

## The effect of mutual interaction anisotropy parameter on the system of collective motion

A. Santhanam, R. Arunchunai Annadurai and M. Balakrishnan\*  
 PG & Research Dept. of Physics, Government Arts College, Tiruvannamalai - 606 603, TN, India.  
 mbkarunmathi\_ro@yahoo.co.in\*

### Abstract

The collective motion of birds is studied numerically. The effect of the anisotropy mutual interaction parameter plays a vital role on the order parameter of the collective motion of biological groups like birds, fishes etc. It is found that, if the interaction with the elements in front of a given element is stronger than with those behind, then the system will move as a single unit.

**Keywords:** Bird, fly, collective motion, anisotropy parameter, disorder parameter.

### Introduction

Many biological organisms form group such as bacterial colony, flocking of birds swarming of fishes etc. The collective motion of such biological systems show varied complex behaviors and have rich dynamics (Wilson, 1975; Jnoue, 1981; Patridge, 1982; Edelstein-Keshet, 1990). These collective motions of biological organisms are very interesting from theoretical point of view (Budrene *et al.*, 1991; Matsuyama *et al.*, 1993; Vicsek *et al.*, 1995). Many mathematical models have been proposed for the collective motion (Niwa, 1994). The objective of the present study is to understand why birds fly in a neat flock and to study the effect of the value of anisotropy of mutual interaction on the order of the collective motion.

### Materials and methods

The mathematical model has been developed by Naohiko *et al.* (1996). This model postulates that individuals are simply particles with the mutual interactions and motive force. The concept of heading vector  $\vec{n}_i$  is introduced which points out the direction of the head of the birds. Large birds often glide. In a glide, the heading vector and the velocity vector need not be parallel, therefore, it is assumed that heading vector and velocity vector relax to parallel with relaxation time  $\tau$ . The state variables for the  $i^{\text{th}}$  particle are the position vector  $r_i$ , the velocity vector  $v_i$  and the heading unit vector  $n_i$ . They obey the following equations of motion,

$$m \frac{d\vec{v}_i}{dt} = -\gamma \vec{v}_i + a \vec{n}_i + \sum \alpha_{ij} \vec{f}_{ij} + \vec{g}_i \quad (1)$$

The term  $m \frac{d\vec{v}_i}{dt}$ , represents the force on  $i^{\text{th}}$  particle. And  $m$  is the mass of any particle which is considered to be constant.

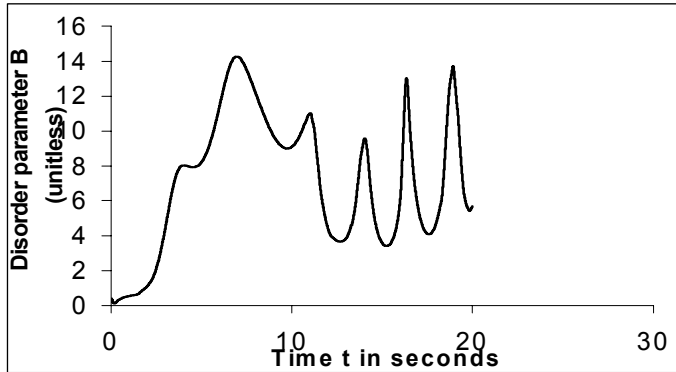
The term  $-\gamma \vec{v}_i$  represents the resistive force based on Stoke's law.  $\gamma$  is the resistive coefficient representing velocity dependent friction. This resistive force restricts the speed of the particle. The term  $a \vec{n}_i$  represents the force motivating to move along the heading vector. Here,  $a$  is the locomotive force which is a constant. This force acts in the direction along the heading unit vector. The term  $\alpha_{ij} \vec{f}_{ij}$  represents short range force between  $i^{\text{th}}$  and  $j^{\text{th}}$  elements. Here the interaction force is given by,

$$\vec{f}_{ij} = -c_i \left[ \left( \frac{|\vec{r}_j - \vec{r}_i|}{r_c} \right)^{-3} - \left( \frac{|\vec{r}_j - \vec{r}_i|}{r_c} \right)^{-2} \right] \left( \frac{\vec{r}_j - \vec{r}_i}{r_c} \right) \exp \left( \frac{-r}{r_c} \right) \quad (2)$$

