

Single machine common flow allowance scheduling with a fuzzy rate-modifying activity

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Abstract

The fuzzy scheduling is a new approach presented in this paper. In classic scheduling, it was assumed that the machines are always available. So, no maintenance time was considered for calculations. Also, assumed that all given times are exact and there is no uncertainty, however in the real world accurate calculation is impossible and always faced with uncertainty. Therefore, scheduling calculations would not be accurate and an optimal solution would not be reached. In this study, rate-modifying activity time considered as the fuzzy number and the optimal solution was calculated in uncertainty condition. A novel type of fuzzy scheduling model is presented. A ROG algorithm for ranking fuzzy numbers was introduced and example was solved to show the effectiveness of the proposed approach.

Keywords: Fuzzy algorithm; machine scheduling; job scheduling rate-modifying activity; single machine..

Introduction

Machine scheduling is concerned with the problem of optimality available resources to process jobs. This is a decision making process that exists in most production environments and information processing systems. In the last five decades, many papers have been published in the scheduling area. Most of them deal with problems assuming that machines are always available at constant speed. However, in today's manufacturing applications, it has become very common for a machine to be in subnormal condition after running for a certain period of time. A production planner can decide to stop the machine and fix it, or can wait and fix it later. For convenience, we call the fixing activity maintenance. On the other hand, if the production planners continue to run the machine without fixing it; it is possible that the machine will break down and will have to be repaired immediately. We call this activity a repair. Both maintenance and repair activity can change machine speed from a sub-normal production rate to a normal one. Hence we call both activities *rate-modifying* activities since they can be expected to change the speed of the machine. Motivated by the problem commonly found in the surface-mount technology lines of electronic assembly lines, Lee and Leon (2001) first considered single machine scheduling with a rate-modifying activity. They studied several single machine scheduling problems in this area: minimizing makespan, flow-time, weighted flow-time and maximum lateness. Lodree and Geiger (2010) studies single machine scheduling problem characterized by both time-dependent processing times and a rate-modifying activity. They proved that under certain conditions, the optimal policy is to schedule the rate-modifying activity in the middle of the job sequence. Zhao *et al.* (2009) considered the parallel machine scheduling problem with rate-modifying activities. For the total completion time minimization problem, they provided a polynomial algorithm to solve the problem optimally. Gordon and Tarasevich (2009) considered the single machine common due date assignment and scheduling problem with the possibility to perform a rate-modifying activity for changing the processing times of the jobs

following this activity. The objective is to minimize the total weighted sum of earliness, tardiness and due date costs. Mosheiov and Oron (2006) considered maintenance activity scheduling and due-date assignment simultaneously. They indicated that the problem remains solvable in polynomial time. Lee and Lin (2001) investigated single machine scheduling with maintenance and repair rate-modifying activity.

In the conventional scheduling problem, the parameters such as job processing time, ready times, due-date and rate-modifying activities have been assumed to be deterministic. However, in the real-world situations, these parameters are often encountered with uncertainties. Accordingly, scheduling problems have been mainly branched into two categories: deterministic scheduling and uncertain (stochastic, fuzzy, etc.) scheduling. In facts, various factors involved in the scheduling problems are often imprecise or uncertain in nature when we formulate scheduling problems in the real-world. This is especially true when human-made factors are incorporated into the problems. In these cases, it seems more appropriate to consider fuzzy processing times, fuzzy due-date, fuzzy rate-modifying activities and so on.

