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On Bi-Magic labeling of 4-regular graphs

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Abstract

In this paper we present an algorithm and show that for any finite j, where j ≥ 3 and 2j - 1 ≤ n, there exists a (n, 2n) 4-regular bimagic graph of girth j with magic constants 4n - (j-2) and 5n - (j-2).

AMS Subject classification: 05C78

Keywords: Graph labeling, k-regular graph, girth**.**

1 Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray, Crystallography, radar, astronomy, circuit design, communication networks and data base management (Gallian, 2009). Hence, in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, mean labeling, arithmetic labeling etc., have been studied in over 1100 papers (Gallian, 2009). In particular antimagic labeling for class of digraphs has been studied. Hartsfield and Ringel (1990) made a conjecture on vertex-antimagic labeling and Baca posed a conjecture about edge-antimagic vertex labeling (2001). In particular, super vertex (a,d)-antimagic labeling for digraphs was introduced in (Thirusangu *et al.,* 2009). Moreover, the existence of super vertex (a,d) antimagic labeling and vertex magic total labeling for a certain class of Cayley digraph has been investigated in the literature. The original concept of bi-magic labeling is due to Baskar Babujee in 2004.

Let G be a (n,2n)connected graph of girth j. An injective function f: E → {1, 2, 3,….2n} is called the labeling of G. Any such labeling induces f*: V→N defined by $f^*(v_i) = \sum f(v_i, v_j)$ where v_i is adjacent with v_j. If there exists a labeling whose induced map on V(G) has two constants ,we say that f is a Bi-magic labeling. In this paper we present an algorithm and prove that for any finite j, where j ≥ 3 and 2j - 1 ≤ n, there exists a (n, 2n) 4-regular bimagic graph of girth j with magic constants 4n - (j-2) and 5n - (j-2).

2 Preliminaries

In this section we give the basic notation relevant to this paper. Let G = G (V, E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper we deal with the labeling with domain set of all edges.

Definition2.1: A regular graph is a graph without loop and multiple edges where each vertex has the same degree or valency.

Definition2.2: A regular graph with vertices of degree *k* is called a *k*-regular graph or regular graph of degree *k*.

Definition2.3: A graph G is said to be a vertex magic graph if the edges can be labeled with nonnegative real numbers such that (i) different edges have distinct labels, and (ii) the sum of the labels of edges incident to each vertex is a constant.

Definition2.4: From [2], we have the definition of Bi-magic labeling as follows: A graph G is said to be a bi-magic graph if the sum of the labels on the edges incident at the vertices are k_1, k_2 where k_1& k_2 are constants.

Definition2.5: Let G be a connected graph. The girth of a graph G is defined as the length of smallest cycle in the graph.

3 Main results

In this section, we present an algorithm to get Bi-magic labeling for 4-regular graph of girth j where j ≥ 3

Algorithm 1:

Input: n, the number of vertices of a connected graph with girth j where j = 3.

Begin.

Step 1: V = { v_1 , v_2 , v_3…….v_n}.

Step 2: E = { (v_i ,v_{i+1}),1≤ i ≤ n-1}U{(v_i ,v_{i+2}),1≤ i ≤ n-2}U { (v_n , v_1)}U{(v_{n-1} , v_1)}U{ (v_n , v_2)}.

End.

Output: 4-Regular graph with n vertices and 2n edges of girth 3.

Algorithm 2:

Input: n, the number of vertices of a connected graph with girth j where j = 4.

Begin.

Step 1: V = { v_1 , v_2 , v_3…….v_n}.

Step 2: E = {(v_i , v_{i+1}),1≤ i ≤ n-1}U{ (v_i , v_{i+3}), 1 ≤ i ≤ n-3} U (v_n , v_1) U (v_{n-1} , v_2) U (v_{n-2} , v_1) U (v_n , v_3).

End.

Output: 4-Regular graph with n vertices and 2n edges of girth 4

Definition3.1: The structure of the 4-regular graph with girth 3 is defined as follows. By using step 2 of Algorithm 1 edges are constructed. Thus v_1 is adjacent with v_2, v_3, v_{n-1}, v_n; v_2 is adjacent with v_1, v_3, v_4, v_n ; v_i is adjacent with v_{i-2}, v_{i-1}, v_{i+1}, v_{i+2} where i = 3 to n-2; v_{n-1} is adjacent with v_{n-3}, v_{n-2}, v_n, v_1 and v_n is adjacent with v_{n-2}, v_{n-1}, v_1, v_2. Clearly each vertex is of degree 4. Hence the graph has 2n edges. Thus from the construction, we have a 4-regular graph of girth 3 with n vertices and 2n edges.