Where,  $|\vec{r}_j - \vec{r}_i|$  is the distance between the positions of two particles considered. Here the exponential term is introduced to weaken the strength of the interactions without changing the direction multiplier term. It is necessary to weaken the strength because this creates dynamic patterns rather than aggregate grouping. This force avoids each particle to come very closer to each other. Here,  $r_c$  is the optimal distance between neighbors as well as the range of force. The interaction need not be isotropic; the interaction with the elements in front of a given element is stronger than with those behind. Therefore, we introduce,  $\alpha_{ij}$  the direction sensitivity factor,

$$\alpha_{ij} = 1 + d \left[ \frac{\vec{n}_i \cdot (\vec{r}_j - \vec{r}_i)}{|\vec{r}_j - \vec{r}_i|} \right]; \quad 0 \leq d \leq 1 \quad (3)$$

Fig. 1. The system oscillates between wandering and swarming state (for  $d = 1$ )



Here  $d$  is the anisotropy mutual interaction parameter. If the factor  $d = 0$ , the interaction is isotropic. This direction sensitive factor avoids the closeness of the particle in front. The term  $\vec{g}_i$  is the globally attractive force, given

$$\text{by, } \vec{g}_i = \frac{c_g (\vec{g} - \vec{r}_i)}{N |\vec{g} - \vec{r}_i|} \quad (4)$$

Here,  $N$  is the number of particles in the system and

$$\vec{g} \text{ is the centre of the group. } \vec{g} = \frac{\sum_i \vec{r}_i}{N} \quad (5)$$

This force acts on each particle towards the centre and thus motivates the grouping. In the following discussions we assume these two interaction forces have the same degree of magnitude i.e.  $c = c_1 = c_g$ . To investigate the qualitative properties of this model, numerical simulations for various control parameters were carried out and observed the collective motions. The group velocity of the cluster at a moment 't' be,

$$\vec{v}(t) = \frac{1}{N} \sum_i \vec{v}_i(t) \quad (6)$$

The fluctuation in velocity can be evaluated by averaging the r.m.s. velocity deviation,

$$\langle (\Delta v)^2 \rangle = \left\langle \frac{1}{N} \sum_i |\vec{v}_i(t) - \vec{v}(t)|^2 \right\rangle \quad (7)$$

The magnitude of the quantity  $\langle (\Delta v)^2 \rangle^{1/2}$  is zero for ordered motions and non zero for disordered motions. To characterize the different collective motions quantitatively, a disorder parameter is introduced,

$$B = \frac{\langle (\Delta v)^2 \rangle^{1/2}}{\langle v^2 \rangle^{1/2}} \quad (8)$$

The equations of motion are solved numerically using Ruge-Kutta method and the disorder parameter is calculated at each step by changing the value of anisotropy mutual interaction factor. Graphs are drawn between disorder parameter and time for different values of mutual interaction parameter and shown in Fig. 1- 4.

**Results and discussion**

*Collective Motion*

The wandering and swarming occurs alternatively (Fig.1). Here, the small peaks represent wandering and higher peaks correspond to swarming states.

The value of anisotropy mutual interaction  $d$  is decreased to 0.8 (Fig.2). We can find that  $B_{\min}$  decreases and  $B_{\max}$  increases. This means, the possibility for ordered phase increases during the initial counts and also the maximum value for swarming get increased.

It is observed that as the value of  $d$  is decreased (Fig.1-4) the value of  $B_{\min}$  decreases. This means the system tends to well-ordered pattern during the initial time count. And also the value of  $B_{\max}$  increases. Hence, irregular motion inside the cluster increases

Fig. 2. The system is in the swarming state during the initial time and then goes to wandering state (for  $d = 0.8$ )

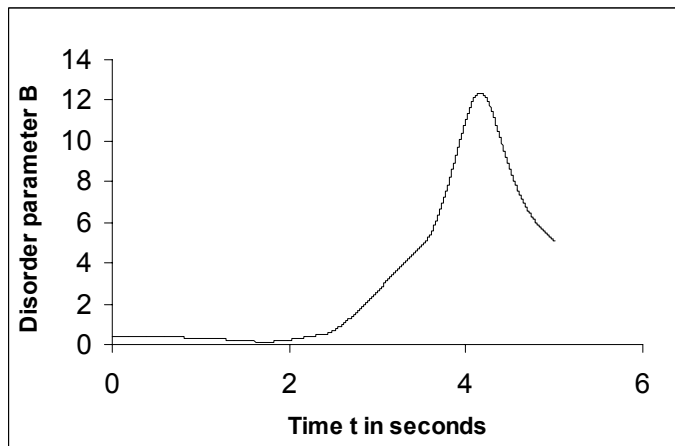


Fig.3. The system is in the ordered state initially as in the previous case (for  $d = 0.6$ )