So far, much of research work has been performed on fuzzy scheduling problems (Prade, 1979). Ishii *et al.* (1992) first investigated scheduling problems with fuzzy due-dates. Han *et al.* (1994) considered single machine scheduling problem with fuzzy due-dates. Ishibuchi *et al.* (1994) studied flow shop scheduling with fuzzy processing times. The fuzzy job shop scheduling problem was analyzed by Kuroda and Wang (1996) and Lee *et al.* (2002). They used linguistic values to evaluate each criterion (e.g., very poor, poor, fair, good, and very good) and to represent its relative weights (e.g., very unimportant, unimportant, moderately important, important, and very important). Also, a tabu search method is used as a stochastic tool to find the near-optimal solution for an aggregated fuzzy objective function. Chanas and Kasperski (2003) considered two single-machine scheduling problems with fuzzy

processing times and fuzzy due dates. They defined the fuzzy tardiness of a job in a given sequence as a fuzzy maximum of zero and the difference between the fuzzy completion time and the fuzzy due date of this job. In the first problem, they minimized the maximal expected value of a fuzzy tardiness. In the second one, they considered minimizing the expected value of a maximal fuzzy tardiness. Adamopoulos and Pappis (1996a) presented a fuzzy-linguistic approach to a multi-criteria sequencing problem. They considered a single machine, in which each job is characterized by fuzzy processing times. The objective was to determine the processing times of jobs and the common due dates as well as to sequence the jobs on the machine where penalty values are associated with due dates assigned, earliness, and tardiness. For a recent survey on fuzzy scheduling, the readers are referred to Dubois et al. (2003). In this paper objective is to determine optimal sequence, optimal common flow allowance, the location of a rate-modifying activity and the same time to minimize a total penalty function based on earliness, tardiness and flow allowance in uncertainty condition. In this paper the method presented by Wang and Wang (2010), Adamopoulos and Pappis (1996 b) and Lee and Leon (2001) considered in fuzzy set theory. As this study represents one of the first fuzzy rate- modifying activities of scheduling, there is very little data with which to directly compare it.

Preliminary problem formulation

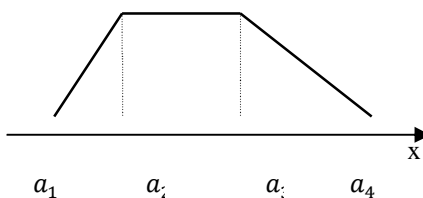
Some basic notations used in this paper are introduced. Also, the problem under consideration is formulated.

Preliminary

The fuzzy numbers are considered in trapezoid form (Zadeh, 1978). Fig.1 shows the trapezoidal fuzzy number of \tilde{A} , the fuzzy number of \tilde{A} is set in $[0, 1]$.a trapezoidal fuzzy number is shown as (a_1, a_2, a_3, a_4) . Membership function of \tilde{A} is defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3} & a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

Fig. 1. Trapezoidal fuzzy number of $\tilde{A} = (a_1, a_2, a_3, a_4)$



$\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two fuzzy numbers, arithmetic operations is calculated by the following equations:

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$\lambda * \tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4), & \lambda > 0 \\ (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1), & \lambda < 0 \end{cases}$$

Fuzzy number ranking

For comparison of fuzzy numbers the Radius of Gyration (ROG) method will presented as follows:

Consider two fuzzy numbers of $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ for comparing these two numbers, first, the number should calculated and then r_x and r_y will be determined and finally, r_A and r_B would be compared and the larger number will be determined.

$$Ix_1 = \frac{1}{12} (a_2 - a_1) w^3$$

$$Iy_1 = \frac{1}{36} (a_2 - a_1)^3 w + \left[\frac{(a_2 - a_1)w}{2} \right] \left(a_1 + \frac{2}{3} (a_2 - a_1) \right)^2$$

$$Ix_2 = \frac{1}{3} (a_3 - a_2) w^3$$

$$Iy_2 = \frac{1}{12} (a_3 - a_2)^3 w + (a_2 - a_3) w \left(\frac{a_2 + a_3}{2} \right)^2$$

$$Ix_3 = \frac{1}{12} (a_4 - a_3) w^3$$

$$Iy_3 = \frac{1}{36} w (a_3 - a_4)^3 + \left(\frac{2a_3 + a_4}{3} \right)^2 \left(\frac{(a_4 - a_3)w}{2} \right)$$

$$r_x = \sqrt{\frac{Ix_1 + Ix_2 + Ix_3}{(a_3 - a_2) + (a_4 - a_1)w}}$$

$$r_y = \sqrt{\frac{Iy_1 + Iy_2 + Iy_3}{(a_3 - a_2) + (a_4 - a_1)w}}$$

$$R = \sqrt{(r_x)^2 + (r_y)^2}$$

Since the fuzzy number was used in this paper is normal, so $w=1$. The comparison between two fuzzy numbers is as follows:

