Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

STH: A highly scalable and economical topology for massively parallel systems

Jahangir Alam*[1] and Rajesh Kumar[2]

[1]Faculty of Engineering and Technology, Aligarh Muslim University, Aligarh, U.P. - 202002, India
[2]School of Mathematics and Computer Applications, Thapar University Patiala, P.B. - 147004, India
jahangiralam@zhcet.ac.in*

Abstract

Highly parallel systems are receiving significant attention to solve the large and complex problems. This has resulted in the emergence of many attractive interconnection network topologies. This paper introduces a new processor interconnection topology called STH (Scalable Twisted Hypercube) to counter the poor scalability of twisted hypercube. Its suitability for use as multiprocessor interconnection networks has also been explored. The various properties of the proposed topology have been analyzed and it has been compared with some other highly scalable topologies of interest on a number of interconnection networks evaluation parameters. With reduced diameter, better average distance, low traffic density, low cost, maximum number of links, high bisection width and tremendous scalability, STH is more suitable for Massively Parallel Systems. Procedures for routing and broadcasting on the proposed topology have also been discussed and a simple routing algorithm has been presented. The proposed interconnection network provides a great architectural support for parallel computing due to the concurrent existence of multiple LST(m) and TQn.

Keywords: Parallel Systems, Processor Topology, Scalability, LST, Routing.

Introduction

High speed parallel computing is essential for modern research as the demand for more and more computing power is continuously increasing. In the resent past several high performance parallel computing platforms have been installed with more than 10,000 processing elements. Some examples of such installations are BlueGene/L at Lawrence Livermore National Laboratory, Blue Gene Watson at IBM and Columbia at NASA. Research projects in the varied application areas such as environmental simulation, astronomy and engineering design have been undertaken to utilize this tremendous amount of computing power.

Massive parallel systems are placing a major emphasis on scalable processor topologies with low degree and diameter (Dongarra *et al.,* 1997). Many processor topologies such as Hyperstar (Ayyoub & Day, 1998) Hyper-Mesh (Abuelrub, 2008) and Hex-Cell (Sharieh *et al.,* 2008) have been proposed in the literature for the purpose of connecting hundreds or thousands of processors. Each of these topologies has some attractive features as well as some inherent limitations.

The Hypercube graph is a topology with logarithmic diameter, simple node designation scheme, good connectivity, fault tolerance, vertex/ edge symmetry, partitionability, simple routing and existence of node disjoint parallel paths (Saad & Schultz, 1998; Hwang, 2004). From the research point of view Hypercube and its variants like Twisted Hypercube (Abuelrub, 2007) have always been the popular choice of researchers and a large number of processor interconnection topologies and effective computation algorithms based on them have been reported in the literature. (Day & Tripathi, 1994) compared topological properties of hypercube with Star Graphs. Al-Sadi et al. (2002) proposed a fault tolerant routing algorithm and Chiu & Chon (1998) proposed efficient multicasting procedure for binary hypercube. Klasing (1998) discussed efficient compression of CCC networks and a dynamic interconnection network named as 'Scalable Optical Hypercube' has been reported by Louri & Sung (1994). Consequently a large number of commercial installations of parallel computers like Intel's iPSC Series (can have up to 128 processors), NCUBE/ 10 (can have up to 1024 processors) and FPS's T Series (can have up to 4096 processors) and nCube are based on Hypercube architecture.

A parallel architecture is said to be scalable if it can be expanded (reduced) to a larger (smaller) system with a linear increase in its performance (Louri *et al.,* 1998; Rewini & Barr, 2005). This general rule indicates the desirability for providing equal chance for scaling up a system for improved performance and for scaling down a system for greater cost-effectiveness and/ or affordability. In spite of their admitted superiority the Hypercube and its variants architectures like Twisted Hypercube have a major disadvantage related to scalability. It grows to its next higher dimension by a factor of 2. For example the nine-dimensional hypercube (HC(9)) has $2^9 = 512$ nodes, whereas a ten-dimensional hypercube (HC(10)) has $2^{10} = 1024$ nodes. This significant gap between the two consecutive sizes of the Hypercube is considered a major drawback of this topology and needs further attention for its improvement. Moreover, in Hypercube like architectures moving form a particular dimension to the next higher dimension, the number of communication paths (wires) and the number of ports per processor increase. Considering the above mentioned problems with the Hypercube like architectures a number of topologies like Cross Cube (Efe, 1992), dBCube (Chen *et al.,* 1993), Hierarchical Hypercube (HHC) (Malluhi &

Bayoumi, 1994), Cube Connected Cycles (CCC) (Preparata & Vuillemin, 1981) have been proposed in the literature. These topologies in one way or other are the modifications of Hypercube architecture and suffer from similar problems. For example Cube Connected Cycles (CCC) topology, is obtained by replacing each node of an n-dimensional hypercube with a ring of size n. The result of this replacement is a constant degree (three) topology (*i.e.,* CCC). But, such a replacement introduces some unwanted features in the Hypercube architecture like large diameter and complex routing.

A recently proposed topology (Sharieh *et al.,* 2008), which is not based on Hypercube architecture has been named as Hex-Cell. The maximum degree of Hex-Cell is 3 (constant) and it offers simple routing. But, the diameter of a Hex-Cell consisting of N nodes is given by $4\sqrt{(N/6)} - 1$ which is sufficiently large as compared to the diameter of Hypercube. Also, it is irregular, vertex asymmetric and its bisection width is too low. So, Hex-Cell can't be used to design massively parallel systems.

Other attempts for scaling the Hypercube architecture have been made by means of Hybrid Topologies. Hyper-Star (Ayyoub & Day, 1998), Hype-Mesh (Abuelrub, 2008), Arrangment Star Network (Awwad *et al.,* 2003) and Double-Loop Hypercube (Youyao *et al.,* 2008) are some examples of such attempts. A Hybrid topology is derived from two or more existing topologies using a graph theoretic operator. Researchers have extensively used the Cartesian Product operator while designing the Hybrid Topologies. Star-Hypercube Hybrid Interconnection Networks (Zeng *et al.,* 1993), Hyper-Mesh Multicomputers (Abuelrub, 2002), Banyan-Hypercube Networks (Youssef & Narahari, 1990) and Hyper Petersen Network (Das & Banerjee, 1992) are some examples of topologies based on cartesian product of graphs. The generalized results for the Cartesian Product of topology graphs were first derived by Youssef (1995) and later extended by Day & Ayyoub (1997). The most recently reported topology in this category has been named as Double Loop Hypercube (DLH) (Youyao *et al.,* 2008). It has been derived as the Cartesian Product of Double Loop topology with Hypercube.

This paper introduces and analyses a new hybrid processor interconnection topology called STH (Scalable Twisted Hypercube). To establish suitability of STH for massively parallel systems, it has been compared with two recently proposed highly scalable topologies (Hex-Cell and DLH) on several parameters. It has been observed that STH is scalable and economical. A simple routing scheme that makes good use of STH architecture has also been proposed.

Materials and methods

Preliminary remarks and graph theoretic terms

This paper uses the standard graph theoretic terminology (Chartrand & Lesniak, 1986) and definitions used by Alam & Kumar (2010).

Definition 1: The interconnection network topology is a finite undirected graph G = (V, E), where $V = \{v_1, v_2 \ldots v_n\}$ is the set of nodes (vertices) and $E = \{e_1, e_2 \ldots e_m\}$ is the set of edges. Each vertex represents a processor and each edge a communication link between processors.

