

## A sequential approach with matrix framework for economic thermal power dispatch problems

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### Abstract

The objective of the economic dispatch problem is to schedule the committed generating units output so as to meet the required load demand while satisfying the system equality and inequality constraints. The improvements in the scheduling of generating units can lead to significant cost savings. This paper presents a sequential approach for solving the economic dispatch of thermal units. This is a maiden attempt has been developed to obtain the optimal dispatch solutions for all achievable load demands of a system in single execution. The feasibility of the proposed method is demonstrated for suitable test systems. The generator operational constraints such as ramp rate limits and prohibited operating zones and transmission loss are considered. The simulation results are compared with the recent reports in terms of solution quality. Numerical results indicate an improvement in the cost saving and hence the superiority of the proposed method is also revealed for economic dispatch problems.

**Keywords:** Economic load dispatch; matrix framework; ramp rate limit; sequential approach; transmission loss.

### Introduction

The primary objective of the economic dispatch problem is to schedule the generations of thermal units so as to meet the required load demand at minimum operating cost while satisfying the system operating constraints. Traditionally, the cost function for generating units has been approximated as a quadratic function. The efforts on solving the economic dispatch problems have employed various mathematical programming methods and optimization techniques. A variety of optimization techniques has been applied in solving economic dispatch problems. The classical optimization methods are reported that to be highly sensitive to the selection of the starting points and sometimes converge to a local optimum.

The methods based on the operational research and artificial intelligence concepts such as genetic algorithm, evolutionary algorithms, fuzzy and artificial neural networks have been given attention for solving economic dispatch problems because of their ability to find the solution near global optimal. Simulated annealing technique (SA) had been applied to determine the optimal generation schedule for economic dispatch problem in a power system (Wong & Wang, 1993). The SA technique is similar to the local search technique in optimization, which can only guarantee a local optimum solution. Genetic algorithm (GA) has been applied to find the optimal generations of thermal units considering the prohibited operating zones of some generating units (Orero & Erving, 1996). Particle swarm optimization method (PSO) has been applied for solving economic dispatch problems with various operating constraints like prohibited operating zones and ramp rate limits (Gaing, 2003). The two-phase neural network based modelling framework has been presented for solving economic

dispatch problems with unit operational constraints (Naresh *et al.*, 2004). The authors developed the method based on the solution set of differential equations obtained from transformation of augmented Lagrangian energy function.

Evolutionary programming (EP) technique has been applied to solve economic dispatch problems. Three different evolutionary programming techniques based on different kind of mutation techniques have been developed and their effectiveness has been tested with different economic dispatch problems (Jayabarathi *et al.*, 2005). Particle swarm optimization is applied to solve various kinds of economic dispatch problems (Jeyakumar *et al.*, 2006). Partition approach algorithm has been applied to solve economic dispatch problems considering the physical limitations of the system (Lin *et al.*, 2007). It uses the multi-section divided and interval elimination method to provide solutions. Differential evolution (DE) has been applied for solving economic load dispatch problems with different characteristics (Noman & Iba, 2008). A variable scale hybrid differential evolution has been developed to overcome the demerit of the fixed and random scaling factor in differential evolution and has been applied for solving large scale economic dispatch problems (Chiou, 2007). Enhanced Hopfield neural network has been developed by introducing an adaptive correction factor in the conventional Hopfield neural network and has also been applied to obtain the economic schedule of generation (Abdelaziz *et al.*, 2008).

A modified particle swarm optimization (MPSO) has been suggested to solve this problem (Kuo, 2008). The author introduced a coding scheme to prevent obtaining infeasible solutions through the applications of stochastic search methods, thereby improving search efficiency and solution quality. The improved version of genetic

algorithm, namely directional search genetic algorithm has been reported for solving this type of problems (Theerthamalai & Maheswarapu, 2008). The authors suggest a directional search approach to reach the final best solution instead of searching in the entire search space. A simplified recursive approach has been proposed for solving economic dispatch problems with various operating constraints (Balamurugan & Subramanian, 2008). The authors developed a generalized equation which simplifies the evaluation of optimal generation scheduling of committed units. Hybrid Bacterial Foraging - Nelder Mead algorithm has been developed to solve economic dispatch problems (Panigrahi & Pandi, 2008). In this article, simplified strategy has been developed for solving economic dispatch problems. The generator operational constraints such as ramp rate (RR) limits and prohibited operating zones (POZ) have been considered to validate the proposed methodology.