If $R_a < R_b$ then $\tilde{A} < \tilde{B}$

If $R_a = R_b$ then $\tilde{A} = \tilde{B}$

If $R_a > R_b$ then $\tilde{A} > \tilde{B}$

Problem formulation

Notations in this paper are described as follows:

- n: the number of jobs to be scheduled.
- [j]: job in the jth position in any sequence.
- \tilde{p}_j : the normal processing time of job J_j .
- δ_j : the modifying rate of job J_j .
- \tilde{q} : a non-negative number representing the common flow allowance.
- \tilde{d}_j : the due date for job J_j .
- \tilde{c}_j : the completion time of job J_j .

\tilde{w}_j : the waiting time of job J_j , that is $w_j = c_j - 1$

$\tilde{E}_j = \max \{0, d_j - c_j\}$: earliness for job J_j .

$\tilde{T}_j = \max \{0, c_j - d_j\}$: tardiness for job J_j .

α : per unit time earliness penalty for each job.

β : per unit time tardiness penalty for each job.

γ : per unit time common flow allowance penalty for each job

\tilde{t} : rate - modifying activity fuzzy time

Here, we define the considered problem, and formulate a new fuzzy method based on just-in-time scheduling problem where preemption is not allowed. The proposed model deals with the optimal sequence, the optimal common flow allowance and the location of rate modifying activities to minimize a total penalty function based on earliness, tardiness and common flow allowance.

The problem can be stated as follows: A set of n independent jobs $\{j_1, j_2, \dots, j_n\}$ has to be schedule on a single machine which can handle at most one job at a point time. The machine is assumed to be simultaneously available for processing time. The scheduler has an option to schedule a maintenance activity which last \tilde{t} unit of time and during maintenance no production is performed.

There is a due date \tilde{d}_j for each job j_j which is to be processed on a single machine. Each of the n jobs is available for processing at time zero and has a determined processing time \tilde{p}_j if the job is processed prior to the maintenance activity, and $\delta_j \tilde{p}_j$ ($0 < \delta_j \leq 1$) if schedule after it, where δ_j is the rate modifying of job j_j ($j = 1, 2, \dots, n$). The due date \tilde{d}_j for job j_j is equal to processing time plus the common flow allowance, if the job is processed before the maintenance activity, and $\tilde{d}_j = \delta_j \tilde{p}_j + \tilde{q}$ if it is schedule after it.

In this paper $\tilde{p}_j, \tilde{q}, \tilde{d}_j, \tilde{c}_j, \tilde{w}_j, \tilde{E}_j, \tilde{T}_j$ and \tilde{t} considered as the trapezoidal fuzzy numbers, so according to Wang and Wang (2010) the problem modeling modifies as follows:

The objective function is to make a decision on when to schedule the rate-modifying activities and the sequence of jobs on each machine to minimize $\tilde{Z} = \sum_{j=1}^n (\alpha \tilde{E}_j + \beta \tilde{T}_j + \gamma \tilde{q})$. As in lee and leon (2001) the completion or waiting times of jobs are as follows:

$$\tilde{C}_j = \tilde{C}_{j-1} + \tilde{P}_j, \quad j = 1, 2, \dots, l,$$

$$\tilde{C}_j = \tilde{C}_{j-1} + \tilde{t} + \delta_j \tilde{P}_j, \quad j = i + 2$$

$$\tilde{C}_j = \tilde{C}_{j-1} + \delta_j \tilde{P}_j, \quad j = i + 2, \dots, n.$$

$$\tilde{C}_0 = 0, i \in Z^+, \quad 0 \leq i \leq n.$$

If $\tilde{C}_j \geq \tilde{d}_j$ then $\tilde{C}_{j+1} \geq \tilde{d}_{j+1}$, also if $\tilde{C}_j \leq \tilde{d}_j$ then $\tilde{C}_{j-1} \leq \tilde{d}_{j-1}$.

