Indian Journal of Science and Technology Vol. 3 No. 7 (July 2010) ISSN: 0974- 6846

K-release inventory model in manpower planning

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Abstract

A deterministic inventory model is developed to investigate the optimum number of persons to be trained by the organization; here we assume the replenishment rate is infinite with no shortages. In our model, we consider an organization with two centres; one is the training centre for the beginners and the other for the experienced which is treated as inventory house. Our aim is to obtain the optimal number of recruits Q for the organization and the optimal number of K employees trained and sent from the training centre to inventory house. The solution of the model is illustrated with the help of sensitivity analysis on the optimal average cost and the batch width. Also the effect of change in demand parameter on the optimal average cost is graphically presented.

Keywords: Deterministic inventory model, manpower planning, 2000 mathematics subject classification: 90B05.

Introduction

Recruitment and training of human resources are made in order to meet the changing trends of the organization's requirements. We consider an organization which recruits persons and trains them for its job requirement and hold them as an inventory. The inventory of human resources is necessary because the delay in recruitment leads to a heavy loss to the organization. Hence, it is the must for the organization to hold the inventory in the form of trained persons.

The problem of finding the order quantity for an inventory system with two-warehouse facility has been discussed by Hartley (1976). Sarma (1983) has considered the aspect of transportation cost per unit from the rented warehouse (RW) to the owned warehouse (OW) and proposed the K release rule according to which the units from RW to OW are shipped in bulk of size K. The necessary and sufficient conditions for the K-release rule to be optimum are established by Sarma (1983). Also Bhunia and Maiti (1994) in their paper have discussed a two-warehouse inventory model for a linear trend in demand. In this model, we assume the trained persons to be an inventory of an organization.

The organization selects some persons with required qualification through interview and trains them for its job requirements along with the bond undertaken. They are classified as beginners and experienced persons. We assume that the number of persons in T_1 centre is Z and the number of persons in T_2 is W (Q=Z+W). In T_1 centre the persons who are totally new to the job are given training and then sent to T_2 centre. The persons in T_2 are treated as human resource inventory until they are absorbed by the organization.

The organization grants a flat rated travel allowance for all the trained persons while taking up jobs. Here we assume that the persons from T_1 to T_2 are transferred in 'n' groups with a fixed size of optimum number of K persons per group. Again the number of persons transferred in each group is always less than the total number of people in T_2 centre (K<W). The time interval between each transfer is considered to be constant and it is economical to transfer a constant number of persons from T_1 to T_2 centre because they can be trained in batches and also can be allotted in groups for working for a project. Hence, we assume that the number of persons, K is fixed. Unlike the inventory models which we are familiar with here the inventory are trained persons. During the inventory period they are paid some stipend by the organization, which accounts for administrative cost.

Assumptions and notations

A deterministic model for two training centre is presented under the following assumptions and notations:

1. Demand D is fixed for the time period T.
2. The fixed cost of advertisement is A per advertisement.
3. The holding cost is 'E' per person in centre T_2 and 'B' per person in centre T_1, B>E, since the beginners stay for a longer period in T_1 for training than the trained persons in T_2.
4. The travel allowance cost per person (for one cycle 'T') is C_T which is constant.
5. The replenishment rate is infinite, so that there is no lead time.
6. Shortages are not allowed.
7. The planning period 'T' is one year.
8. Q is the number of persons who have been selected through interview for training.
9. K is the number of successful persons to be transferred from T_1 to T_2.
10. Let μ be the administrative cost for the period 'T'.
11. The requirement of employees for the organization is assumed to be 'K' during the time interval 't'.
12. We assume that Z is the multiple of 'K'.
13. H is the fixed training cost of persons in T_1.

Indian Journal of Science and Technology Vol. 3 No. 7 (July 2010) ISSN: 0974- 6846

14. *n* is the number of batches (the number of groups of people transferred from T_1 to T_2 during the time period T).
15. Z and W are the number of persons in T_1 and T_2 respectively at the beginning of the time period T.
16. *c*- total inventory cost.
17. C (Q, K) - optimal average cost.

Model development

In the development of the model, we assume that an organization recruits Q persons (Q > W) of which Z persons are trained in training centre T_1 and W in T_2 where Q = Z + W. The requirement of employees are met by absorbing persons from T_2 centre, when the inventory level reaches (W-K) at the end of 't' time the organization transfers K person from T_1 to T_2 as the result the inventory level of T_2 again becomes W. This process is continued until the inventory level of T_1 is fully exhausted. The problem is to decide the optimal value of Q and K which minimizes cost of recruitment which includes costs for advertisement, holding inventory, travel allowance and administrative cost of the organization. The inventory situations in both the training centre are shown in Fig. 1 and 2.

