Indian Journal of Science and Technology Vol. 3 No. 4 (Apr. 2010) ISSN: 0974- 6846

Thermal radiation on linearly accelerated vertical plate with variable temperature and uniform mass flux

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Abstract

Radiative heat transfer effects on unsteady flow of viscous incompressible fluid past a uniformly accelerated infinite vertical plate with variable temperature and uniform mass flux has been studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is linearly accelerated with a velocity $u = \frac{u_0^3}{\nu} t'$ in its own plane. The plate temperature is raised linearly with time and the concentration level near the plate is also raised at a uniform rate. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, radiation parameter and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. But the trend is just reversed with respect to the thermal radiation parameter.

Keywords: Linearly; accelerated; vertical plate; radiation; mass flux.

Introduction

Thermal radiation is key to many fundamental phenomena surrounding us, from solar radiation to fire incandescent lamp, and has played a major role in combustion and furnace design, design of fins, steel rolling, nuclear power plants, cooling of towers, gas turbines and various propulsion device for aircraft, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications.

England and Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along an isothermal vertical plate was studied by Hossain and Takhar (1996). Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al.* (1996; 2009) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Sahin Ahmed (2010) have considered the unsteady three-dimensional flow and heat transfer of a viscous incompressible fluid over an infinite vertical flat porous plate in presence of viscous dissipative heat.

Gupta *et al.* (1979) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Basant Kumar *et al.* (1991) have analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

It is proposed to study the thermal radiation effects of on unsteady flow past a uniformly accelerated infinite vertical plate with variable temperature in the presence of uniform mass flux. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

Mathematical analysis

Here the flow of a viscous incompressible fluid past a uniformly accelerated infinite vertical plate with variable temperature and uniform mass flux in the presence of thermal radiation is studied. Consider the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3}{\nu} t'$ in its own plane. The plate temperature is raised linearly with time and the mass is diffused from the plate to the fluid at a uniform rate. The fluid considered here is a gray, absorbing-emitting radiation butanon-scattering medium. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

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$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_P \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$\begin{aligned} &u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad for \;\; all \;\; y, t' \le 0 \\ t' > 0: \; &u = \frac{u^3{}_0}{\nu}t', \quad T = T_\infty + (T_w - T_\infty)\; A\; t', \quad \frac{\partial C'}{\partial y} = -\frac{j''}{D} \quad at \quad y = 0 \\ &u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad as \quad y \to \infty \end{aligned} \quad (4)$$

Where, $A = \frac{u^2{}_0}{\nu}$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\,\sigma(T_\infty{}^4 - T^4) \quad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_P \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\,\sigma T_\infty{}^3(T_\infty - T) \quad (7)$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0{}^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{\left(\frac{j''\nu}{Du_0}\right)}, \quad Gc = \frac{\nu g\beta^*\left(\frac{j''\nu}{Du_0}\right)}{u_0^3}, \quad (8)$$

$$\Pr = \frac{\mu C_P}{k}, \quad Sc = \frac{\nu}{D}, \quad R = \frac{16a^*\nu^2\sigma T_\infty{}^3}{k u_0{}^2}$$

In equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (11)$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} &U = 0, \quad \theta = 0, \quad C = 0 \quad for \;\; all \;\; Y, t \le 0 \\ t > 0: \; &U = t, \quad \theta = t, \quad \frac{\partial C}{\partial y} = -1 \quad at \quad Y = 0 \\ &U \to 0, \quad \theta \to 0, \quad C \to 0 \quad as \quad Y \to \infty \end{aligned} \quad (12)$$

All the physical variables are defined in the nomenclature. The dimensionless governing equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} \theta = \frac{t}{2}&\left[\exp(2\eta\sqrt{Rt})\; erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp(-2\eta\sqrt{Rt})\, erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \\ &- \frac{\eta \Pr\sqrt{t}}{2\sqrt{R}}\left[\exp(-2\eta\sqrt{Rt})\, erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right) - \exp(2\eta\sqrt{Rt})\, erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right)\right] \end{aligned} \quad (13)$$

$$C = 2\sqrt{t}\left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}\sqrt{Sc}} - \eta\; erfc\left(\eta\sqrt{Sc}\right)\right] \quad (14)$$