$$\tilde{q} = \tilde{C}_{k-1} = \tilde{W}_k$$

The total cost is given by:

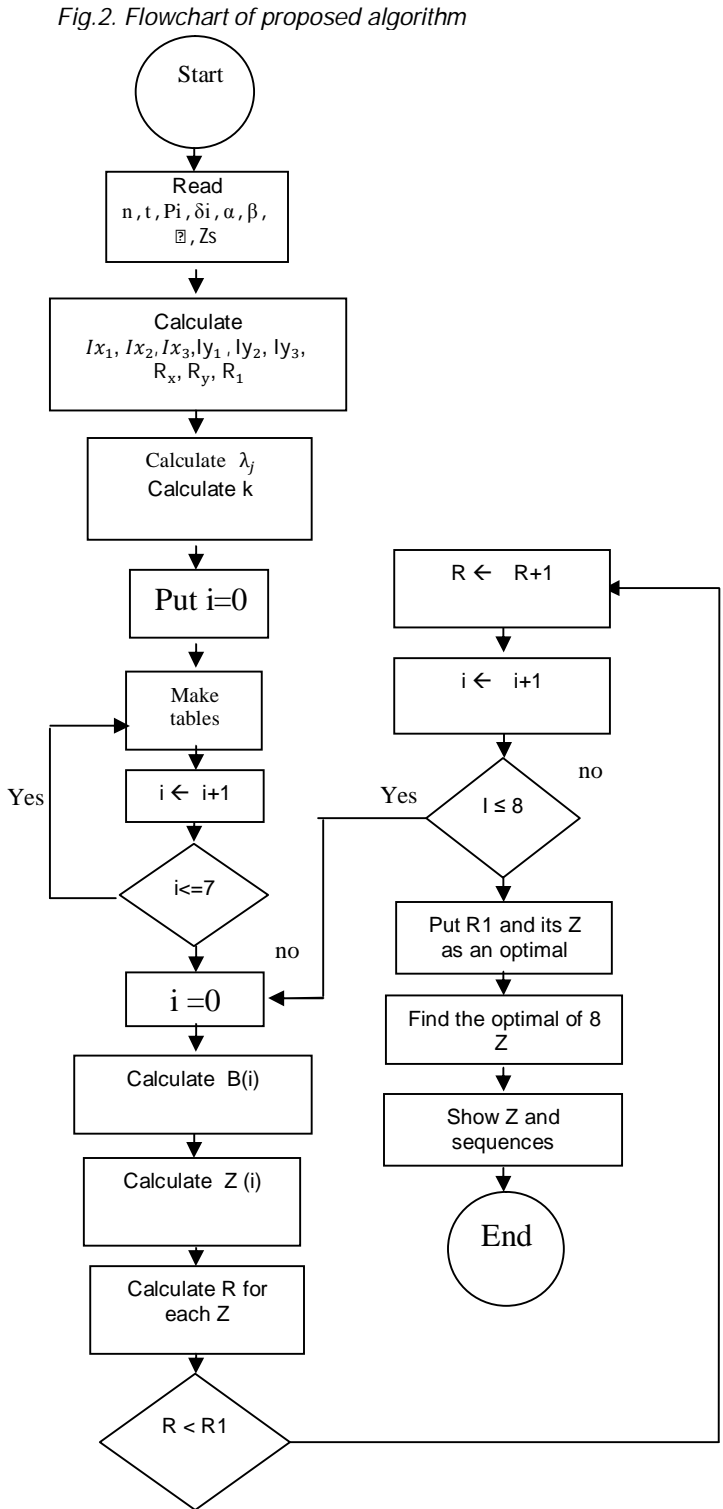
$$\tilde{Z} = \sum_{j=1}^n \tilde{Z}_j + \tilde{Z}_q \rightarrow \tilde{Z} = A\Delta + B$$