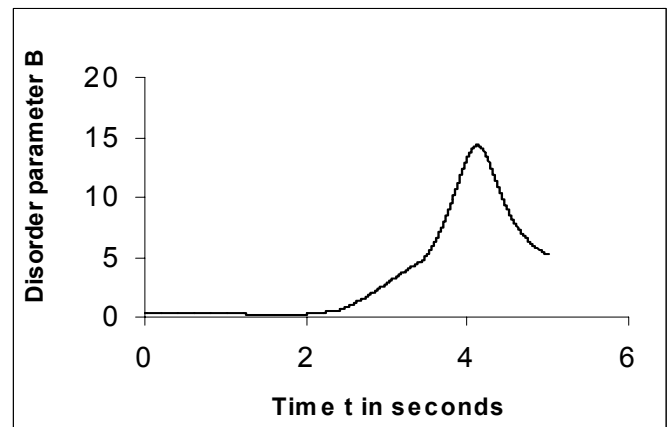
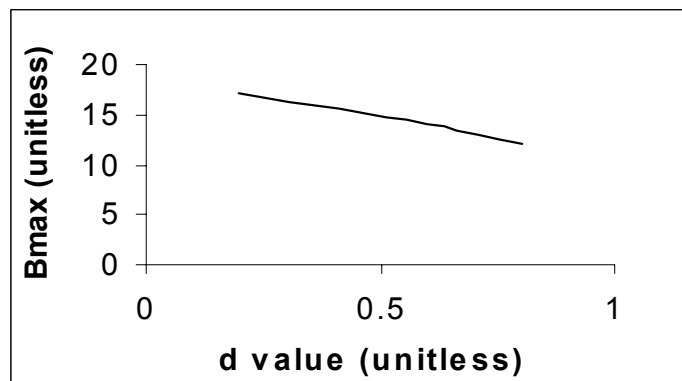




Fig.5. As  $d$  increases, the maximum value of disorder parameter decreases

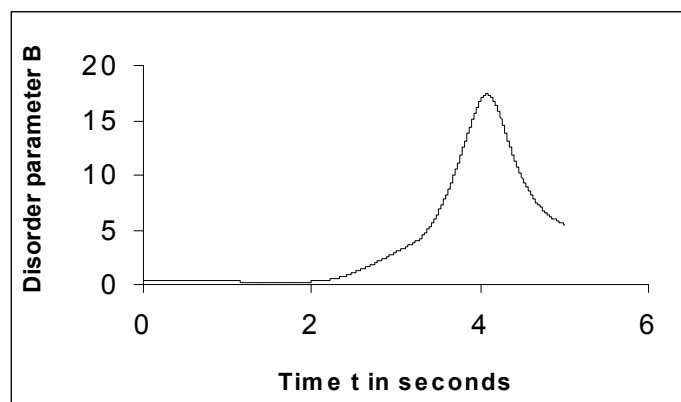


although cluster persists. This shows that the value of anisotropy mutual interaction plays a vital role in the transition from ordered phase to disordered phase. As  $d$  increases, the value of  $B_{max}$  decreases whereas  $B_{min}$  increases (Fig. 5 & 6).

### Conclusion

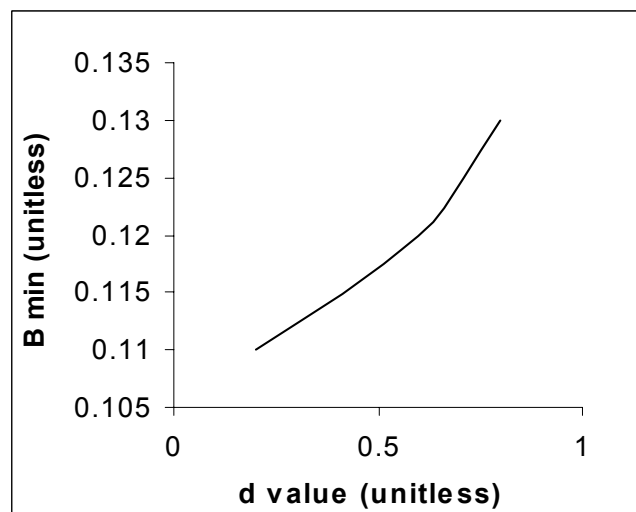
For a given  $d$ , the system oscillates between two states swarming and wandering. As the mutual interaction parameter  $d$  increases,  $B_{max}$  decreases whereas  $B_{min}$  increases. Ultimately, the system moves from bi-stable state to the mono-stable stationary state, due to the increase of the anisotropy force among the particles. If the interaction with the elements in front of a given element is stronger than with those behind, then the system will move as a single unit.

Fig.4 The system is in the ordered state initially as in the previous case (for  $d = 0.2$ )



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Fig.6. As  $d$  increases, the minimum value of disorder parameter increases



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