#### Problem formulation

The objective function corresponding to the production cost (\$/h) can be approximated to be a quadratic function of the active power outputs from the generating units can be represented as,

$$\min F_t = \sum_{i=1}^n F_i(P_i) = \sum_{i=1}^n a_i P_i^2 + b_i P_i + c_i \quad (1)$$

Where,  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of unit  $i$ ,  $P_i$  is the real power generation of unit  $i$  in MW,  $F_t$  is the total fuel cost of generation in \$/h and  $n$  is the number of thermal units.

This constrained optimization problem is subjected to a variety of constraints. These include power balance constraints, maximum and minimum generation limits of generating units and other operational constraints such as ramp rate limits and prohibited operating zones. These constraints are discussed as follows.

#### (i) Power balance constraint

This constraint is based on the principle of equilibrium that the total generation should satisfy the total system demand and transmission losses.

$$\sum_{i=1}^n P_i = P_D + P_L \quad (2)$$

Where,  $P_D$  is the total power demand in MW and  $P_L$  is the transmission loss.

The transmission losses can be approximately expressed as a quadratic function of the generation level of each generator through Kron's loss coefficients,

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j + \sum_{i=1}^n B_{0i} P_i + B_{00} \quad (3)$$

Where,  $B_{ij}$ ,  $B_{0i}$  and  $B_{00}$  are the B coefficients.

#### (ii) Generator capacity constraints

The real power output of each generator has a lower and upper bound so that the generations of generating

unit lies within this limit. This inequality can be given as follows.

$$P_{i,\min} \leq P_i \leq P_{i,\max} \quad (4)$$

Where,  $P_{i,\min}$  and  $P_{i,\max}$  are the unit minimum and maximum generation limit of unit  $i$  respectively in MW.

#### (iii) Ramp rate limit constraints

The inequality constraints due to ramp rate limits for unit generation changes are given

1) as generation increases

$$P_i - P_i^0 \leq UR_i \quad (5)$$

2) as generation decreases

$$P_i - P_i^0 \leq DR_i \quad (6)$$

The generator operation constraint after including ramp rate limit of generators can be described as,

$$\max(P_i^{\min}, P_i^0 - DR_i) \leq P_i \leq \min(P_i^{\max}, P_i^0 + UR_i) \quad (7)$$

Where,  $P_i^0$  is the real power output of generator  $i$  in before dispatched hour in MW,  $UR_i$  and  $DR_i$  are the up ramp and down ramp limit of unit  $i$  respectively in MW/h.

#### (iv) Prohibited operating zone constraints

In practical operation, the generation output  $P_i$  of a unit should not lie in the prohibited zones. The feasible operating zones of unit  $i$  can be described as follows,

$$P_{i,\min} \leq P_i \leq P_{i,1}^l$$

$$P_{i,j-1}^u \leq P_i \leq P_{i,j}^l, \quad j = 2, 3, \dots, n_i$$

$$P_{i,n_i}^u \leq P_i \leq P_{i,\max} \quad (8)$$

Where,  $P_{i,j}^l$  is  $P_{i,j}^u$  are the lower and bound generation of unit  $i$  in prohibited operating zone  $j$  respectively in MW and  $n_i$  is the number of prohibited operating zones.

#### Solution methodology

##### Demonstration of sequential approach with matrix framework

Sequential approach with matrix framework is proposed for solving economic dispatch problems. This is the first method developed to obtain the optimal dispatches for all possible load demands in a system. The demonstration of the solution methodology is presented in this section.

The electric power production in a power plant is allowed to vary from minimum technical limit ( $P_{\min}$ ) to maximum technical limit ( $P_{\max}$ ). Initially the  $P_{i,\min}$  of all generating units in a power plant are considered as initial state input values and is represented by a single dimensional matrix as,

$$s = [P_{1,\min}, P_{2,\min}, P_{3,\min}, \dots, P_{n,\min}] \quad (9)$$

Based on the above single dimensional matrix, a square matrix ( $I$ ) is developed to identify the economic schedule of generation. The formation of the square matrix is as follows. The process starts with a step increment in generation by  $\Delta$  MW in  $P_{1,\min}$  by keeping the remaining units at its input value. This will form first row of the square matrix.

$$I_1 = [ P_{1,min} + \Delta \ P_{2,min} \ P_{3,min} \ , \dots \ , \ P_{n,min} ] \quad (10)$$