Where $A = \alpha(k-1) - \beta(n-k+1) + n\gamma$ and $B = \alpha(\sum_{j=1}^i j\tilde{P}_j + i\tilde{t} + \sum_{j=i+1}^{k-2} j\delta_j\tilde{P}_j) + \beta \sum_{j=k-1}^n (n-j)\delta_j\tilde{P}_j + n\gamma(\tilde{P}_1 + \tilde{P}_2 + \dots + \tilde{P}_i + \tilde{t} + \delta_{i+1}\tilde{P}_{i+1} + \dots + \delta_{k-2}\tilde{P}_{k-2})$

$$\tilde{q} = \tilde{C}_{k-1} = \tilde{W}_k, \text{ where } k = \left\lceil \frac{n(\beta-\gamma)}{\alpha+\beta} \right\rceil$$

For different values of k the cost of assigning job j in position m , λ_{jm} calculated as follow:

If $i < k$ then:



$\lambda_{jm} = n\gamma + m\alpha, \quad m = 1, 2, \dots, i,$
 $\lambda_{jm} = n\gamma\delta_j + m\alpha\delta_j, \quad m = i+1, i+2, \dots, k-i,$
 $\lambda_{jm} = \beta(n-m)\delta_j, \quad m = k, k+1, \dots, n.$ and,
 $\tilde{Z} = \alpha \sum_{j=1}^n \tilde{E}_j + \beta \sum_{j=1}^n \tilde{T}_j + n\gamma\tilde{q} = \alpha \sum_{j=1}^i j\tilde{P}_j + i\alpha\tilde{t} + \alpha \sum_{j=i+1}^{k-1} j\delta_j\tilde{P}_j + \beta \sum_{j=k}^n (n-j)\delta_j\tilde{P}_j + n\gamma(\tilde{P}_1 + \tilde{P}_2 + \dots + \tilde{P}_i + \tilde{t} + \delta_{i+1}\tilde{P}_{i+1} + \dots + \delta_{k-1}\tilde{P}_{k-1})$
 If $i=0$ then

$$\tilde{Z}(0) = \sum_{j=1}^n \lambda_j \delta_j \tilde{P}_j + n\gamma \tilde{t}$$

Where

$$\lambda_j = \begin{cases} n\gamma + j\alpha & \text{for } j = 1, 2, \dots, k-1 \\ \beta(n-j) & \text{for } j = 1, 2, \dots, k-1 \end{cases}$$

If $i > k$ then

$$\lambda_{jm} = n\gamma + m\alpha$$

$$\lambda_{jm} = \beta(n-m)$$

$$\lambda_{jm} = \beta(n-m)\delta_j \text{ and also } \tilde{Z} \text{ calculated as:}$$

$$\begin{aligned} \tilde{Z} &= \alpha \sum_{j=1}^n \tilde{E}_j + \beta \sum_{j=1}^n \tilde{T}_j + n\gamma \tilde{q} \\ &= \alpha \sum_{j=1}^{k-1} j\tilde{P}_j + (n-i)\beta\tilde{t} + \beta \sum_{j=k}^i (n-j)\tilde{P}_j + \beta \sum_{j=i+1}^n (n-j)\delta_j\tilde{P}_j \\ &\quad + n\gamma(\tilde{P}_1 + \tilde{P}_2 + \dots + \tilde{P}_{k-1}) \end{aligned}$$

Finally for $i=n$, the objective function and λ_{jm} are given by

$$\lambda_j = \begin{cases} n\gamma + j\alpha & \text{for } j = 1, 2, \dots, k-1 \\ \beta(n-j) & \text{for } j = 1, 2, \dots, k-1 \end{cases}, Z = \sum_{j=1}^n \lambda_j \tilde{P}_j$$

The flowchart of the proposed algorithm for the full fuzzy process procedures mentioned above is illustrated in Fig.2.

Numerical example

A numerical example is presented to show the effectiveness of the proposed approach.