Definition 2: The degree of a vertex v in G, denoted as d_v is the number of edges incident on v. The minimum degree of a graph G, $\min\{d_v(G) \mid v \in V\}$ is denoted by $\delta(G)$. The maximum degree of G, $\max\{d_v(G) \mid v \in V\}$ is denoted by $\Delta(G)$. The degree of the graph is the maximum of the degrees of all vertices in the graph. Moreover, a graph is called regular if all of its vertices have the same degree.

Definition 3: The distance between two nodes u and v of a graph G denoted by $s_G(u,v)$ is the number of edges in G on the shortest path connecting i and j.

Definition 4: The diameter of a graph G(V, E) denoted by D(G), is the maximum distance between any two nodes in G. The diameter provides a bound on communication between any two nodes. Mathematically:

$D(G) = \max\{ s_G(u,v) \mid u, v \in V\}$

Definition 5: A graph G(V, E) is vertex-symmetric, if for every pair of vertices u and v, u, v є V, there exists an automorphism of the graph that maps u into v.

Definition 6: The (κ-1) - fault diameter of a κ-connected topology graph G(V, E), D κ(G), is defined as:

$D_\kappa(G) = \max\{d(G - F) \mid F \subset V(G), |F| = \kappa - 1\}$, Where F is an induced subgraph of G.

Definition 7: An n-dimensional binary hypercube, Q_n, consists of 2^n nodes. Each node v, for $0 \le v \le 2^n-1$, is labeled as n-bit binary string, L(v). There is an edge between two nodes, u and v, if, and only if, their labels differ in exactly one bit position.

Definition 8: An n-dimensional twisted hypercube, TQ_n, is constructed from a Q_n as follows. Two distinct edges, say (a, b) and (c, d), which have no nodes in common, in a 4-cycle of the hypercube are selected. Now the two new edges (a, d) and (b, c) are created while the original edges (a, b) and (c, d) are removed. Such a twist doesn't effect the degree of a node (d), but the diameter of TQ_n is one less than the diameter of Q_n, i.e., $D(TQ_n) = n-1$.

The Scalable twisted hypercube (STH) interconnection network

The proposed STH(m,n) network is based on two topologies. An m level Linearly Scalable Topology (LST), which is an improved version of the Linearly Extendible Arm Topology (Alam & Kumar, 2010), and the well known n-dimensional twisted hypercube topology. Whereas the level m LEA topology consists of 6m processors, the LST(m) consists of 8m processors and an altered connectivity formula. Formally it is defined as follows:

Let m be a positive integer such that m≥2, then level m LST, denoted as LST(m), is an undirected graph consisting of N(= 8m) processing elements (vertices) labeled as PE_0, PE_1, $PE_2 \ldots \ldots \ldots PE_{(N-1)}$, for $0 \le i < N$ and arranged in m columns, each of which consists of exactly eight processors. A link (edge) exists between processors

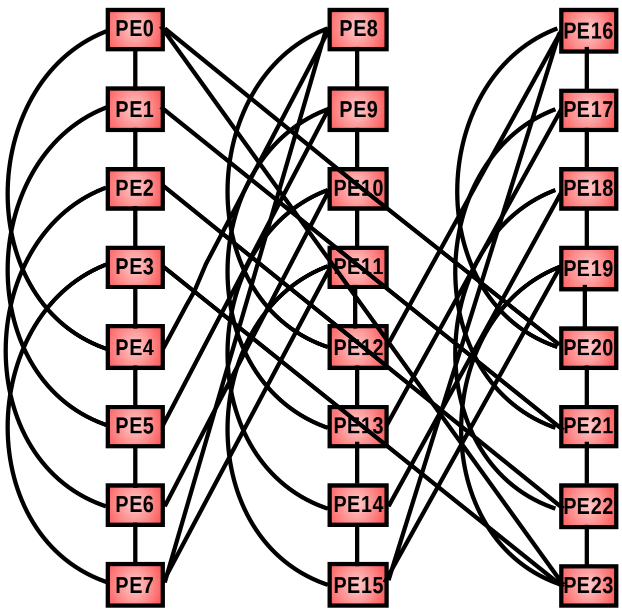
Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

PE_i and PE_j, iff , (i+1) modulo N = j or (i+4) modulo N = j. Alternately, for m ≥2, LST(m) = G(V, E), where,

- V = {PE_i | i є I^+ and 0 ≤ i <N}
- E = {(PE_i, PE_j) | j = ((i+1) modulo N) OR j = ((i+4) modulo N)}.

From the definition of LST it is obvious that each processor on an LST network provides two links to the two distinct processors and receives two links from two distinct processors. An example of a LST(m) is shown in Fig.1, where m equals to 3, which is composed of 3x8 = 24 nodes.

Fig. 1. LST(m) network, where m equals to 3



The topology of LST network is simple, symmetric and scalable in architecture and it is 4-regular vertex (node) symmetric graph.

STH (m, n) topology is obtained as a cartesian product of level m LST topology and n-dimensional twisted hypercube topology, so it can formally be defined as follows:

Let m and n be two positive integers such that m≥2, and the level m LST topology is represented by the undirected graph, $X_1(V_1,E_1)$ and the n dimensional twisted hypercube topology is represented by the undirected graph, $X_2(V_2,E_2)$, then the STH topology, is an undirected graph, X(V,E), where V and E are given by:

- V = {(a,b) | a є V_1 and b є V_2}, and
- For any x = (a,b) and y = (c, d) in V, (x,y) is an edge in E if, and only if, (a,c) is an edge in E_1 and b = d or (b,d) is an edge in E_2 and a = c.

Fig.2 shows STH (2, 3), which has been obtained as a Cartesian Product of LST (2) and TQ_3. From Fig.2 it is fairly obvious that STH (m,n) can be obtained from an n-dimensional twisted hypercube such that each of its node is replaced by LST (m) and connections are established according to the above definition. For the sake of simplicity, each node in STH (m, n) is represented as two tuples: (u, v), where PE_u is a node in LST network and PE_v is a node in TQ_n network. More precisely, in an STH(m, n) network we define the following two kinds of connections:

LST Connection: A node (u, v) є STH, is adjacent to another node (u', v) є STH, if and only if, u' = ((i+1) modulo N) OR u' = ((i+4) modulo N).

Twisted Hypercube Connection: Twisted Hypercube connection can be obtained using Definition 2.5. A node (u, v) є STH, is adjacent to another node (u, v') є STH if, and only if, L(v) and L(v'), where L(v) and L(v') denote the n-bit binary strings corresponding to v and v' respectively, follow Definition 8.

Topological properties of STH(m,n)

Lemma 1: The total number of nodes in STH(m, n) network is $m.2^{n+3}$.

Proof: From the construction method of STH(m, n), we know that STH(m, n) can be obtained by replacing each of the n nodes of an n-dimensional twisted hypercube network (TQ_n) by an m level LST network consisting of 8m nodes. So, replacement of one node of TQ_n, introduces 8m new nodes in the network. It implies that replacement of all n nodes will introduce $8m.2^n = m.2^{n+3}$ nodes in the network.

Lemma 2: The Degree of STH(m,n) is (n+4).

Proof: It follows directly from the definition of STH(m, n). Two components of STH(m, n) are LST(m). LST is a is 4-regular vertex (node) symmetric graph and the degree of TQ_n is n. Hence the degree of STH(m, n) is (4+n) . Fig.2 further confirms this expression. In STH(2, 3), the degree of each node is 7.

Lemma 3: The Diameter of STH(m,n) is (m+1)+$\lceil (n+1)/2 \rceil$.