Cost model and analysis

From Fig.1 the inventory time units in training house T_1 can be seen to be equal to

$$A_1 = t\{Z+(Z-K)+(Z-2K)+\ldots\ldots)+[Z-(n-1)K]\}$$

$$= t\{nZ - K[1+2+3+\ldots\ldots+(n-1)]\}$$

$$= t\{nZ - K\frac{n(n-1)}{2}\}$$

$$= t\{nZ - Z\frac{(n-1)}{2}\} \text{ since } nK=Z$$

$$= t\{Z\frac{(n+1)}{2}\}$$

Where $t = \frac{K}{D}$, 't' the time taken for transferring K persons from T_1 to T_2.

Since Z=(Q-W) and holding cost in T_1 is 'B' per person, we have

$$B A_1 = B t\{Z\frac{(n+1)}{2}\}$$

$$= B\frac{K}{D}\{Z\frac{(n+1)}{2}\}$$

$$= B K(Q-W)\{\frac{(n+1)}{2D}\} \quad (1)$$

Training cost in T_1 is 'H' per person, we have

$$H K n t \quad (2)$$

The travel allowance cost per group of *K* persons to move from T_1 to T_2 when they are transferred in *n* groups are given by,

$$n C_T = \frac{Z}{K} C_T \quad (3)$$

The holding cost in T_2 corresponding to these *K* persons absorbed by the organization during period t is

$$(\frac{KE}{2} t)$$

Since there are '*n*' such groups, the holding cost for these persons is given by,

$$E t\{K\frac{(n+1)}{2}\} = E\frac{K}{D}\{K\frac{(n+1)}{2}\} = (n+1) E\frac{K^2}{2D} \quad (4)$$

As (W-K) persons will be kept as inventory for the period of (*n*+1) t, hence the cost corresponding to this is given by

$$E (W-K)(n+1) t \quad (5)$$

And the average inventory during absorption from T_2 is $(\frac{W-K}{2})$ persons for a period of

$$((T-(n+1)t) \quad \text{i.e.} \quad \frac{(W-K)}{2}\{\frac{Q}{D}-(n+1) t\} \quad (6)$$

Hence the inventory holding cost in T_2 for these persons is, from (5) and (6)

$$E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)}{2}[\frac{Q}{D}-(n+1) t]\}$$

$$=E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)}{2}[\frac{Q}{D}-(n+1)\frac{K}{D}]\}$$

$$= E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)}{2}[\frac{Z+W}{D}-(n+1)\frac{K}{D}]\}$$

$$= E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)}{2}[\frac{nK+W}{D}-(n+1)\frac{K}{D}]\}$$

$$= E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)}{2}[\frac{nK+W-nK-K}{D}]\}$$

$$= E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)}{2}[\frac{W-K}{D}]\}$$

$$= E\{ (W-K)(n+1)\frac{K}{D} + \frac{(W-K)^2}{2D}\} \quad (7)$$

The fixed ordering cost or advertisement cost per advertisement is 'A', so that the total

Indian Journal of Science and Technology Vol. 3 No. 7 (July 2010) ISSN: 0974- 6846

inventory cost c for the system using (1), (2), (3), (4) and (7) is given by,

$$c = A + (n+1)\{\frac{BK(Q-W)}{2D} + \frac{EK^2}{2D} + E\ (W-K)\frac{K}{D}\}$$

$$+E\frac{(W-K)^2}{2D} + nC_T + \mu + HKnt \qquad (8)$$

Using the values of Z and n, the average inventory cost becomes

$C(Q, K) = \frac{c}{T}$ and $T = \frac{Q}{D}$

From (8)

Thus, $C(Q, K) = \frac{AD}{Q} + \frac{D}{Q}[$

$$\frac{Q-W+K}{K}]\{\frac{BK(Q-W)}{2D} + \frac{EK^2}{2D} + \frac{KE(W-K)}{D}\}$$

$$+HKnt\frac{D}{Q} + \frac{D}{Q}\frac{E(W-K)^2}{2D} + nC_T\frac{D}{Q} + \mu\frac{D}{Q}$$

