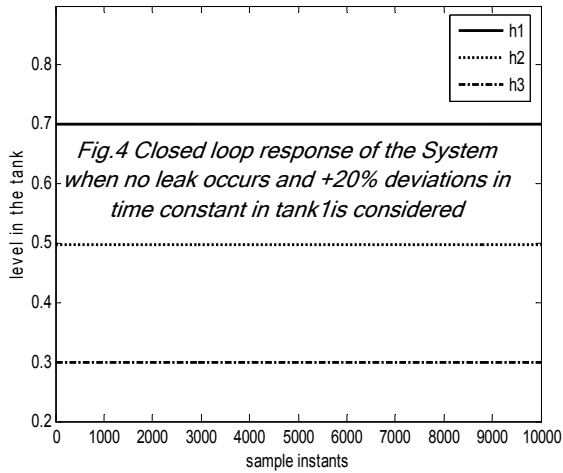
Simulation results

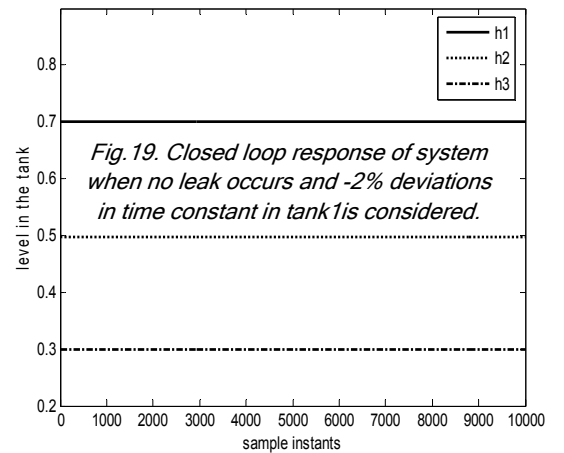
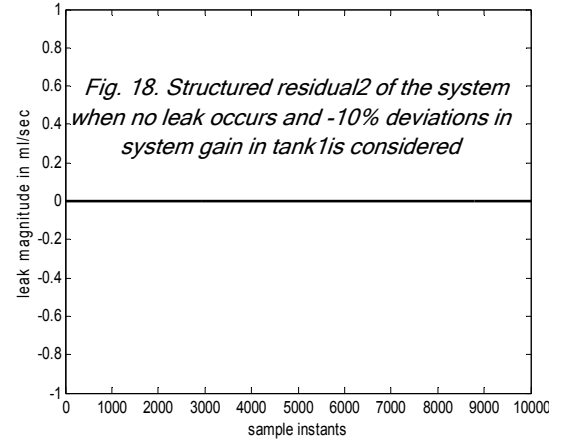
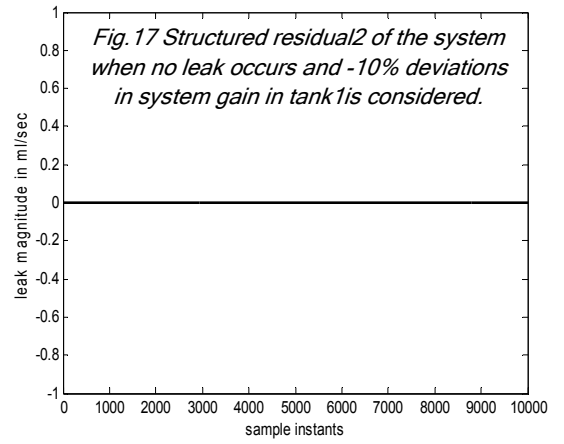
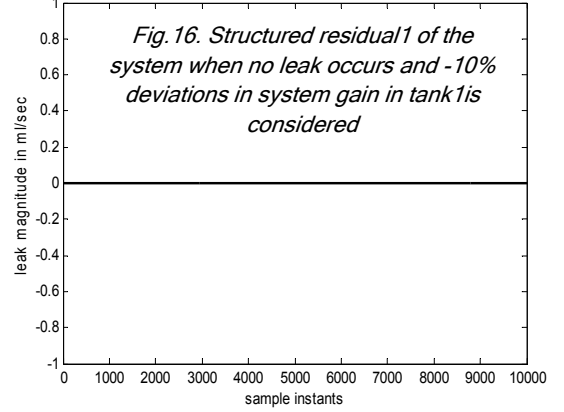
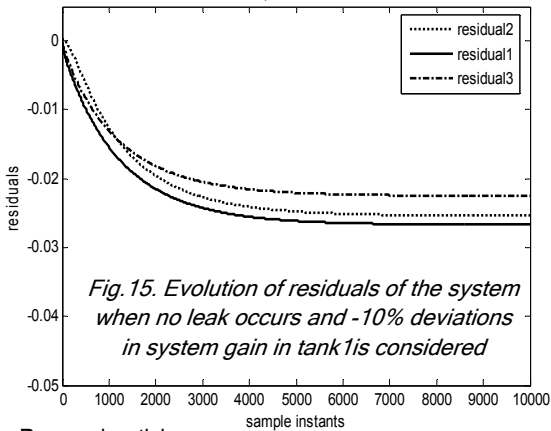
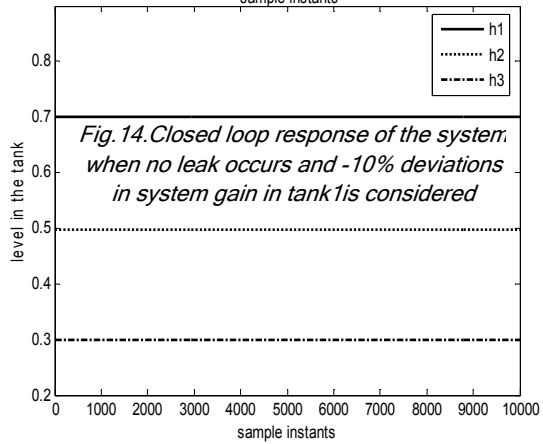
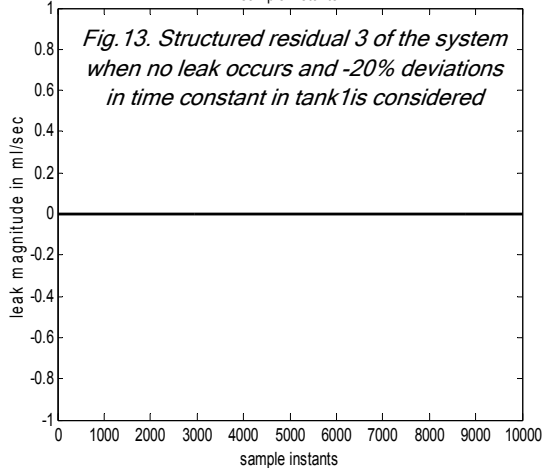
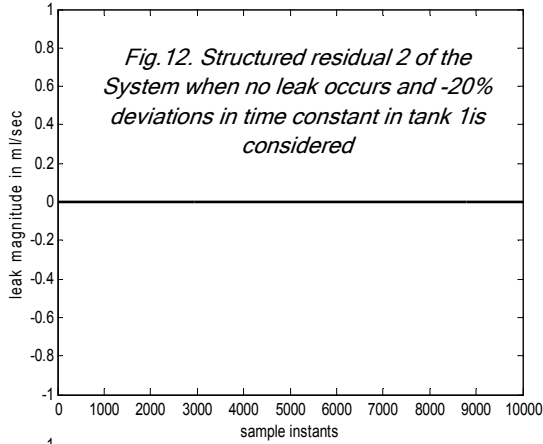
The proposed robust FDI scheme has been implemented on a three-tank system and its performance is observed. The controlled variables are the level of tank1 (h1) and tank3 (h3). Inflow of tank1 (fin₁) and tank3 (fin₃) are chosen as manipulated inputs. Outflow of tank1 (L₁), tank2 (L₂) and tank3 (L₃) are considered as leak variables.