$$\begin{aligned} U = \; & t(1 + 2bc)\left[(1 + 2\eta^2)\, erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] + 2c\; erfc(\eta) \\ & - c\; \exp(bt)\left[\exp(2\eta\sqrt{bt})\; erfc\left(\eta + \sqrt{bt}\right) + \exp(-2\eta\sqrt{bt})\; erfc\left(\eta - \sqrt{bt}\right)\right] \\ & - c(1 + bt)\left[\exp(2\eta\sqrt{Rt})\;\; erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp(-2\eta\sqrt{Rt})\;\; erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \\ & + \frac{bc\eta \Pr\sqrt{t}}{\sqrt{R}}\left[\exp(-2\eta\sqrt{Rt})\, erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right) - \exp(2\eta\sqrt{Rt})\, erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right)\right]\} \\ & + \frac{Gc\, t\sqrt{t}}{3(Sc - 1)\sqrt{Sc}}\left[\frac{4}{\sqrt{\pi}}(1 + \eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1 + \eta^2 Sc)\exp(-\eta^2 Sc) \right. \\ & \left. - \eta(6 + 4\eta^2)erfc(\eta) + \eta\sqrt{Sc}\;\; (6 + 4\eta^2 Sc)erfc(\eta\sqrt{Sc})\right] \end{aligned}$$

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$$+ c\ \exp(b\,t)\Big[\exp(-2\eta\sqrt{\Pr(a+b)\,t})\ erfc\,(\eta\sqrt{\Pr} - \sqrt{(a+b)\,t}\,) + \exp(2\eta\sqrt{\Pr(a+b)\,t})\ erfc\,(\eta\sqrt{\Pr} + \sqrt{(a+b)\,t}\,)\Big]$$

Where,

$$a = \frac{R}{\Pr},\ b = \frac{R}{1-\Pr},\ c = \frac{Gr}{2b^2(1-\Pr)} \quad and \quad \eta = \frac{Y}{2\sqrt{t}}.$$

Results and discussion

For physical understanding of the problem numerical computations are carried out for different parameters depends upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which