(Since $n = \frac{Z}{K} = \frac{Q-W}{K}$, $n+1 = \frac{Q-W+K}{K}$)

Hence,

$$C(Q, K) = \frac{AD}{Q} + [\frac{Q-W+K}{2}]B - [\frac{Q-W+K}{2Q}]\ BW +$$

$$[\frac{Q-W+K}{2Q}]EK + [\frac{Q-W+K}{Q}]E(W-K) +$$

$$\frac{E(W^2+K^2-2WK)}{2Q} + nC_T\frac{D}{Q} + \mu\frac{D}{Q} + HKn\frac{K}{D}\frac{D}{Q}$$

$$= \frac{AD}{Q} + \frac{QB}{2} - \frac{WB}{2} + \frac{KB}{2} - \frac{QBW}{2Q} + \frac{W^2B}{2Q} -$$

$$\frac{KWB}{2Q} + \frac{QEK}{2Q} - \frac{WEK}{2Q} + \frac{K^2E}{2Q}$$

$$+\frac{QEW}{Q} - \frac{QEK}{Q} - \frac{W^2E}{Q} + \frac{WEK}{Q} + \frac{EWK}{Q} - \frac{EK^2}{Q} +$$

$$\frac{EW^2}{2Q} + \frac{EK^2}{2Q} - \frac{E2WK}{2Q}$$

$$+\frac{D}{Q}\frac{Z}{K}C_T + \mu\frac{D}{Q} + HKn\frac{K}{Q}$$

$$= \frac{AD}{Q} + \frac{QB}{2} - \frac{WB}{2} + \frac{KB}{2} - \frac{QBW}{2Q} + \frac{W^2B}{2Q} -$$

$$\frac{KWB}{2Q} + \frac{QEW}{Q} - \frac{QEK}{2Q} - \frac{W^2E}{2Q}$$

$$+\frac{WEK}{2Q} + \frac{(Q-W)D}{KQ}C_T + \mu\frac{D}{Q} + H\frac{Z}{K}\frac{K^2}{Q}$$

Thus,

$$C(Q,K) = \frac{AD}{Q} + \frac{QB}{2} + \frac{W^2(B-E)}{2Q} - \frac{KW(B-E)}{2Q} -$$

$$W(B-E) + \frac{K(B-E)}{2}$$

$$+\frac{(Q-W)D}{KQ}C_T + \mu\frac{D}{Q} + H$$

$$\frac{K(Q-W)}{Q}$$

$$C(Q,K) = \frac{AD}{Q} + \frac{QB}{2} + \frac{W^2(B-E)}{2Q} - \frac{KW(B-E)}{2Q} -$$

$$W(B-E) + \frac{K(B-E)}{2}$$

$$+\frac{(Q-W)D}{KQ}C_T + \mu\frac{D}{Q} + HK - \frac{HKW}{Q} \qquad (9)$$

The optimal values of Q and K which minimizes (9) are obtained by solving

$$\frac{\partial C(Q,K)}{\partial Q} = 0 \text{ and } \frac{\partial C(Q,K)}{\partial K} = 0$$

Consider,

$$\frac{\partial C(Q,K)}{\partial Q} = -\frac{AD}{Q^2} + \frac{B}{2} - \frac{W^2(B-E)}{2Q^2} + \frac{KW(B-E)}{2Q^2}$$

$$+\frac{WD}{KQ^2}C_T - \mu\frac{D}{Q^2} + H\frac{KW}{Q^2}$$

$\frac{\partial C(Q,K)}{\partial Q} = 0$ gives

$$-\frac{AD}{Q^2} + \frac{B}{2} - \frac{W^2(B-E)}{2Q^2} + \frac{KW(B-E)}{2Q^2} + \frac{WD}{KQ^2}C_T -$$

$$\mu\frac{D}{Q^2} + H\frac{WK}{Q^2} = 0$$

Indian Journal of Science and Technology Vol. 3 No. 7 (July 2010) ISSN: 0974- 6846

$-AD + \frac{B}{2}Q^2 - \frac{1}{2}W^2(B-E) + \frac{1}{2}KW(B-E) + \frac{WD}{K}C_T - \mu D + HWK = 0$

$2AD + W^2(B-E) - KW(B-E) - 2\frac{WD}{K}C_T + 2\mu D - 2HWK = BQ^2$

$$Q^2 = \frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E)}{B} - 2C_T\frac{WD}{BK} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}$$