The synthesis method is used for the design of PI controller. The PI controllers are designed so that the closed loop process behaves like a first order system with unity gain and time constant same as the open loop time constant. The resulting parameter for controlling the height of tank1 using the inflow of tank1 is given by Kc = 2.54e-04 (ml/sec/m) and Ti=222 seconds and that of tank3 using the inflow of tank3 is given by Kc=7.69e-4 (ml/sec/m) and Ti=200 seconds. The process is simulated using the non-linear first principles model, whereas the FDI is based on the time invariant linearized model (Transfer function model).

The proposed robust residual approach is tested for modeling errors that is +20% deviations in time constant in tank1 is considered. The closed loop behavior of the process in absence of leak in the system is shown in Fig.4. The behavior of the residuals is shown in Fig.5 from which one can infer that even though there is no leak in the system, all the three residuals get affected because of modeling error. By simply monitoring either the process output or the residual it is not possible to identify the location of the fault. The structured residual output is shown in Fig. 6-8 from which one can conclude that there is no leak in the system.

The proposed robust residual approach is tested for modeling errors that is -20% deviations in time constant in tank1 is considered. The closed loop behavior of the process when no leak present in the system is shown in Fig.9. The behavior of the residuals is shown in Fig.10 from which one can infer that even though there is no leak in the system, all the three residuals get affected





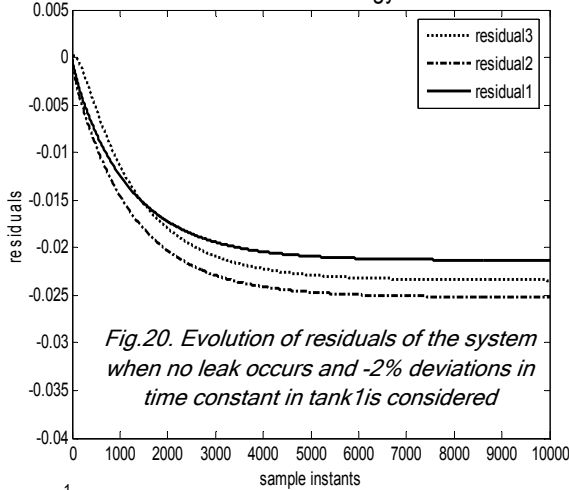


Fig.20. Evolution of residuals of the system when no leak occurs and -2% deviations in time constant in tank1 is considered

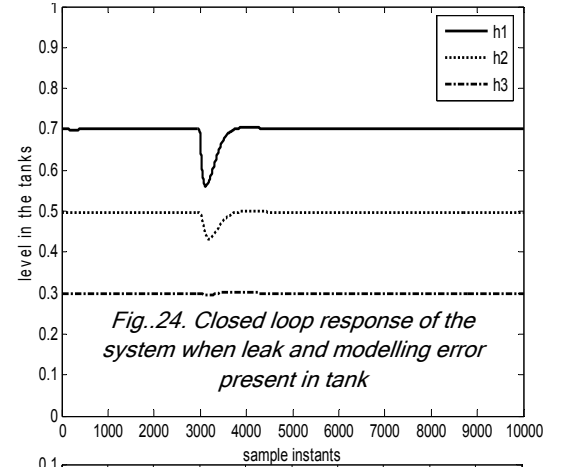


Fig..24. Closed loop response of the system when leak and modelling error present in tank

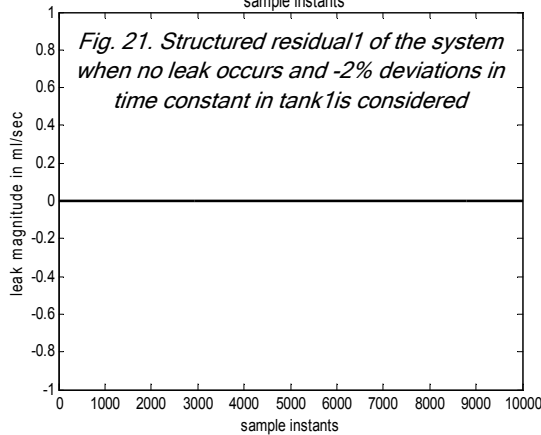


Fig.21. Structured residual1 of the system when no leak occurs and -2% deviations in time constant in tank1 is considered