The increment in generation is made in the second element by keeping the other elements at its input value that leads to the development of second row of the square matrix.

$$I_2 = [ P_{1,min} \ P_{2,min} + \Delta \ P_{3,min} \ , \dots \ , \ P_{n,min} ] \quad (11)$$

In the same logic, an increment is made for remaining units one at a time and a square matrix with a dimension equal to the number of units has been developed. For every step increment in the operating range of the plant the unit one at a time is allowed to experience the change in generation thus leads to the formation of the square matrix.

$$I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} P_{1,min} + \Delta & P_{2,min} & \dots & P_{n,min} \\ P_{1,min} & P_{2,min} + \Delta & \dots & P_{n,min} \\ \vdots & \vdots & \ddots & \vdots \\ P_{1,min} & P_{2,min} & \dots & P_{n,min} + \Delta \end{bmatrix} \quad (12)$$

Each element in the square matrix represents the generation of a unit corresponding to the column that should satisfy the unit capacity constraints. In addition, the operational constraints such as ramp rate limits and prohibited operating zones are also enforced. The operating regions of the unit after including ramp rate limits are identified as mentioned in Eq. (7). The operating regions of the units having prohibited operating zones are separated into isolated sub regions and it is identified using Eq. (8). The operating regions of the units having prohibited operating zones and ramp rate constraints are obtained as mentioned earlier. The units are allowed to operate in the one of the operating zones. If the generation of a unit falls in a prohibited operating zone, the feasible optimal level would most likely to be located in any one of the adjacent feasible operating regions, that is, the operating region above or below the prohibited operating zone.

In the square matrix the unit generations of each row that satisfy the constraints are identified and total fuel cost of generation is evaluated. The desired economic schedule of generation is identified by analyzing fitness of each row. The fitness function of each row is calculated as,

$$fit(j) = \frac{F_T(j)}{(pd + \Delta)} \quad j = 1, 2, \dots, n \quad (13)$$

Where, pd is the total of input values.

The schedule with the minimum fitness is chosen as the successive state input values. This process is repeated till all the generating units reach their maximum generation capacity. The feasible solutions for every increment from  $P_{min}$  to  $P_{max}$  are obtained and hence the best solution for any load demand falls in the operating boundary can be easily sited.

In practical applications, the total generation must be

equal to the power demand and transmission loss. In such cases, the power balance constraint is exactly met by calculating the diagonal unit generation as follows.

$$P_{i,i} = (pd + \Delta) + P_L - \sum_{\substack{j=1 \\ j \neq i}}^n P_{i,j} \quad i = 1, 2, \dots, n \quad (14)$$

The detailed computational flow of the proposed method is presented in Fig. 1. The proposed methodology in the form of matrix framework to support the demonstration is as follows:

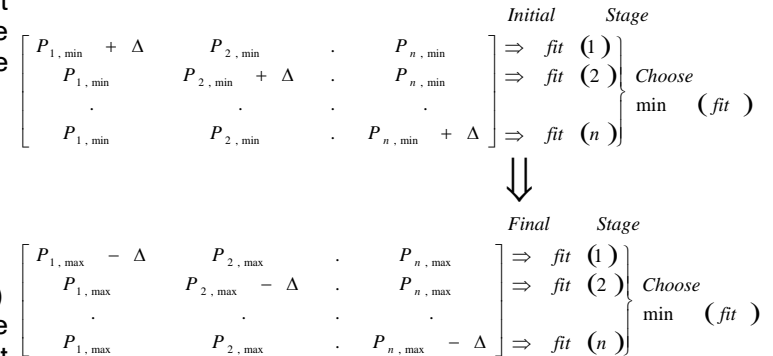


Fig. 1. Computational flow of proposed method

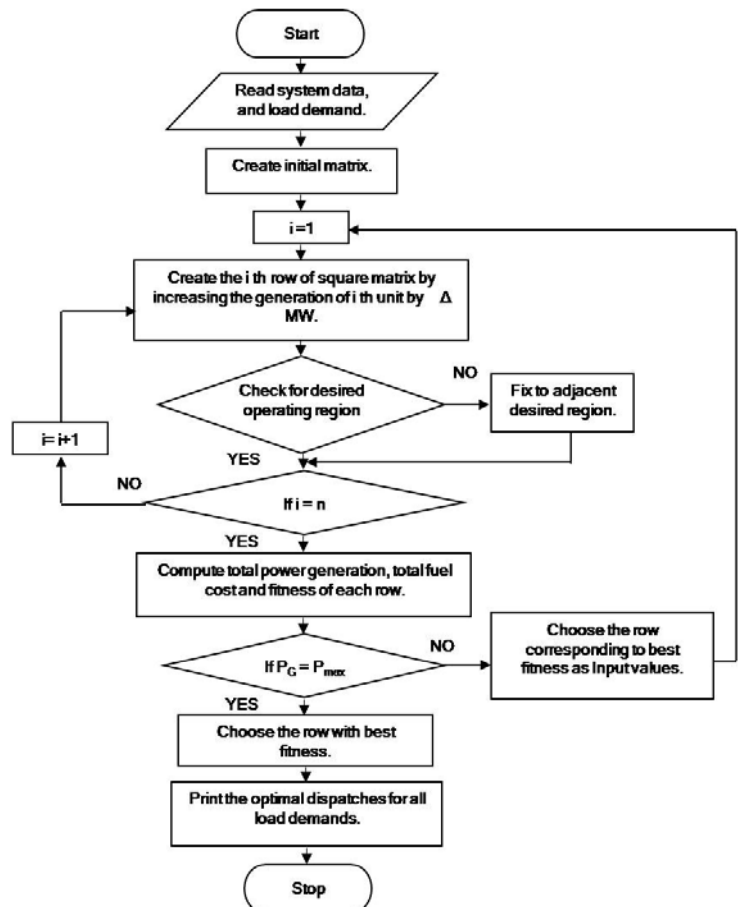


Table 1. Economic dispatch results & comparison for 6 unit system

Unit power output (MW)	Load demand 1263 MW				
	GA	PSO	MPSO	ABFNM	Proposed method
P <sub>1</sub>	474.81	447.50	446.71	449.46	447.1637
P <sub>2</sub>	178.64	173.32	173.01	172.88	173
P <sub>3</sub>	262.21	263.47	265.00	263.41	264
P <sub>4</sub>	134.28	139.06	139.00	143.49	139
P <sub>5</sub>	151.90	165.48	165.23	164.91	166
P <sub>6</sub>	74.18	87.13	86.78	81.252	86
Line loss (MW)	13.022	12.958	12.733	12.4437	12.1637
Total cost (\$/h)	15459	15450	15447	15443.8164	15425.70

### Simulation results and discussions

The applicability and viability of the proposed approach for practical applications have been tested with different power system cases. Three different scale of power systems were simulated in this section. The algorithms for solving the examples were implemented in Matlab 7.0 platform and executed with Pentium IV, 2.8 GHz personal computer. The proposed methodology provides the optimal schedule of generations for all possible load demands which is varied from minimum technical limit by a small increment to maximum technical limit of the system. The selection of increment is also an important factor. Too large increment may end up with unfeasible solution and too small increment may take long execution time. Based on experience, the desired increment is chosen as 1 MW.

#### Case I: Six unit system

The system contains six thermal units, 26 buses and 46 transmission lines. The details of the cost coefficients, generation limits and prohibited operating zones, ramp rate limit of six unit sample system are given in the literature (Gaing, 2003). The generator operational constraints such as ramp rate limits and prohibited operating zones and transmission loss are considered. The transmission loss is calculated using B coefficients. The optimal dispatches are obtained for all load demands from minimum technical limit to maximum technical limit. The optimal generation schedule for a load demand of 1263 MW has been considered and the simulation results of total fuel cost and transmission loss are compared with GA (Gaing, 2003), PSO (Gaing, 2003), Modified PSO (Kuo, 2008) and adaptive bacteria foraging with Nelder-Mead technique (ABFNM) (Panigrahi & Pandi, 2008). The comparisons of results are presented

in Table 1. From the comparison, it is clear that the proposed approach offers better schedule than the recent reports.