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E)}{B} - 2C_T\frac{WD}{KB} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}\right]^{\frac{1}{2}} \quad (10)$$

$$\frac{\partial C(Q,K)}{\partial K} = -\frac{W(B-E)}{2Q} + \frac{(B-E)}{2} - C_T\frac{(Q-W)D}{K^2Q} + H\frac{(Q-W)}{Q}$$

$\frac{\partial C(Q,K)}{\partial K} = 0$ gives,

$-W(B-E)K^2 + K^2Q(B-E) - 2C_TD(Q-W) + 2HK^2(Q-W) = 0$

$(Q-W)(B-E)K^2 - 2C_TD(Q-W) + 2HK^2(Q-W) = 0$

$(Q-W)(B-E)K^2 + 2HK^2(Q-W) = 2C_TD(Q-W)$

$(Q-W)[(B-E)K^2 + 2HK^2 - 2C_TD] = 0$

$K^2[(B-E) + 2H] = 2C_TD$

$K^2 = \frac{1}{(B-E)+2H}[2C_TD]$

$$K = \left[\frac{2C_TD}{(B-E+2H)}\right]^{\frac{1}{2}} \quad (11)$$

Consider equation (10)

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E)}{B} - 2C_T\frac{K}{K}\frac{WD}{KB} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}\right]^{\frac{1}{2}}$$

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E)}{B} - 2C_T\frac{WDK}{K^2B} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}\right]^{\frac{1}{2}}$$

Substituting value of K for the fourth term alone we get,

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E)}{B} - 2C_T\frac{(B-E+2H)}{2C_TD}\frac{WDK}{B} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}\right]^{\frac{1}{2}}$$

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E)}{B} - KW\frac{(B-E+2H)}{B} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}\right]^{\frac{1}{2}}$$

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{KW(B-E+B-E+2H)}{B} + 2\mu\frac{D}{B} - 2H\frac{WK}{B}\right]^{\frac{1}{2}}$$

Substituting these values of K in Q in equation (9) the value of Q is,

$$Q = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{W(2B-2E+2H)}{B}\sqrt{\frac{2C_TD}{(B-E+2H)}} + 2\mu\frac{D}{B} - 2\frac{HW}{B}\sqrt{\frac{2C_TD}{(B-E+2H)}}\right]^{\frac{1}{2}} \quad (12)$$

The optimal values of Q and K are given by,

$$Q^* = \left[\frac{2AD}{B} + \frac{W^2(B-E)}{B} - \frac{2W(B-E+H)}{B}\sqrt{\frac{2C_TD}{(B-E+2H)}} + 2\mu\frac{D}{B} - 2\frac{HW}{B}\sqrt{\frac{2C_TD}{(B-E+2H)}}\right]^{\frac{1}{2}}$$

$$K^* = \left[\frac{2C_TD}{(B-E+2H)}\right]^{\frac{1}{2}}$$

The minimum cost can be obtained by substituting the value of Q^* and K^* in (9).

Sensitivity analysis

For the Inventory model mentioned above, sensitivity analysis is performed to study the changes in the values of the optimal average cost for different values of D keeping the other parameters fixed and the results are displayed graphically.

Indian Journal of Science and Technology Vol. 3 No. 7 (July 2010) ISSN: 0974- 6846