Proof: It is easy to know that:
Diameter of 16 node LST, i.e., D(LST(2)) = 3.
Diameter of 24 node LST, i.e., D(LST(3)) = 4.
Diameter of 32 node LST, i.e., D(LST(5)) = 5.
So, the Diameter of 8m node (level m) LST i.e. D(LST (m)) = m+1
The diameter of an n-dimensional twisted hypercube is $\lceil (n+1)/2 \rceil$. So, the diameter of STH(m, n) is given by the following expression (Youssef, 1995; Day & Ayyoub, 1997):

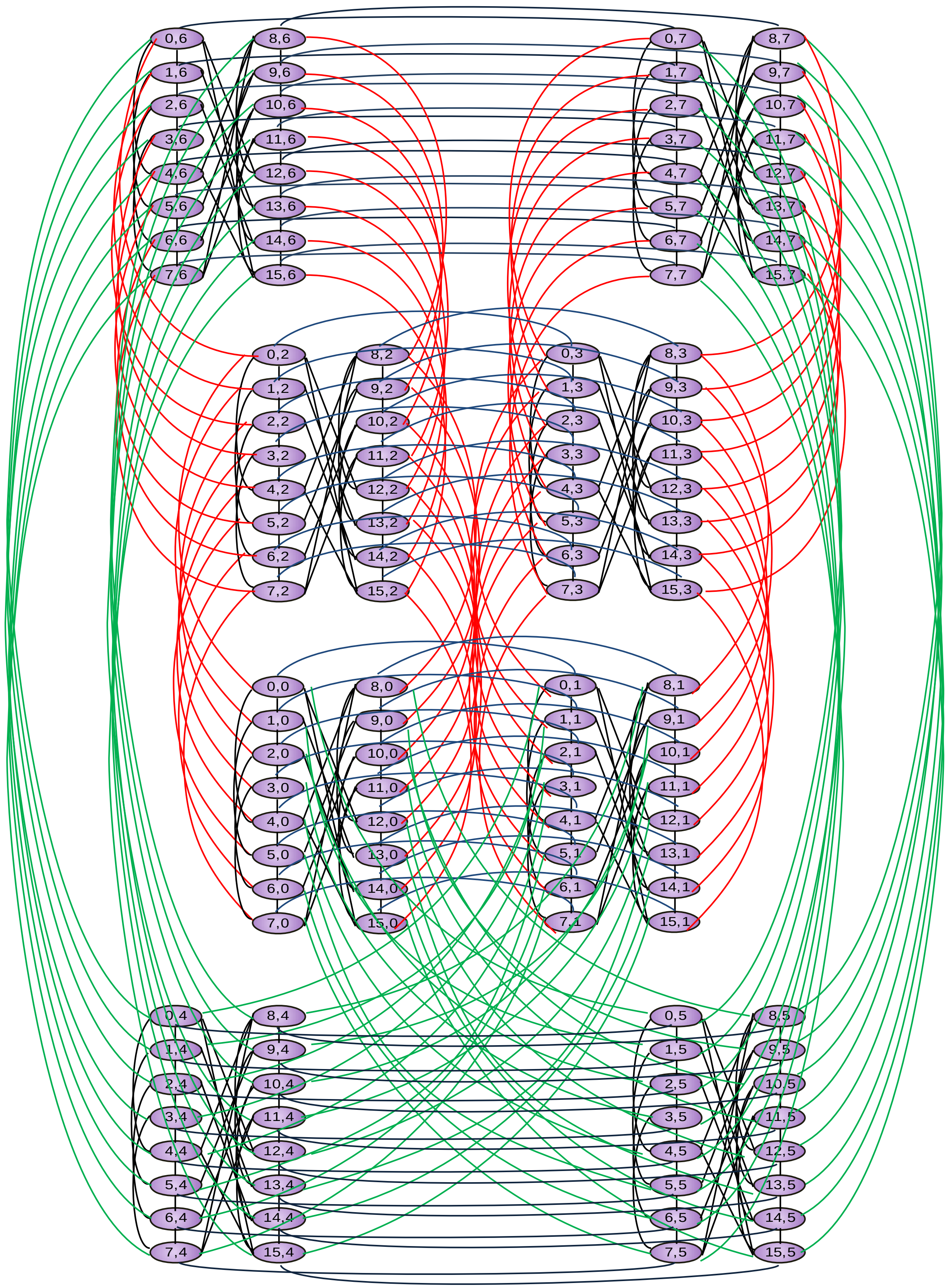
$$D(STH(m,n)) = (m+1)+ \lceil (n+1)/2 \rceil$$

The proof can also be obtained using the analytical method as follows:

Let x = (p, q) and y = (r, s) be two vertices of STH graph. x would be at a maximum distance from y in STH if, and only if, p is at a maximum distance (m+1) from r in LST and q is at a maximum distance $\lceil (n+1)/2 \rceil$ from s in TQ_n. Clearly, the maximum distance (diameter) of STH(m, n) is (m+1)+ $\lceil (n+1)/2 \rceil$.

Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

Fig. 2. 128-node STH Network (STH(2,3))



Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

Lemma 4: STH(m, n) is a regular vertex (node) symmetric graph.

Proof: Each node of STH(m,n) has the degree 4+n so, its obvious that STH(m, n) is a regular graph. Let LST(m) network is represented by a graph X(V, E), and ϕ be a permutation of the vertex set V, of LST(m). Then, for any edge x = (a, b), x є E and a, b є V, we have ϕ(x) = (ϕ(a), ϕ(b)), is also an edge. It shows that graph X (representing LST(m)) is isomorphic to itself (automorphism). Clearly LST is a vertex symmetric (Definition 5).

Twisted Hypercube belongs to the family of graphs known as Cayley Graphs and every Cayley Graph is vertex symmetric (Day & Ayyoub, 1997). Hence, Twisted Hypercube graph is also vertex symmetric. If two given graphs X and Y are vertex symmetric then X.Y is also vertex symmetric (Youssef, 1995). From this discussion, it is easy to conclude that STH(m, n) is vertex symmetric.

Lemma 5: The Bisection Width (ω) of STH(m ,n) is given by $(3m-1).2^{n+1}$ or $(3N-8).2^{n-2}$.

Proof: The Bisection Width of an interconnection network is defined as the minimum number of edges (wires) cut to split a network into two parts each having the same number of nodes.

Bisection Width of LST(k) can be calculated as follows (Table 1):

Table 1. Bisection Width Calculation for LST(k)

m	N = 8*m	No. of edges cut to split LST(m) into two Parts
2	16	20 = (12*2 - 4)
3	24	32 = (12*3 - 4)
4	32	44 = (12*4 - 4)
5	40	56 = (12*5 - 4)
…	…	…
…	…	…
m	8m	(12*m - 4)

STH(m, n) is obtained by replacing each of the 8m nodes of LST(m) by an n-dimensional twisted hypercube. We have already proved that the bisection width of LST(m) is (12m-4). The bisection width of TQ_n is 2^{n-1}.Hence the Bisection Width of STH(m, n) is $(12m-4).2^{n-1}$, i.e.