Definition3.2: The structure of the 4-regular graph with girth 4 is defined as follows. By using step 2 of algorithm 2, the edges are constructed. Thus the vertex v_1 is adjacent with v_2, v_4, v_{n-2}, v_n; v_2 is adjacent with v_3, v_1, v_5 , v_{n-1}; v_3 is adjacent with v_4, v_2, v_6 , v_n ; v_4 is adjacent with v_5, v_3, v_7 , v_1; v_i is adjacent with v_{i-1}, v_{i+1}, v_{i-3}, v_{i+3} where i = 5 to n-3; v_{n-2} is adjacent with v_{n-1}, v_{n-3}, v_1 , v_{n-5} ; v_{n-1} is adjacent with v_n, v_{n-2}, v_2, v_{n-4} and v_n is adjacent with v_{n-1}, v_1, v_3, v_{n-3} . Clearly each vertex is of degree 4. Hence the graph has 2n edges. Thus we have constructed a 4-Regular graph of girth 4 with n vertices and 2n edges.

Theorem3.1: For every n ≥ 5, there exists a 4-regular (n, 2n)-Bimagic graph of girth 3 with constants 4n-1 and 5n-1.

Proof: From the structure of 4-regular graph of girth j where j = 3, we have n vertices and 2n edges. To prove that for every *n ≥ 5,* there exists a 4-regular graph of girth 3 which admits bi-magic labeling with magic constants 4n-1 and 5n-1. i.e., we have to show that the induced function f*:V → N is defined as $f^*(v_i) = \{\sum f(v_i v_j)$ = either k_1 or k_2, where k_1 & k_2 are constants and $v_i v_j \in E\}$.

Define the map f on E as follows:

Let f: E → {1,2,3……..2n} such that

(i) $f(v_i , v_{i+1}) = i$, $1 \le i \le n-1$;

(ii) $f(v_n, v_1) = n$;

(iii) $f(v_{i+2} , v_i) = 2n - i - 1$, $1 \le i \le n-2$;

(iv) $f(v_1 , v_{n-1}) = 2n$

(v) $f(v_2 , v_n) = 2n-1$.

Thus the Number of labels assigned to edges = (n-1) + 1 + (n-2) +1 + 1 = 2n. In order to get the labels on vertices define the induced map f* on V as f*: V → N is defined such that $f^*(v_i) = \{\sum f(v_i v_j) / v_i v_j \in E\}$. Now for every $v_i \in V$,

$f^*(v_2)$ = 1 + 2 + (2n-3) + (2n-1) [By (iii), (iv)]

$f^*(v_i)$ = [2n - (i-2) - 1] + (i-1) + i + (2n-i-1) where i = 3 to n-2. [By (iii), (i), (iii)]

$f^*(v_1)$ = 1 + (2n-2) + 2n + n [By (iii), (iv), (ii)]

$f^*(v_{n-1})$ = 2n + (n-2) + (n + 2) + (n-1) [By (iv), (i), (iii]

$f^*(v_n)$ = n + (2n-1) + (n-1) + (n +1) [By (ii), (v), (i), (iii)]

Thus $f^*(v_2)=f^*(v_3)$ = …..$=f^*(v_{n-2})$= 4n-1and $f^*(v_1)= f^*(v_{n-1}) = f^*(v_n)$ = 5n-1.

Example3.1: 4- Regular (8, 16) graph of girth 3 admits bi-magic labeling is shown in (Fig.1)

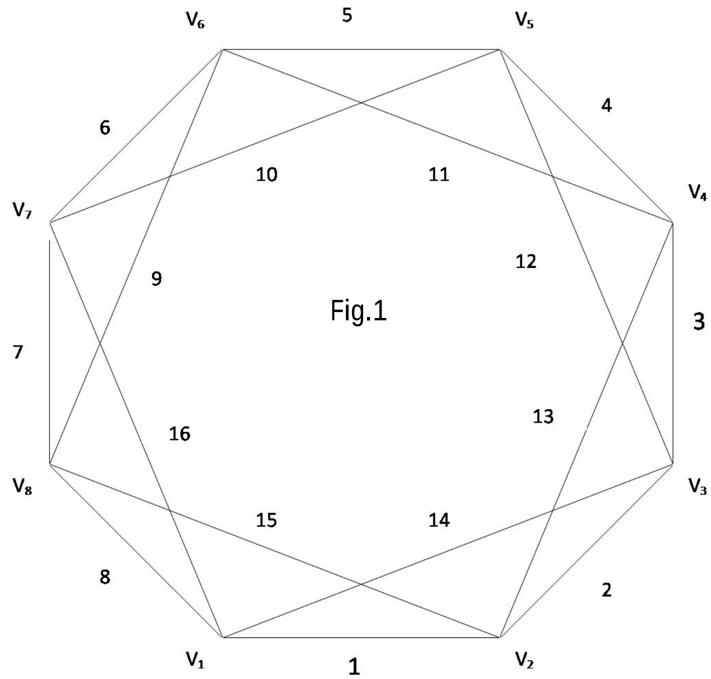

Fig.1

Hence the 4- Regular (8, 16) graph of girth 3 admits bi-magic labeling with magic constants 31 and 39.