Fig. 1. The inventory situation in training house T_1

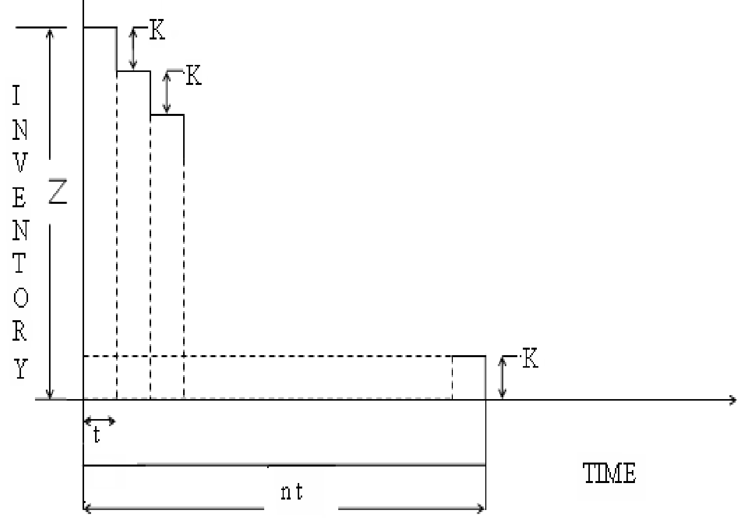
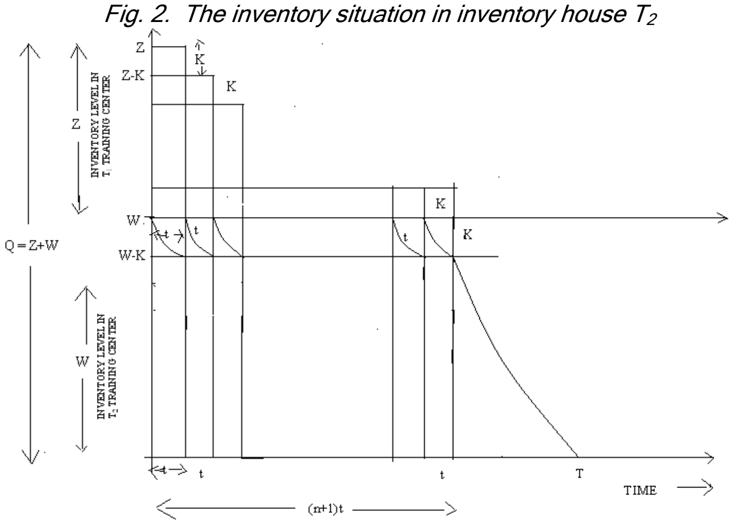


Fig. 2. The inventory situation in inventory house T_2



For an inventory system with, A= 1000, B=200, E=150, C_T =50, W=25, D=45 μ=5000, H=50. The value of Q is 51.48, the values of *n*, K and optimal average cost are calculated for *n*=1, 2, 3, 4, 5, 6 & 7. From Table 1 we get the optimal values by truncating the decimal values and taking the integer values, for Q we get Q = 51 and K values are as follows:

The optimal values becomes *n* = 5, Q = 52, K = 5, optimal average cost is 9871.08.

From above Table 1 we infer that whenever Q<W the model reduces to a single training house model. For the other case Q>W the results of our model are feasible only when K$\leq$W.

Fig. 3 depicts the relation between the batch widths and the optimal average cost when the capacity of training house T_2-W is fixed. Also it is assumed that the other parameters are fixed. For an inventory system with, A= 1000, B=200, E=150, C_T =50, W=25, D=45 μ=5000, H=50. The value of Q is 51.48, the values of *n*, K and optimal average cost are calculated for *n*=1, 2, 3, 4, 5, 6 & 7. From Table 2 we get the optimal values by rounding off to the nearest integer values, for Q we get Q = 52 and K values are as follows:

The optimal values becomes *n* = 5, Q = 52, K = 6, optimal average cost is 9871.15. The values of Q, K and optimal average cost obtained by both the methods, are shown in Table1 & 2.

1. Truncating the decimal values and taking the integer values.
2. Rounding off to the nearest integer values.

From both the methods we find negligible difference in the optimal values. So, either of the methods can be used. From Fig. 4 it is clear that optimal cost changes for change in demand. This situation indicates that the optimal cost depends linearly on demand.

Conclusion

A deterministic inventory model is being formulated. We infer that whenever Q<W the model reduces to a single training house model. And when Q>W the results of our model are feasible only when K$\leq$W. This model will help the organization which recruits its employees with different capacities