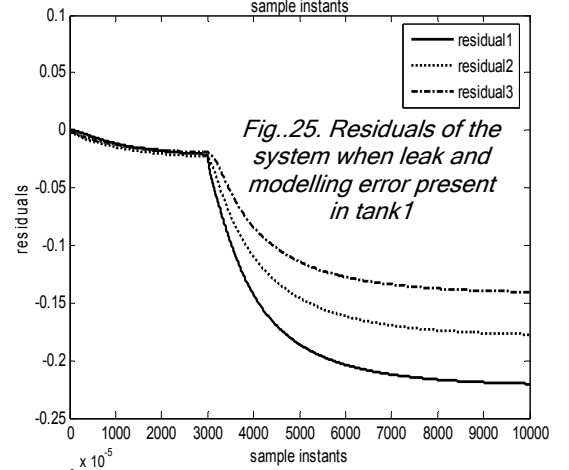


Fig..25. Residuals of the system when leak and modelling error present in tank1

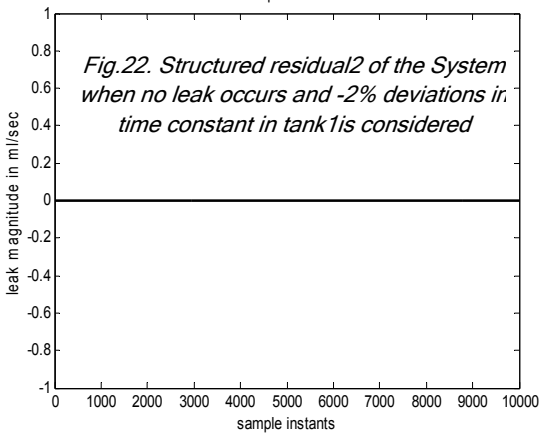


Fig.22. Structured residual2 of the System when no leak occurs and -2% deviations in time constant in tank1 is considered

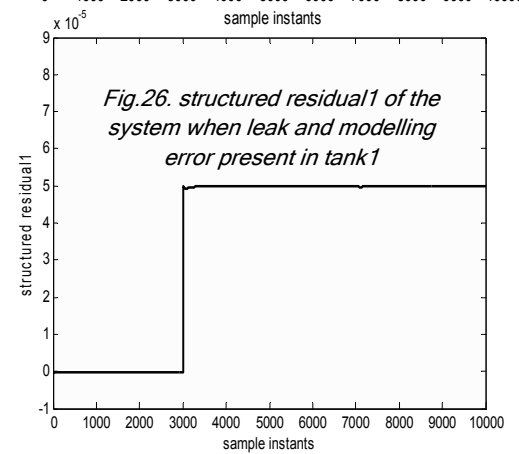


Fig.26. structured residual1 of the system when leak and modelling error present in tank1

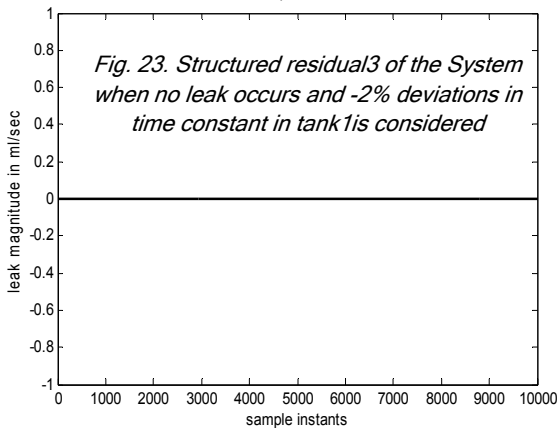


Fig. 23. Structured residual3 of the System when no leak occurs and -2% deviations in time constant in tank1 is considered