Fuzzy number of \tilde{p}_j which $j=7$ considered as:

$$\tilde{p}_1 = (2, 3, 4, \text{ and } 5), \tilde{p}_2 = (3, 4, 5, 6), \tilde{p}_3 = (5, 6, 7, 8), \tilde{p}_4 = (8, 9, 9, 10), \tilde{p}_5 = (13, 14, 14, 15),$$

$$\tilde{p}_6 = (15, 18, 19, 21), \tilde{p}_7 = (17, 20, 21, 23),$$

$$\delta_1 = 0.6, \delta_2 = 0.4, \delta_3 = 0.7, \delta_4 = 0.8,$$

$$\delta_5 = 0.4, \delta_6 = 0.5, \delta_7 = 0.2,$$

$$\alpha = 11, \beta = 18, \gamma = 5,$$

$$k = [(n(\beta - \gamma)) / (\alpha + \beta)] \rightarrow k = [(7(18 - 5)) / (11 + 18)] = 4$$

$$\lambda_j = (46, 57, 68, 54, 36, 18, 0)$$

In Table 1 the cost of scheduling jobs in all positions are given ($i=1$).

The fuzzy numbers in the table are the result of multiplication $\lambda_{jm} \tilde{p}_j (\lambda_{jm} \delta_j \tilde{p}_j)$. The bold numbers shown the optimal sequence, and $Z = (2200, 2414.6, 2532.8, 2736.6)$. For solving this assignment problem the ROG method combined with the Hungarian method. For ranking the fuzzy numbers in Table 1, R must be calculated. Because of prevent the repetition, R calculated for one of the fuzzy cost number in Table 1. The fuzzy number of cost of assigning the job in 1th position (j_1) in sequence 1, is (55.2, 82.8, 110, 138), R is calculated as below:

$$\begin{aligned} Ix_1 &= \frac{1}{12} (82.8 - 55.2) \\ Iy_1 &= \frac{1}{36} (82.8 - 55.2)^3 \\ &\quad + \left[\frac{(82.8 - 55.2)}{2} \right] \left(55.2 + \frac{2}{3} (82.8 - 55.2) \right)^2 \end{aligned}$$

$$Ix_2 = \frac{1}{3} (110 - 82.8)$$

$$Iy_2 = \frac{1}{12} (110 - 82.8)^3 + (82.8 - 110) \left(\frac{110 + 82.8}{2} \right)^2$$

$$Ix_3 = \frac{1}{12} (138 - 110)$$

$$Iy_3 = \frac{1}{36} (110 - 138)^3 + \left(\frac{2 * 110 + 138}{3} \right)^2 \left(\frac{138 - 110}{2} \right)$$

$$r_x = \sqrt{\frac{Ix_1 + Ix_2 + Ix_3}{(110-82.8) + (138-55.2)}} = 0.499$$

$$r_y = \sqrt{\frac{Iy_1 + Iy_2 + Iy_3}{(110-82.8) + (138-55.2)}} = 98.142$$

$$R = \sqrt{(r_x)^2 + (r_y)^2} = 98.143.$$

The results of our computational testing are summarized in Table 2. It can be seen that algorithm finds the best sequence and results for our fuzzy objective function Z .

Table 1. Fuzzy cost job J_i in position m ($i=1$).