$\omega = (12m-4).2^{n-1} = (3m-1).2^{n+1} = (3N-8).2^{n-2}$

Lemma 6: The average distance of STH(m, n) is given by:

$$\bar{d}(STH) = \frac{m(4m+7)}{(8m-1)^2} + \frac{n.2^{n-1}-1}{2^n-1}$$

Proof: The average distance $\bar{d}(G)$ of a graph G, consisting of N nodes is given by the following equation:

$$\bar{d}(G) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} d(i,j)}{N(N-1)}, \text{ where } i \neq j$$

The average distance $\bar{d}v$ of a node v is obtained from the following equation:

$$\bar{d}v = \frac{\sum_{j=1}^{N} d(i,j)}{(N-1)}, \text{ where } i \neq j$$

Average Distance of LST can be calculated as follows. The average distances for node 1 in different level LST networks are shown in the Table 2:

Table 2. LST Average Distances for First Node

m	No. of Nodes	$\bar{d}1$
2	16	(1+1+1+1+2+2+2+2+2+2+2+3+3+3+3)/ 15 = (4x1+7x2+4x3)/15 = 30/15
3	24	(1+1+1+1+2+2+2+2+2+2+2+3+3+3+3+ 3+3+3+3+4+4+4+4+4)/23= (4x1+8x2+7x3+4x4)/23 = 57/23
4	32	(4x1+8x2+8x3+7x4+4x5)/31 = 92/31
…	…	…
…	…	…
k	8k	(4x1+8x2+8x3+ … +8x(k-1)+7xk+4x(k+1)/(8k-1)

Clearly, the average distance of first node in LST(m) is be given by:

$$\bar{d}1 = \frac{(4x1+8x2+8x3+.......+8x(m-1)+7xm+4x(m+1))}{(8m-1)} = \frac{m(4m+7)}{(8m-1)}$$

As LST is a 4-regular, node symmetric graph, average distance for each node is same and can be given by the above expression.

Thus, the average distance of LST graph i.e. $\bar{d}1$ can be calculated as follows.

$$\bar{d}(LST) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} d(i,j)}{N(N-1)} = \frac{m(4m+7)}{(8m-1)^2}$$

The average distance of an n-dimensional Twisted Hyper Cube is given by (Abuelrub, 2007):

$$d(TQ) = \frac{n.2^{n-1}-1}{2^n-1}$$

We know that if G_1 and G_2 are two graphs with average distances $\bar{d}(G_1)$ and $\bar{d}(G_2)$ respectively, then the average distance $\bar{d}(G\ )$ of the Cartesian Product graph G, of G_1 and G_2 is given by :

$$\bar{d}(G) = \bar{d}(G_1) + \bar{d}(G_2)$$

So, for STH(m, n) we have:

$$\bar{d}(STH) = \frac{m(4m+7)}{(8m-1)^2} + \frac{n.2^{n-1}-1}{2^n-1}$$

Lemma 7: The (n+3) - fault diameter of STH(m, n) is bounded by the following inequality:

$$D_{(n+4)}(STH) \leq (m+n+2)$$

Proof: It has been shown (Xu *et al.,* 2005) that if G_1 and G_2 are two undirected graphs such that G_1 is κ1 connected with $D_{\kappa 1}$ as (κ-1) fault diameter and G_2 is κ2 connected with $D_{\kappa 2}$ as (κ-1) fault diameter then if, the product graph of G_1 and G_2 ($G_1.G_2$) is (κ1+ κ2) connected, its diameter is bounded by the following inequality:

$$D_{(\kappa 1+\kappa 2)}(G_1.G_2) \leq D_{\kappa 1}(G_1) + D_{\kappa 2}(G_2) + 1$$

Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

STH is a product graph of LST and TQ. Using Whitney's Theorem (Balakrishnan, 2007), it is easy to prove that LST is 4-connected i.e. κ(LST) = 4 and its 3 - fault diameter *i.e.* D_4(LST) is (m+2).

The connectivity of n-dimensional Twisted Hypercube is n (Abuelrub, 2007) *i.e.* κ(TQ) = n and its (n-1) - fault diameter is (n-1) i.e. D_n(TQ) = n-1.

Also, κ(STH) = κ(LEA)+ κ(TQ) = n+4

Hence, using the results of Xu *et al.* (2005) it is easy to prove that (n+3) - fault diameter of STH(m, n) is bounded by the following inequality:

$$D_{(n+4)}(STH) \leq (m+n+2)$$

Routing on STH Network

Routing Algorithm is an important factor which affects the performance of an interconnection network. Routing involves the process of identification of a set of permissible paths that may be used by a message to reach its destination, and a function that selects one path from the set of permissible paths. We have two approaches to develop the routing algorithms for interconnection networks (Rewini & Barr, 2005) - Adaptive Routing and Deterministic or Oblivious Routing. This paper favours the deterministic routing techniques due to their simplicity. The following sub-sections describe the unicast and broadcast routing algorithms for STH interconnection networks.

Unicast routing algorithm for STH(m,n)

Let on an STH(m,n) network a node u = (a,b) wants to send a message to another node v = (a',b'). We propose the following optimal (O(m+n) in worst case) algorithm for the required message routing.

Notations used

u, v: Source and Destination Nodes
m: Level of LST Network
n: Dimension of Twisted Hypercube
R_i: Variable to record movements along path i in LST network.
K_i: List of nodes traversed along path i.
add(K, p): A function that inserts a node p in the list L.
follow(K): A function that forwards a message from source to destination along the path K.
spath: Shortest Path
splength: Length of Shortest Path
ℓ(K): Last node in the list K.
procedure_route( ): Main Routing Procedure
sub_procedure_routLST( ): Sub procedure for dealing routing on LST part of STH Network.
sub_procedure_routTH( ): Sub procedure for dealing routing on TQ part of STH Network.

Main Routing Procedure

```
procedure_rout(u,v,m,n)
begin
if (a!=a' AND b=b') then
   begin
   call sub_procedure_routLST(a,a',m)
   exit( )
   end
if (a=a' AND b!=b') then
   begin
   call sub_procedure_routTH(b,b',n)
   exit( )
   end
if(a!=a' AND b!=b') then
   begin
   call sub_procedure_routLST(a,a',m)
   call sub_procedure_routTH(b,b',n)
   end
end
```

```
sub_procedure_routLST(x,y,p)
begin
    splength ← 1;
    m=8*p;
    for i =1 to 4 do
    add(K_i, PE_x);
    end for
    R1 ← x-4; if (R1 < 0) then R1 ← m+R1;
    R2 ← x+4; if (R2 >= m) then R2 ← |m-R2|;
    R3 = x+1; if (R3 >= m) then R3 ← |m-R3|;
    R4 ← x-1; if (R4 < 0) then R4 ← m+R4;
    for i =1 to 4 do
    add(K_i,PE_Ri);
    end for
    while true do
    begin
    if (R1= y  OR  R2 = y OR R3 = y OR R4  = y) then
    break;
    hopsreq ← (y-X1)
    if (hopsreq >= 4) then R1 ← R1-4;
    else
    {
    if (hopsreq > 0) then R1 ← R1+1;
    else R1 ← R1-1;
    }
    if (R1 < 0) then R1 ← m+R1;
    hopsreq ← (y-R2)
    if (hopsreq >= 4) then R2 ← R2+4;
    else
    {
    if (hopsreq > 0) then R2 ← R2+1;
    else R2 ← R2-1;
    }
    if (R2 >= m) then R2 ← |m-R2|;
    R3 ← R3+1; if (R3 >= m) then R3 ← |m-R3|;
    R4 ← R4-1; if (R4  < 0) then R4 ← m+R4;
    for i =1 to 4 do
    add(K_i PE_Ri);
    end for
    splength ← splength + 1;
    end while
    for i=1 to 4 do
    if (ℓ(K_i) = PE_y) then spath ← K_i;
    end for
    follow(spath);
    end
```

Table 3. LST Shortest Path Route Calculations

R_1	R_2	R_3	R_4	K_1	K_2	K_3	K_4	Splength
21	10	4	0	PE_2 $PE_{22}PE_{21}$	PE_2 PE_6PE_{10}	PE_2 PE_3PE_4	PE_2 PE_1PE_0	2
20	14	5	23	PE_2 $PE_{22}PE_{21}$ PE_{20}	PE_2 PE_6PE_{10} PE_{14}	PE_2 $PE_3PE_4PE_5$	PE_2 $PE_1PE_0PE_{23}$	3
19	18	6	22	$PE_2PE_{22}PE_{21}PE_{20}$ PE_{19}	PE_2 PE_6PE_{10} PE_{14} PE_{18}	$PE_2PE_3PE_4PE_5$ PE_6	$PE_2PE_1PE_0PE_{23}$ PE_{22}	4

Twisted hypercube routing has been discussed extensively in the literature (Agrawal & Ravikumar, 1996; Abuelrub, 2007) so we are not giving algorithmic code for sub_procedure_routeTH( ).