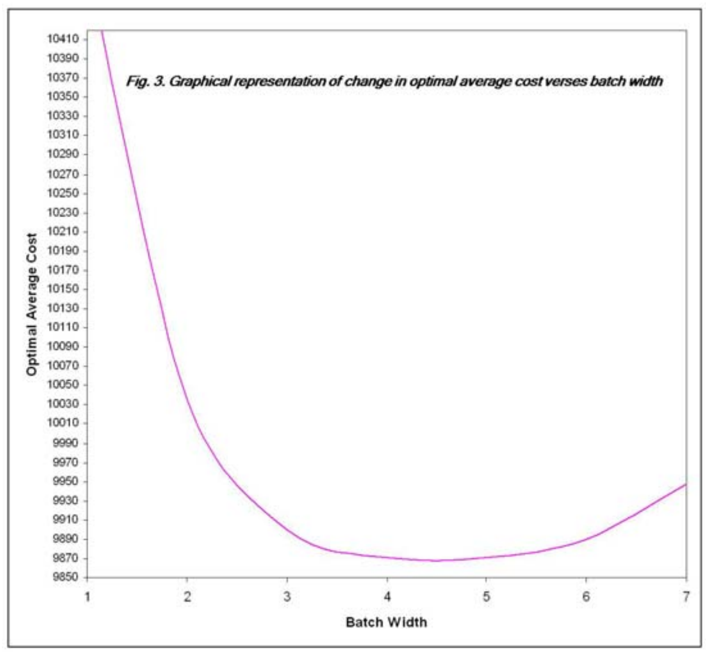


Fig. 3. Graphical representation of change in optimal average cost verses batch width

(experienced & inexperienced) and trains them as per requirement. It also helps the organization to determine the optimal number of people to be released from one training centre to another during the time period "t'. This also suggests the organization the optimal number of people to be recruited for the organization which minimizes the average Inventory cost.

Future work

A similar analysis can be carried out with this model with shortage in T_2 inventory house.

Indian Journal of Science and Technology Vol. 3 No. 7 (July 2010) ISSN: 0974- 6846

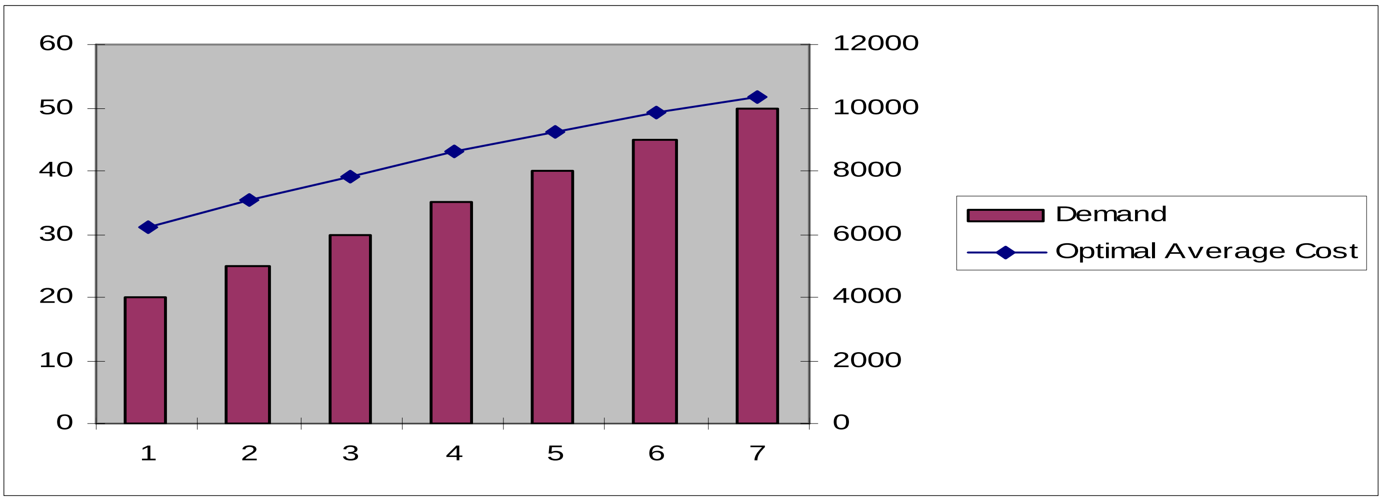
Table 1. Optimal solution for the model after truncation

n-Batch width	K	Optimal average cost
1	26	10488.7
2	13	10035.8
3	8	9899.75
4	6	9871.08
5	5	9871.08
6	4	9890.2
7	3	9947.55

Table 2. Optimal solution for the model after rounding off

n-Batch width	K	Optimal average cost
1	27	10537.5
2	14	10071.4
3	9	9923.08
4	7	9882.28
5	6	9871.15
6	5	9871.15
7	4	9890.63

Fig. 4. Graphical representation of change in optimal average cost vs demand



References

1. Bhunia AK and Maiti M (1994) A two warehouse inventory model for a linear trend in demand. *Opsearch.* 31(4), 318-329.
2. Hartley RV (1976) Operations research-a managerial emphasis. *Santa Monica, CA: Good year Publ. Co., California*, pp315- 317.
3. Sarma KVS (1983) A deterministic inventory model with two levels of storage and an optimum release rule. *Opsearch.* 20(3), 175-180.