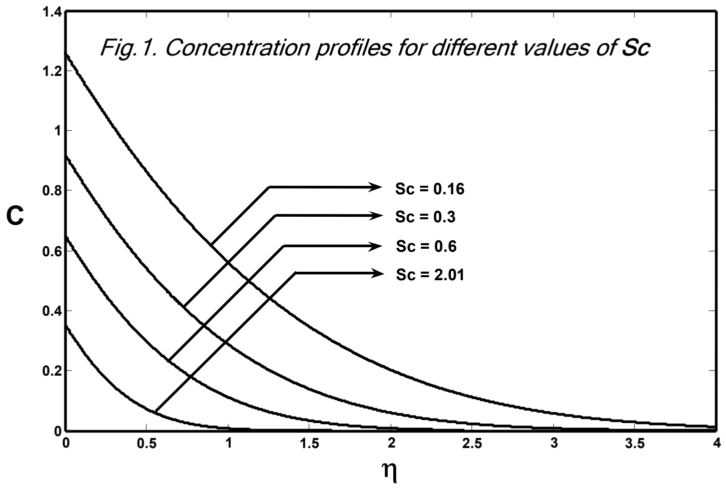

Fig.1. Concentration profiles for different values of ***Sc***

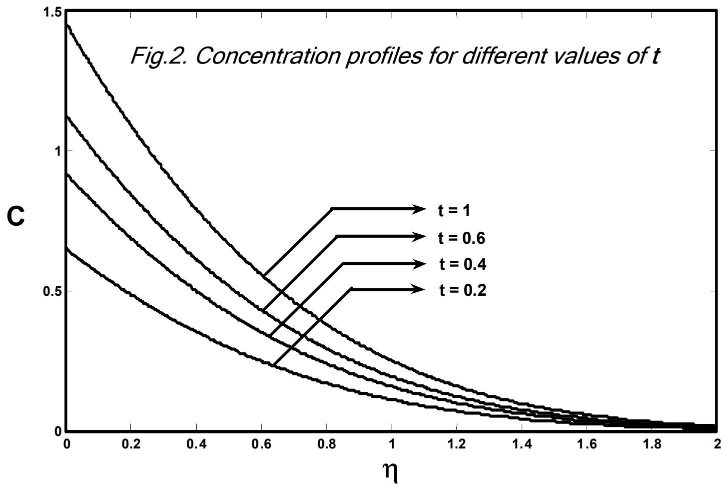

Fig.2. Concentration profiles for different values of ***t***

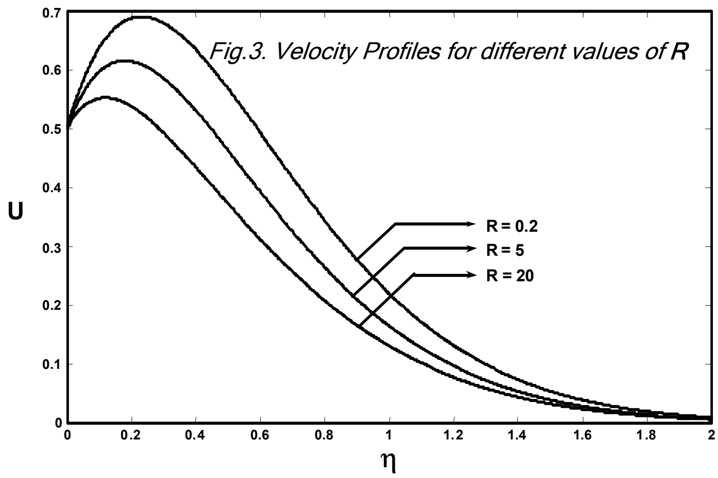

Fig.3. Velocity Profiles for different values of ***R***

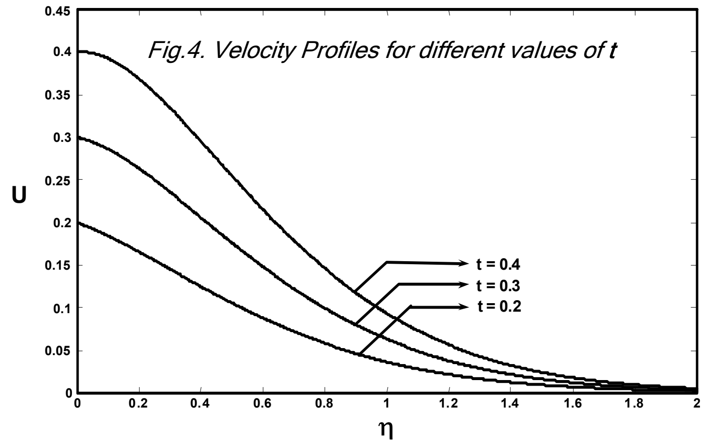

Fig.4. Velocity Profiles for different values of ***t***

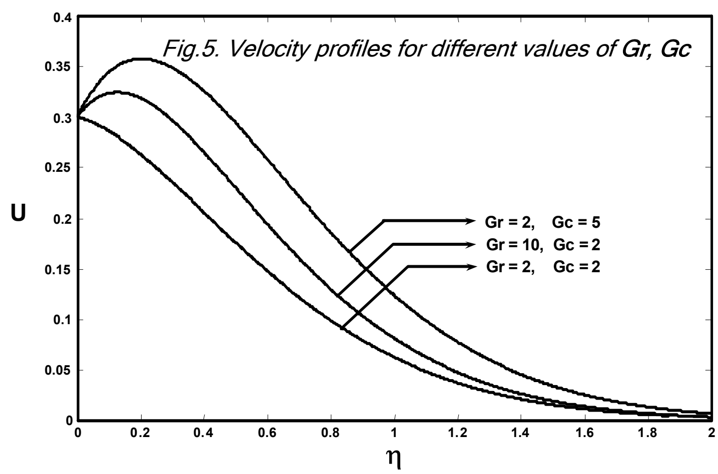

Fig.5. Velocity profiles for different values of ***Gr, Gc***

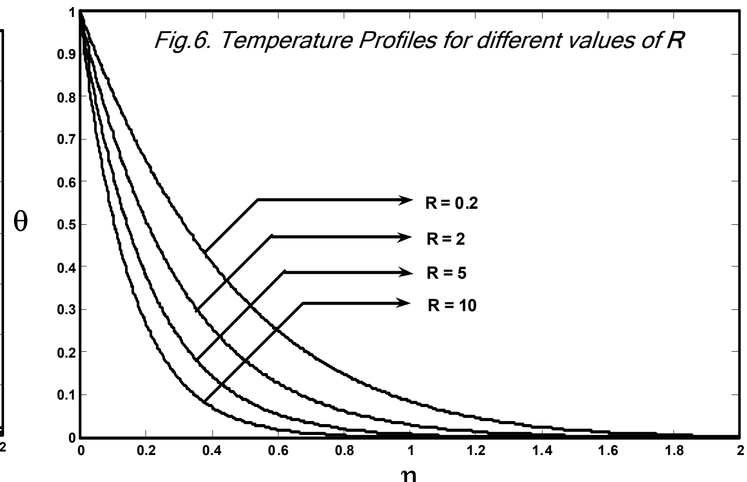

Fig.6. Temperature Profiles for different values of ***R***

corresponds to water vapour. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr=0.71). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like thermal radiation parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The concentration profiles for different values of the Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01) and time t = 0.2 are presented in Fig. 1. The effect of Schmidt number is very important in concentration field. It is observed that the concentration increases with decreasing values of the Schmidt number. It is observed that the relative variation of the concentration with the magnitude of the Schmidt number. Fig. 2 represents the effect of concentration profiles for different time (t = 0.2, 0.4, 0.6, 1) in the present of water vapour. The trend shows that the wall concentration increases with increasing time t.