Example: Let in a 192-node STH network (STH(3,3)), node u(2,6) wants to send a message to node v(19,3). Then according to the above algorithm the message is routed as follows:

A. Here, a=2, b=6, a'=19 and b'=3 therefore the sub_procedure_routLST(2,19,3) is executed as follows:

Steps 1 - 12:

splength = 1, m = 8*3 = 24, $K_1 = K_2 = K_3 = K_4 = PE_2$, $R_1 = 22$, $R_2 = 6$, $R_3 = 3$, $R_4 = 1$.

$K_1 = PE_2.PE_{22}$, $K_2 = PE_2.PE_6$, $K_3 = PE_2.PE_3$, $K_4 = PE_2.PE_1$.

Steps 13 - 38:

Clearly, List K_1 is selected as it consists the shortest path. The message is now routed along the path

Contained in K1. And now the message is available at (19, 6)

B. From node(19,6) the message is sent to node (19,3) using sub_procedure_routTH(6,3,3).

Broadcasting on STH (m,n)

Let node X wants to broadcast a message on STH (m,n) network, then such a broadcasting will be performed as follows(Table 3):

1. X will send message to all the nodes of that LST (m) network on which X itself resides.
2. Then on every twisted hypercube of STH (m,n), the node received the messages will send them to all of the nodes in the same twisted hypercube network.

Results and discussion

Comparative Study

Designing a high performance interconnection network is the most challenging task in the field of parallel computers. This section presents the comparative study of STH network with some recently proposed topologies of interest. The various performance parameters used here are degree, diameter, cost factor, bisection width, average distance, message traffic density, cost of one to all broadcasts and cost of all-to-all broadcasts. Table 4 presents some basic parameters for STH and other topologies of interest. Tables 5-7 present calculated values of several parameters related to the topologies under consideration. Fig.3-11 compare various parameters shown in Tables 5-7 graphically. For a justifiable comparison we have chosen a hypercube of fixed degree (7).

Following observations can be easily made:

Degree (d) and Diameter (D): For networks of any size, the node degree of STH(m, n) is admittedly larger than that of any of the other networks under consideration. For a fixed value of hypercube degree (7), the degree of DLH is 10 and that of STH is 11.

Fig.3 compares the diameters of all the topologies of interest. It has been observed that the diameter of STH is smallest among all the topologies even up to more than 10^6 nodes, when we scale up the network for a fixed hypercube dimension. The diameter may further be reduced by choosing the large dimensions of hypercube part.

Cost Factor (ξ) and Cost (ζ): A network with high node degree is expensive and a network with large diameter suffers form high latency. It is always desirable to have a topology with both small degree (low cost) and small diameter (low latency). Thus, for an interconnection network the product of degree (d) and diameter (D) is defined as the cost factor (ξ) (Amawy & Latifi, 1991). Another metrics used to describe the cost (ζ) of an interconnection network topology is the product of the number of Links (L) and the diameter (Louri & Neocleous, 1997).

Fig.5 & 6 compare Cost Factor and Cost for the topologies of interest. From these figures it is evident that STH outperforms other topologies when topologies are either evaluated using Cost Factor or Cost.

No. of Links (L): Fig.4 confirms that, in spite of its lowest cost, among all the topologies of interest, STH has maximum number of links for a given network size. In other words it can also be said that among all topologies of interest STH is maximal fault tolerant.

Table 4. Comparison of Basic Parameters

Topological Property	DLH(m, n)	HX(i)	STH(m, n)
No. of Nodes (N)	$m.2^{n+2}$	6(2i-1)	$m.2^{n+3}$
Degree (d)	3+n	≤ 3	4+n
Diameter (D)	m+n+1	$4\sqrt{2i-1}$ - 1	(m+1)+ $\lceil (n+1)/2 \rceil$
No. of Links (L)	$m.2^{n+1}.(3+n)$	$3(3i^2 - i)$	$m.2^{n+2}.(4+n)$
Cost Factor (d*D)	(3+n)(m+n+1)	$3.(4\sqrt{2i-1}$ - 1)	(4+n).(m+n+1)
Cost (L*D)	(m+n+1). $m.2^{n+1}.(3+n)$	$(4\sqrt{2i-1}$ - 1).(18i-12)	((m+1) + $\lceil (n+1)/2 \rceil$). $m.2^{n+2}.(4+n)$
Bisection Width (ω)	$m.2^n$	2. i	$(3m-1).2^{n+1}$

Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

Table 5. Basic topological properties comparison

No. of Nodes	Diameter (D)			No. of Links (L)			Cost Factor(d*D)			Cost (L*D)		
	DLH (m,7)	HX	STH (m,7)	DLH (m,7)	HX	STH (m,7)	DLH (m,7)	HX	STH (m,7)	DLH(m,7)	HX	STH(m,7)
1024 *STH(m,6)	10	51	7	5120	1497	5120	100	153	70	51200	76337	35840
2048	12	72	7	10240	3017	11264	120	216	77	122880	217193	78848
4096	16	103	9	20480	6066	22528	160	309	99	327680	624758	202752
8192	24	146	13	40960	12178	45056	240	438	143	983040	1777863	585728
16384	40	208	21	81920	24420	90112	400	624	231	3276800	5079200	1892352
32768	72	294	37	163840	48931	180224	720	882	407	11796480	14385507	6668288
65536	136	417	69	327680	97991	360448	1360	1251	759	44564480	40862024	24870912
131072	264	590	133	655360	196165	720896	2640	1770	1463	173015040	115737111	95879168

Table 6. Bisection width, cost of one-to-all broadcasts and cost of all-to-all broadcasts comparison

No. of Nodes	Bisection Width (ω)			Cost of one to all Broadcast			Cost of All to All Broadcast		
	DLH(m,7)	HX	STH(m,7)	DLH(m,7)	HX	STH(m,7)	DLH(m,7)	HX	STH(m,7)
1024 *STH(m,6)	256	26.127891	640	11022.400000	58603.697805	7670.073435	10102.300000	51341.000000	7102.300000
2048	512	36.950417	1280	13225.039866	81187.076915	7587.808882	12204.700000	72682.333333	7186.090909
4096	1024	52.255781	2816	17581.109342	114142.350239	9819.042557	16409.500000	104365.000000	9372.272727
8192	2048	73.900834	5888	26171.732175	159411.633022	14206.941338	24819.100000	148730.333333	13744.636364
16384	4096	104.511562	12032	43099.198719	224152.758020	22822.060078	41638.300000	213461.000000	22489.363636
32768	8192	147.801669	24320	76495.135855	313342.399972	39754.387252	75276.700000	304922.333333	39978.818182
65536	16384	209.023125	48896	142538.528025	440173.636341	73125.061425	142553.500000	438845.000000	74957.727273
131072	32768	295.603338	98048	273481.455834	617699.424477	139103.939645	277107.100000	633690.333333	144915.545455

Table 7. Average internode distances and message traffic density with respect to hypercube and DLH