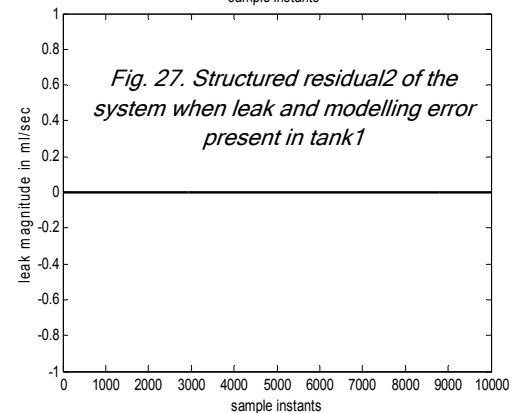


Fig. 27. Structured residual2 of the system when leak and modelling error present in tank1

because of modeling error. By simply monitoring either the process output or the residual it is not possible to identify the location of the fault. The Structured residual output is shown in Fig. 11-13 which concludes that there is no leak in the system.

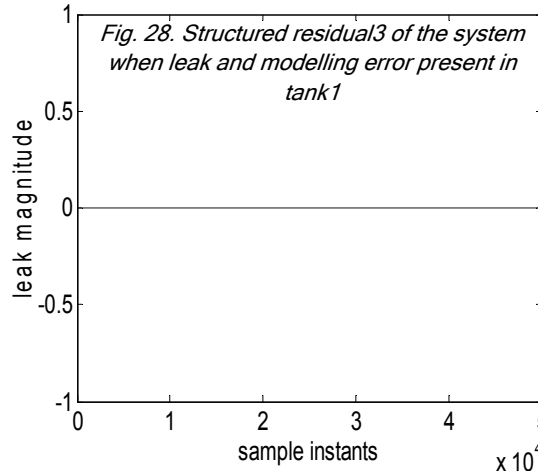
The proposed robust residual approach is tested for modeling errors that is -10% deviations in system gain in tank1 is considered. The closed loop behavior of the process in absence of leak in the system is shown in Fig.14. The behavior of the residuals is shown in Fig.15 revealing that even though there is no leak in the system, all the three residuals get affected because of modeling error. By simply monitoring either the process output or the residual it is not possible to identify the location of the fault. The structured residual output is shown in Fig.16-18 which concludes that there is no leak in the system.

The proposed robust residual approach is tested for modeling errors *i.e* -2% deviations in time constant in tank1 is considered. The closed loop behavior of the process when no leak is present in the system shown in Fig.19. The behavior of the residuals is shown in Fig.20 from which one can infer that even though there is no leak in the system all the three residuals get affected because of modeling error. By simply monitoring either the process output or the residual it is not possible to identify the location of the fault. The Structured residual output is shown in Fig. 21-23 which led to the conclusion that there is no leak in the system.

The proposed robust residual approach is tested for leak in the system with modeling errors *i.e* -20% deviations in time constant in tank1 is considered. The closed loop behavior of the process when leak of magnitude 50ml/sec is introduced at time $t=3000\text{sec}$ in tank1 is shown in Fig.24. The behavior of the residuals is shown in Fig.25 from which one can infer that there is leak in the system and all the three residuals get affected because of it. By simply monitoring either the process output or the residuals it is not possible to identify the location of the fault. The Structured residual output is shown in Fig. 26-28. From which one can conclude that there is leak in the tank1.

Conclusions

A theory for robust residual design has been studied where a key element is used in a reference model. The reference model represents desired structure and performance of the synthesized residual generator. It also includes



performance issues such as fault response in the residual. Without considering constraints, it is possible to form unrealistic performance demands and this can de-emphasize the robustness parts of the optimization and lead to a design with unnecessary poor robustness properties. A methodology on how to select realistic reference models is presented where the design freedom available is explicit and intuitive. The optimization algorithms used to synthesize the residual generator rely on

established and efficient methods. The designer of a diagnosis system is thus provided with a tool where it is easy to specify desired behavior.

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Table 1 Steady state operating data

h1, h2,h3 in m	0.7,0.5,0.3
fin ₁ and fin ₃ in ml/sec	100
Outflow coefficient(Az1,Az2,Az3)	2.251e-5,3.057e-5,2.307e-5
Area of tank (S1-S3)in m ²	0.0154
L ₁ ,L ₂ ,L ₃ in ml/sec	0
Acceleration due to gravity in m/sec ²	9.81