| m | 1 | | | | 2 | | | | 3 | | | | 4 | | | |
|-------|------------|------------|------------|------------|-------------|------------|------------|------------|-------------|------------|------------|------------|--|------------|------------|------------|
| j_1 | 55.2 | 82.8 | 110 | 138 | 68.4 | 103 | 137 | 171 | 81.6 | 122 | 163 | 204 | 64.8 | 97.2 | 130 | 162 |
| j_2 | 55.2 | 73.6 | 92 | 110 | 68.4 | 91.2 | 114 | 137 | 81.6 | 109 | 136 | 163 | 64.8 | 86.4 | 108 | 130 |
| j_3 | 161 | 193 | 225 | 258 | 200 | 239 | 279 | 319 | 238 | 286 | 333 | 381 | 189 | 227 | 265 | 302 |
| j_4 | 294 | 331 | 331 | 368 | 365 | 410 | 410 | 456 | 435.2 | 490 | 490 | 544 | 346 | 389 | 389 | 432 |
| j_5 | 239 | 258 | 258 | 276 | 296 | 319 | 319 | 342 | 353.6 | 381 | 381 | 408 | 281 | 302 | 302 | 324 |
| j_6 | 345 | 414 | 437 | 483 | 428 | 513 | 542 | 599 | 510 | 612 | 646 | 714 | 405 | 486 | 513 | 567 |
| j_7 | 156 | 184 | 193 | 212 | 194 | 228 | 239 | 262 | 231.2 | 272 | 286 | 313 | 184 | 216 | 227 | 248 |
| | 5 | | | | 6 | | | | 7 | | | | According to ROG method and Table 2 the optimal Z is | | | |
| | 43.2 | 64.8 | 86.4 | 108 | 21.6 | 32.4 | 43.2 | 54 | 0 | 0 | 0 | 0 | $Z_0 = (1812, 2001.8, 2106.2, 2285.2)$ with sequences of ($j_3 j_1 j_2 j_7 j_5 j_4 j_6$). | | | |
| | 43.2 | 57.6 | 72 | 86.4 | 21.6 | 28.8 | 36 | 43.2 | 0 | 0 | 0 | 0 | Conclusion | | | |
| | 126 | 151 | 176 | 202 | 63 | 75.6 | 88.2 | 101 | 0 | 0 | 0 | 0 | In this paper, we discussed fully about fuzzy single machine common flow allowance with a rate modifying activity. The rate-modifying activity time, normal processing time, and also completion and | | | |
| | 230 | 259 | 259 | 288 | 115 | 130 | 130 | 144 | 0 | 0 | 0 | 0 | waiting time of jobs considered as the fuzzy times, so the | | | |
| | 187 | 202 | 202 | 216 | 93.6 | 101 | 101 | 108 | 0 | 0 | 0 | 0 | | | | |
| | 270 | 324 | 342 | 378 | 135 | 162 | 171 | 189 | 0 | 0 | 0 | 0 | | | | |
| | 122 | 144 | 151 | 166 | 61.2 | 72 | 75.6 | 82.8 | 0 | 0 | 0 | 0 | | | | |

Table 2. Objective functions and sequences

| $\bar{Z}_i = (a_1, a_2, a_3, a_4)$ | related sequence |
|--|-------------------------|
| $\bar{Z}_0 = (1812, 2001.8, 2106.2, 2285.2)$ | $j_3j_1j_2j_7j_5j_4j_6$ |
| $\bar{Z}_1 = (2200, 2414.6, 2532.8, 2736.6)$ | $j_3j_1j_2j_7j_5j_4j_6$ |
| $\bar{Z}_2 = (2564.6, 2813, 2954, 3191.6)$ | $j_3j_1j_2j_7j_5j_4j_6$ |
| $\bar{Z}_3 = (2995, 3295.2, 3477, 3766.4)$ | $j_3j_2j_1j_7j_5j_4j_6$ |
| $\bar{Z}_4 = (2727, 3034.8, 3221, 3521.6)$ | $j_4j_2j_1j_3j_7j_5j_6$ |
| $\bar{Z}_5 = (2518.2, 2826, 3008.6, 3312.8)$ | $j_4j_2j_1j_3j_5j_7j_6$ |
| $\bar{Z}_6 = (2205, 2538, 2735, 3050)$ | $j_4j_2j_1j_3j_5j_6j_7$ |
| $\bar{Z}_7 = (1683, 1998, 2195, 2492)$ | $j_4j_2j_1j_3j_5j_6j_7$ |

formulation are calculated in a fuzzy situation and times are presented in the form of trapezoidal fuzzy number. The ROG method was introduced for ranking of fuzzy numbers and the optimal sequence for rate modifying activity was calculated. It is shown that solutions obtained by the proposed method are superior. As a future activity, the fully fuzzy scheduling problem can be solved in determined limitation of weighted sequences.

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