No. of Nodes (N)	Average Distance			Message Traffic Density		
	DLH(m,7)	HC	STH(m,7)	DLH(m,7)	HC	STH(m,7)
1024 *STH(m,6)	3.489320	5.004888	3.165079	0.697864	1.000978	0.633016
2048	3.485549	5.502687	3.653018	0.697110	1.000489	0.664185
4096	3.523719	6.001465	3.615419	0.704744	1.000244	0.657349
8192	3.773481	6.500794	3.598294	0.754696	1.000122	0.654235
16384	4.899130	7.000427	3.590117	0.979826	1.000061	0.652749
32768	9.215962	7.500229	3.586121	1.843192	1.000031	0.652022
65536	23.698827	8.000122	3.584146	4.739765	1.000015	0.651663
131072	65.815249	8.500065	3.583163	13.163050	1.000008	0.651484

Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

Fig. 3. Diameter (D) Comparison

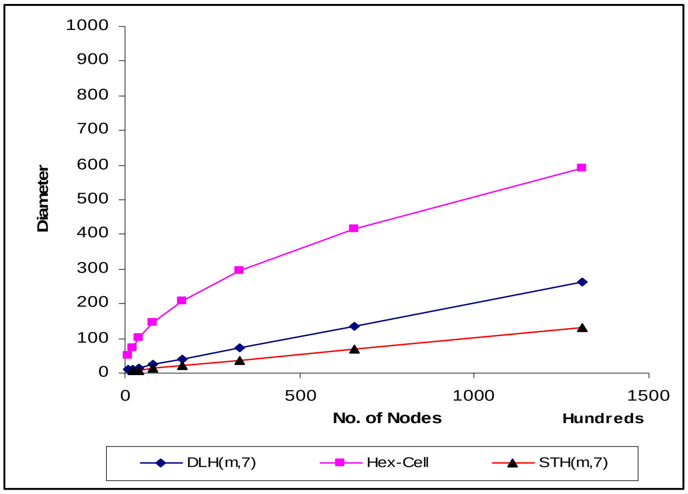


Fig.4 Comparison of No. of Links

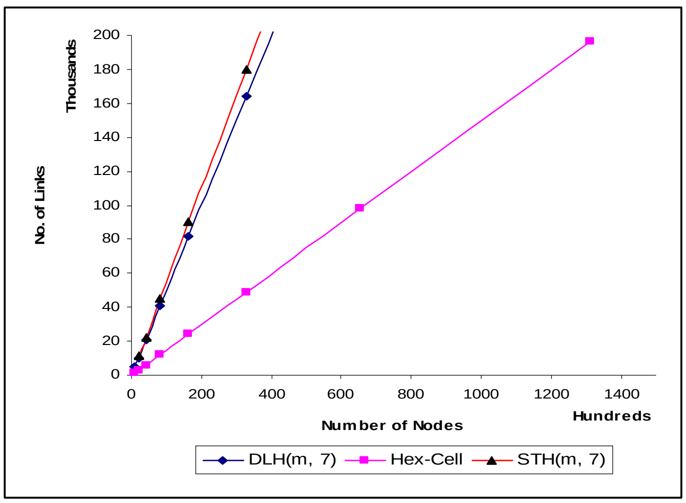


Fig. 5. Cost Factor Comparison

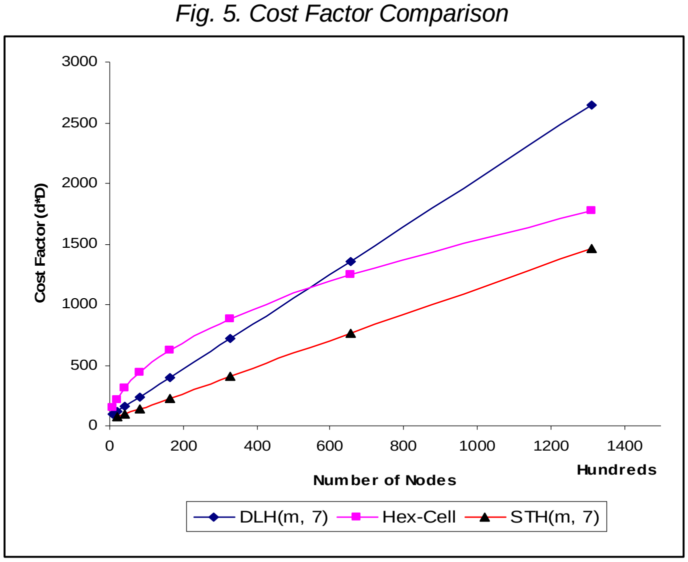


Fig. 6 Cost Comparison

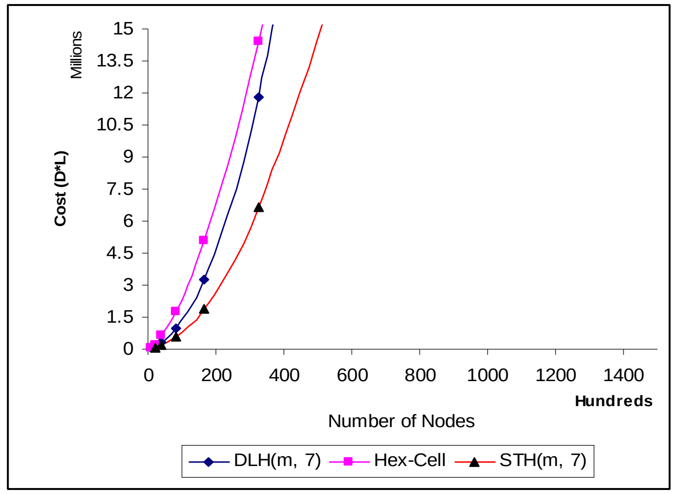


Fig. 7. Bisection Width Comparison

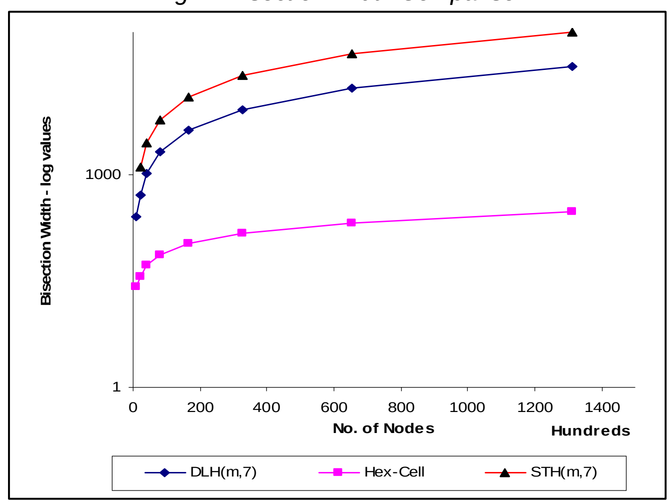
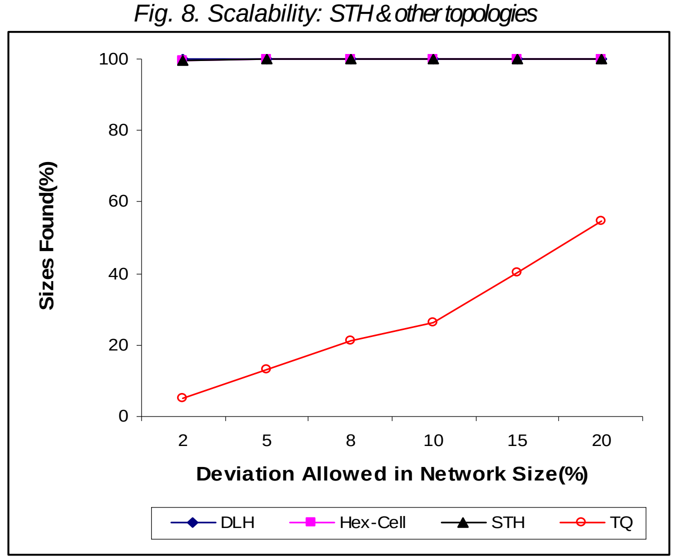


Fig. 8. Scalability: STH & other topologies



Indian Journal of Science and Technology Vol. 4 No. 12 (Dec 2011) ISSN: 0974- 6846

Bisection Width (ω): The Bisection Width of an interconnection network is defined as the minimum number of edges (wires) cut to split the network into two parts each having the same number of nodes. Bisection width reflects the wiring density of an interconnection network and provides a good indicator of the maximum communication bandwidth along the bisection of an interconnection network. Interconnection network designers strive for high bisection width.