Theorem3.2: For every n ≥ 7, there exists a 4-regular (n, 2n) Bi-magic graph of girth 4 with magic constants 4n-2 and 5n-2.

Proof: From the structure of 4-regular graph of girth j where j = 4, we have n vertices and 2n edges. To prove that, for n ≥ 7 with girth j where j = 4 there exists a 4-regular graph which admits bi-magic labeling with magic constants 4n-2 and 5n-2. i.e., we have to show that the induced function f* : V → N is defined as $f^*(v_i) = \{\sum f(v_i v_j)$ = either k_1 or k_2 where k_1 & k_2 are two constants and $v_i v_j \in E\}$.

Define the map f on E as follows:

Let f: E → {1, 2, 3……..2n} such that

(i) $f(v_i , v_{i+1}) = i$, $1 \le i \le n-1$;

(ii) $f(v_n, v_1) = n$;

(iii) $f(v_{i+3} , v_i) = 2n - i - 2$, $1 \le i \le n-3$;

(iv) $f(v_1 , v_{n-2}) = 2n$

(v) $f(v_2 , v_{n-1}) = 2n-1$.

(vi) $f(v_3 , v_n) = 2n-2$.

Thus, Number of labels assigned to edges = (n-1) + 1+ (n-3) +1+1+1= 2n. In order to get the labels on vertices define the induced map f* on V as f*: V→N is defined such that $f^*(v_i) = \{\sum f(v_i v_j) / v_i v_j \in E\}$. Now for every $v_i \in V$,

$f^*(v_i)$ = (i-1) + i + (2n -(i-2)-1)+(2n- i- 2) where i = 2 to n - 3 [By (i), (iii)]

$f^*(v_1)$ = 1 + n + 2n + (2n - 3) [By (ii), (iv), (iii)]

$f^*(v_{n-2})$ = n -2 + n -3 + 2n + n+3. [By (i), (i), (iv), (i)]

$f^*(v_{n-1})$ = (n-1) + (n-2) + (2n-1) + (n+2) [By (i), (v), (iii)]

$f^*(v_n)$ = n + (2n-2) + (n-1) + (n+1) [By (ii), (vi), (i), (iii)]

Thus $f^*(v_2) = f^*(v_3) = \ldots\ldots\ldots = f^*(v_{n-3}) = 4n - 2$ and $f^*(v_1) = f^*(v_{n-2}) = f^*(v_{n-1}) = f^*(v_n) = 5n-2$.

Example3.2: 4-Regular (8, 16) graph of girth 4 admits Bi-magic labeling is shown in (Fig.2).

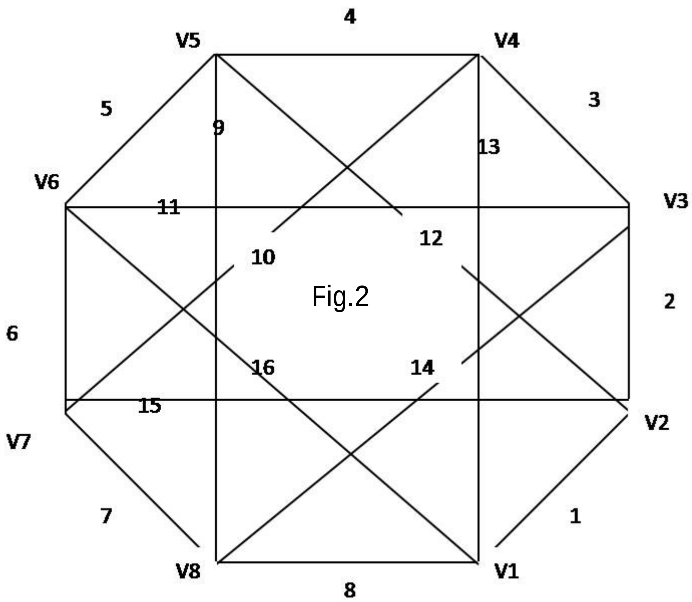

Fig.2

Hence, the 4-Regular (8, 16) graph of girth 4 admits bi-magic labeling with magic constants 30 and 38.

4 Conclusion

From the above two theorems, we conclude that for any finite j, where $j \geq 3$ and $2j - 1 \leq n$, there exists a (n, 2n) 4-regular graph of girth j which admits a bimagic labeling with magic constants $4n - (j-2)$ and $5n - (j-2)$.

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