The effect of velocity for different thermal radiation parameter (R = 0.2, 5, 20), Gr =10, GC = 2 and t = 0.5 are shown in Fig. 3. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation. The variations of velocity profiles for different values of time (t = 0.2, 0.3, 0.4), Gr = Gc = 2 and R = 5 are shown in Fig. 4. It is clear that the velocity increases with increasing values of the time. Fig. 5 demonstrates the effects of different thermal Grashof number (Gr = 2, 10) and mass Grashof number (Gc = 2, 5) on the velocity when R = 5 and t = 0.3. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

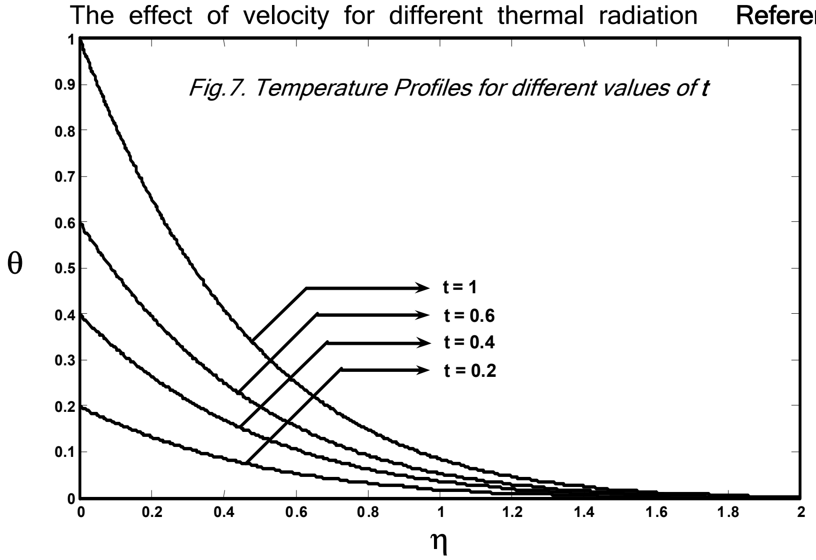

Fig.7. Temperature Profiles for different values of t

The temperature profiles are calculated for different values of thermal radiation parameter(R = 0.2, 2, 5, 10) and time t = 1 are shown in Fig. 6 in the presence of air. The effect of thermal radiation parameter is important in temperature profiles. The trend shows that the temperature increases with decreasing radiation parameter. Fig. 7 is a graphical representation which depicts the temperature profiles for different values of the time (t = 0.2, 0.4, 0.6, 1) in the presence of thermal radiation R =0.2. It is clear that the temperature increases with increasing values of the time t.

Conclusion

An exact solution of flow past a uniformly accelerated infinite vertical plate with variable temperature in the presence of uniform mass flux in the presence of thermal radiation have been analyzed. The dimensionless governing equations are solved by the Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grshof number, thermal radiation parameter and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to thermal radiation parameter. It is also observed that the temperature increases with decreasing values of the thermal radiation parameter.

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