Fig.7 shows that bisection width of STH is best among all the topologies of interest. Hex-Cell (HX) has very low bisection width hence is not suitable for designing massively parallel systems.

Scalability of STH: Fig.8 shows the scalability of DLH, HX and STH with respect to the scalability of twisted hypercube. To determine the relative scalabilities of all the topologies under consideration we define the percentage of deviation allowed in network size representation (ψ) as follows:

$\psi = \frac{|\alpha - \beta|}{\alpha} \times 100$ **,** where α = Requested Network Size,

β = Available Network Size

We start with ψ = 2 (i.e. deviation allowed from actual network size = 2%) and for each topology find out the number and percentage of network sizes available with that particular topology with the flexibility of ψ between 1 and 50000. Then we gradually increase ψ up to 20. The results are presented in Table 8.

From Fig.8 and Table 8, it is clear that scalability of STH network is at par with the scalabilities of DLH and Hex-Cell. These three topologies are capable of representing more than 99% of the network sizes in the range of 1 - 50000, even with a very low deviation (ψ = 2) allowed in the network size representation. Scalability of Twisted Hypercube (TQ) on the other hand is really poor. TQ is capable of representing at the most 55% (approx.) network sizes when maximum deviation in network size representation is allowed (ψ = 20).

Costs of One-to-All and All-to-all Broadcasts: The lower bound on the cost of one-to-all broadcasting on a d-port network (L_G^d), is given by the following equation (Graham & Seidel, 1993):

$$L_G^d = (\sqrt{(Ma/(bd))} - \sqrt{D-1})^2$$

Fig. 9. Cost: One-to-All Broadcast

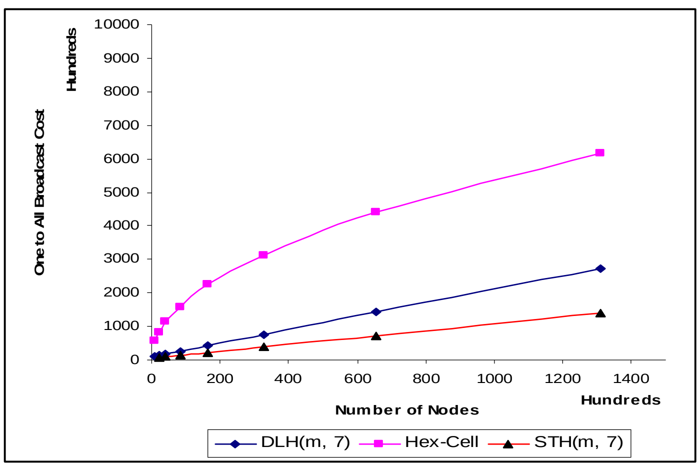


Fig.11. Average Distance Comparison

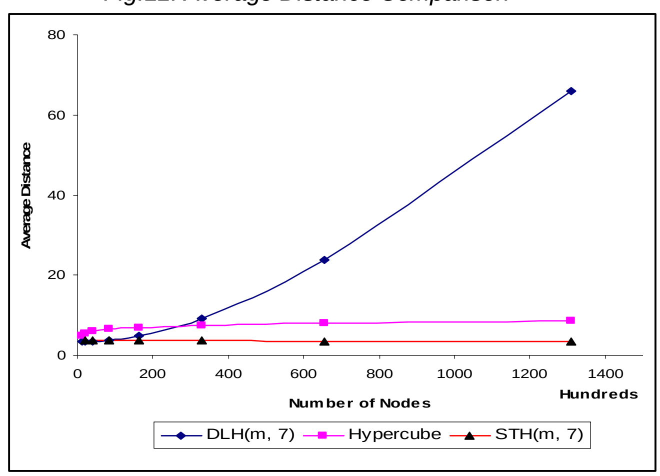


Fig. 10. Cost: All-to-All Broadcast

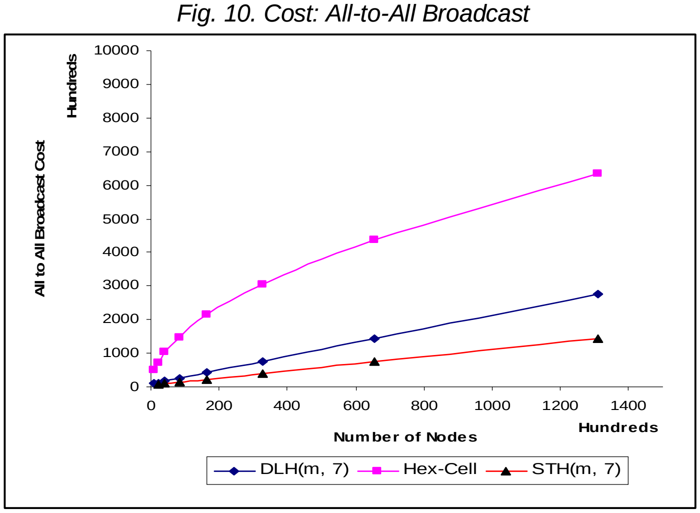


Fig.12. Message traffic density comparison

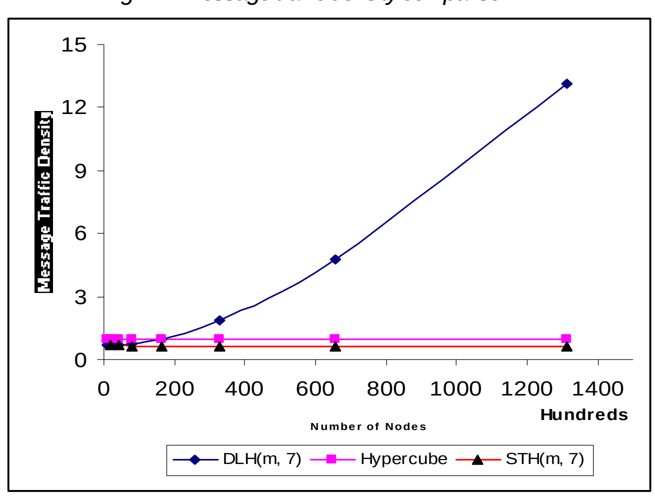


Table 8. Relative Size Representation Capabilities of Topologies

Ψ	DLH		Hex-Cell (HX)		STH		TQ	
	Total No. of sizes available (1-50000)	% of available sizes	Total No. of sizes available (1-50000)	% of available sizes	Total No. of sizes available (1-50000)	% of available sizes	Total No. of sizes available (1-50000)	% of available sizes
2	49898	99.79	49851	99.70	49797	99.59	2627	05.25
5	49959	99.91	49939	99.87	49919	99.83	6572	13.14
8	49974	99.94	49962	99.92	49947	99.89	10554	21.10
10	49978	99.95	49968	99.93	49957	99.91	13239	26.47
15	49985	99.96	49981	99.96	49969	99.93	20115	40.22
20	49986	99.97	49984	99.97	49973	99.94	27299	54.59

and, the lower bound on the cost of all-to-all broadcast is given by the following equation:

$$U_G^d = (N-1)a/d + Db$$

Where: - N= No. of Nodes in the Network, d = Degree of the Network, D = Diameter of the Network, M = Length of the Message, a= Unit Transmission Cost, b = Network Latency.