#### Case II: Fifteen unit system

The cost coefficients, maximum and minimum generation limits, ramp rate limits and prohibited operating zones and the transmission loss coefficients with a base capacity of 100 MVA of the fifteen unit sample system are reported in the literature (Gaing, 2003). The minimum and maximum technical limits of the system are 915 MW and 3542 MW respectively. The operating regions of the unit are identified after incorporating ramp rate limits and prohibited operating zone constraints. The transmission loss is calculated using transmission loss matrix or B coefficients. The power balance is exactly met by evaluating the generation using Eq. (14). The generations of the units neglecting transmission loss are treated as input values for the successive state. The optimum generations of individual thermal units and total fuel cost for the load demand of 2630 MW are presented in Table 2. The simulation result is compared with Genetic Algorithm (Gaing, 2003), Particle Swarm Optimization (Gaing, 2003), Modified PSO (Kuo, 2008) and ABFNM (Panigrahi & Pandi, 2008) and the comparison of results is presented in Table 3. It is clear from the comparison of

Table 2. Economic dispatch results of 15 unit system for 2630 MW

Unit	Power output (MW)	Unit	Power output (MW)	Unit	Power output (MW)
P <sub>1</sub>	455.00	P <sub>6</sub>	460.00	P <sub>11</sub>	80.00
P <sub>2</sub>	380.00	P <sub>7</sub>	430.00	P <sub>12</sub>	80.00
P <sub>3</sub>	130.00	P <sub>8</sub>	60.00	P <sub>13</sub>	25.00
P <sub>4</sub>	130.00	P <sub>9</sub>	70.9033	P <sub>14</sub>	15.00
P <sub>5</sub>	170.00	P <sub>10</sub>	159.00	P <sub>15</sub>	15.00
Transmission losses (MW)					29.9033
Total fuel cost (\$/h)					32696.81

Table 3. Simulation results comparison for 15 unit system

Method	Load demand 2630 MW	
	Transmission loss (MW)	Total cost (\$/h)
Proposed method	29.9033	32696.81
ABFNM	28.9470	32784.5024
MPSO	30.908	32708
PSO	32.431	32858
GA	38.278	33113

results that the proposed method provides better schedule of generations to meet the load demand with existing techniques.

#### Case III: Large scale economic dispatch problem

This sample system consists of forty units in the realistic Taipower system that is a large scale and mixed - generating system where coal-fired, oil-fired, gas-fired, diesel and combined cycle are present. The cost coefficients and maximum and minimum generation limits of the sample system are available in literature (Chiou, 2007). The simulation results for load demands of 9000 MW, 9500 MW and 10500 MW are compared with simulated annealing (SA) (Chiou, 2007), genetic algorithm (GA) (Chiou, 2007), hybrid differential evolution (HDE) (Chiou, 2007), variable scaling hybrid differential evolution (VSHDE) (Chiou, 2007) and direct search genetic algorithm (DSGA) (Theerthamalai & Maheswarapu, 2008) and the comparison of results is presented in Table 4. As seen from comparison, the proposed method provides the minimum generation cost for above mentioned load demands. It also clears that the proposed methodology is

efficient to solve large scale economic dispatch problems. The computational time for the above case studies by the proposed approach is presented in Table 5.

The proposed methodology has following merits.

- From these studies, this approach has the competence to solve various types of economic dispatch problem.
- It is a first method that provides the optimal solution for all possible load demands of system in a single run.
- It provides the schedule with minimum total cost in all cases hence global optimal solution.
- The performance of the proposed approach is independent of the number of generating units in the system and hence it is suitable for system of any size.
- The computational procedure is minimal.
- It offers the solution for all load demands of a system hence it takes a reasonable execution time.

#### Conclusion

In this paper, a simple and efficient approach has been presented to solve economic dispatch problem. The proposed methodology has been demonstrated with three different scale of power system. The six and fifteen unit test systems are with the realistic unit operational constraints such as ramp rate limits, prohibited operating zones and a large scale system consists of 40 generating units are considered for case studies. The simulation results are compared with the recent reports and the comparison illustrates that the proposed methodology outperformed the existing techniques in terms of solution quality. The performance of this methodology is independent of the number of the generating units hence it is suitable for system of any size. The comparison of results concludes that the proposed methodology provides the minimum total fuel cost hence global optimal solution for economic dispatch problems.

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Table 4. Total fuel cost comparison of 40 unit system

Method	Load demand 9000 MW	Load demand 9500 MW	Load demand 10500 MW
Proposed method	121039.18	128219.31	143721.71
VSHDE	121253.01	----	143943.90
HDE	121266.40	----	143955.83
GA	121839.72	----	144486.02
SA	135229.69	----	164069.36
DSGA	----	128424.26	----

Table 5. Total execution time for case studies

Test system	Execution time (s)
6 unit	0.4356
15 unit	0.6642
40 unit	1.037

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