Fig.9 & 10 show that the cost of one-to-all broadcasts and the cost of all-to-all broadcasts are lowest for STH. All the calculations have been made assuming M = 1024, a= 1µs, b = 1000 µs (Graham & Seidel, 1993)

Average node distance ($\bar{d}$): Fig.11 compares the average node distance of DLH and STH with that of Hypercube (HC) From Fig.10 it is clear that average node distance in DLH increases rapidly with the increase in network size. In case of Hypercube it increases gradually when the network is scaled up. However, in case of STH the average node distance remains almost constant, close to 3, even for very large network size.

Message Traffic Density (ρ): The message traffic in a network can be estimated by calculating the message traffic density (ρ). Assuming that in a network of N nodes and L links each node is sending one message to a node at average distance $\bar{d}$, the message traffic density is given by $\rho = (\bar{d} \times N)/L$. Fig.12 shows that for DLH network, message traffic density grows rapidly when the network is scaled up. For Hypercube the message traffic density is almost constant (close to 1) and for STH network it is also constant (close to 0.65). Clearly, STH has the lowest value of ρ.

Conclusions

A new interconnection network topology called Scalable Twisted Hypercube (STH) has been presented in this paper to counter the poor scalability of twisted hypercube. Its suitability for use as multiprocessor interconnection networks has also been explored. The various properties of the proposed topology have been analyzed and it has been compared with some other highly scalable topologies of interest on a number of interconnection networks evaluation parameters. With reduced diameter, better average distance, low traffic density, low cost, maximum number of links, high bisection width and tremendous scalability, STH is more suitable for Massively Parallel Systems. Procedures for routing and broadcasting on the proposed topology have also been discussed and a simple routing algorithm has been presented. To sum up the proposed interconnection network provides a great architectural support for parallel computing due to the concurrent existence of multiple LST(m) and TQn.

References

1. Dongarra JJ, Meuer H and Strohmaier E (1997) TOP500 Supercomputer sites. *Supercomput.* 13, 89-120.
2. Ayyoub A and Day K (1998) The hyperstar interconnection network. *J. Parallel & Distributed Processing*. 48, 175-199.
3. Abuelrub E (2008) A comparative study on topological properties of the hyper-mesh interconnection network. *Proce. World Congress on Engg.* UK, pp: 616-621.
4. Sharieh A, Qatawneh M, Almobaideen W and Sleit A (2008) Hex-cell - modeling, topological properties and routing algorithm. *Eur.J. Sci. Res.* 22(2), 457-468.
5. Saad Y and Schultz M (1988) Topological properties of the hypercube. *IEEE Trans. Comput.* C-37(7), 867-872.
6. Hwang K (2004) Advanced computer architecture: parallelism, scalability, programmability, *6th Reprint, TMH*.
7. Al-Sadi J, Day K and Ould-Khaoua M (2002) Unsafety vectors: A new fault-tolerant routing for binary n-cubes. *J. Sys. Architec.* 47(9), 783-793.
8. Chiu G and Chon K (1998) Efficient fault-tolerant multicast scheme for hypercube multicomputers. *IEEE Trans. Parallel & Distributed Sys.* 9(10), 952-962.
9. Klasing R (1998) Improved compressions of cube-connected cycles networks. *IEEE Trans. Parallel & Distributed Sys.* 19(8), 803-812.
10. Louri A and Sung H (1994) Scalable optical hypercube-based interconnection network for massively parallel computing. *Appl. Optics*. 33, 7588-7598.
11. Louri A, Weech B and Neocleous C (1998) A Spanning multichannel linked hypercube: A gradually scalable optical interconnection network for massively parallel computing. *IEEE Trans. Parallel & Distributed Sys.* 9(5), 497-512.

12. Efe K (1992) The crossed cube architecture for parallel computation. *IEEE Trans. Parallel & Distributed Sys.* 3, 513.
13. Chen C, Agrawal DP and Burke JR (1993) dB-cube: a new class of hierarchical multiprocessor interconnection network with area-efficient layout. *IEEE Trans. Parallel & Distributed Sys.* 4, 1332.
14. Malluhi QM and Bayoumi MA (1994) The hierarchical hypercube: A new interconnection topology for massively parallel systems. *IEEE Trans. Parallel & Distributed Sys.* 5, 17.
15. Preparata FP and Vuillemin J (1981) The cube-connected cycles: A versatile network for parallel computation. *Commun. ACM*. 24(5), 300-309.
16. Youyao L, Jungang H and Huimin DU (2008) A hypercube based scalable interconnection network for massively parallel system. *J. Comput.* 3(10), 58-65.
17. Awwad A, Ayyoub AA and Ould-Khaoua M (2003) On topological properties of arrangement star network. *J. Sys. Architec.* 48, 325-336.
18. Day K and Tripathi A (1994) A comparative study of topological properties of hypercubes and star graphs. *IEEE Trans. Parallel & Distributed Sys.* 5(1), 31-38.
19. Zheng S, Cong B and Bttayeb S (1993) The star-hypercube hybrid interconnection networks. *Proc. ISCA Intl. Conf. Comput. Appl. Design, Simulation & Analysis*. USA. pp: 98-101.
20. Abuelrub E (2002) On hyper-mesh multicomputers. *J. Institute of Maths & Comput. Sci.* 12(2), 81-83.
21. Youssef A and Narahari B (1990) The banyan-hypercube networks. *IEEE Trans. Parallel & Distributed Sys*. 1 (2), 160-169.
22. Das SK and Banerjee AK (1992) Hyper petersen network: Yet another hypercube-like topology. *Proc. Frontiers.* USA. 92, 270-277.
23. Chartrand G and Lesniak L (1986) Graph and diagraph. *2nd Ed., Wadsworth & Books*, California.
24. Youssef A (1995) Design and analysis of product networks. *Proc. Fifth Sym. Frontiers of Massively Parallel Sys. USA.* pp: 521-528.
25. Day K and Ayyoub AA (1997) The cross product of interconnection networks. *IEEE Trans. Parallel & Distributed Sys.* 8(2), 109-118.
26. Rewini HE and Barr MA (2005) Advanced computer architecture and parallel processing. *1st Edition, Wiley-Intersci.,* Canada. pp: 105-106.
27. Abuelrub E (2007) Data communication and parallel computing on twisted hypercubes. *Proc. World Congress on Engg.,* London, UK.
28. Alam J and Kumar R (2010) Linearly extendible arm (LEA) - A constant degree topology for designing scalable and cost effective interconnection networks. *Ubiquitous Computing & Commun. J.* 5(2) .
29. Xu M, Xu JM and Hou X M (2005) Fault diameter of cartesian product graphs. *Info. Processing Letters*. 93, 245-248.
30. Balakrishnan V K (2007) Schaums Outlines Graph Theory, Fifth Reprint, TMH, 135-137.
31. Amawy AE and Latifi S (1991) Properties and performance of folded hypercubes. *IEEE Trans. Parallel & Distrib. Sys.* 2, 31-42.
32. Louri A and Neocleous C (1997) A spanning bus connected hypercube: A new scalable optical interconnection network for multiprocessors and massively parallel systems. *J. Lightwave Technol.* 15, 1241-1252.
33. Graham SW and Seidel SR (1993) The cost of broadcasting on star graphs and *k*-ary hypercubes. *IEEE Trans. Comput.* 42(6), 756-759.
34. Agrawal N and Ravikumar CP (1996) Fault-tolerant routing in nultiply twisted cube topology. *J. Sys. Architecture*. 42(4), 279-288.