A Novel Method of Equivalencing DFIG based Wind Farm for Stability Studies

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Abstract

Objectives: Analyzing the stability of large scale wind power plant (WPP) is very cumbersome and simulation time required performing the analysis is long-drawn-out. Thus a single equivalent model of WPP is developed. Methods/Statistical analysis: Kron’s based network reduction is proposed for the WPP. Two network reduction methodologies are adopted and wind turbine mechanical and electrical parameters are rescaled which is discussed in detail in the paper. Transient stability analysis is performed to validate the equivalence method. Study system consists of 23 buses with 20 DFIG wind turbine generating units connected to the main grid of utility system. Findings: Transient stability analysis is conducted for the study system. Dynamic response of detailed WPP and proposed equivalent system matches accurately for the three phases to ground fault disturbance at point of common coupling (PCC). At PCC the Voltage, Real power and Reactive power values are very close to proposed equivalent system when compared to detailed WPP. In addition, the percentage error computed is least for the proposed system. Time taken to simulate the equivalent system is very less compared to the detailed WPP and existing literature work. All the simulations are carried out in DIgSILENT Power Factory™ software. Application/Improvements: Remarkable simulation time reduction and accuracy is achieved for the proposed method. Thus, the equivalent method is applied for quick assessment of transient stability of large scale WPP.

Keywords: Dynamics Response, Kron’s Reduction, Large Scale Wind Power Plant, Network Reduction, Transient Stability

1. Introduction

The need for energy demand is growing in a faster way. In developing countries like India, renewable energy plays a key role because of two reasons. Firstly, the country driven to reduce greenhouse gases opting for renewable energy. Secondly the load demand is always greater than generation. Wind energy plays a major role in renewable energy. Thus to analyze the effect of wind energy on to the power system in a simpler way; the entire wind plant with huge number of Wind Turbine Generating (WTG) units represented as a single equivalent. A simple aggregation is investigated in paper1. Where two equivalent wind turbines are developed. One for aggregated wind turbine with same wind speed and other is for different incoming wind speed. Reduced order dynamic equivalent proposed for SCIG based wind farm in paper2, which provide good accuracy with detailed model. DFIG is preferred over fixed speed wind turbine generators as it offers numerous advantages3. Then today comes the era of DFIG wind turbine generating units. They are energy efficient and their operating conditions optimized. Thus, dynamic equivalent formed for wind farm consisting of DFIG wind turbines. Since DFIG consists of number of controllers like pitch controller, real power controller and reactive power controller which posed a great challenge in forming dynamic equivalent. These are challenges addressed and simple aggregation techniques were illustrated in paper4, the order of the aggregated model is reduced by using selective modal analysis. It focuses on the most relevant modes and variables. Further dynamic equivalent is formed using parameter identification5. The model is

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written in the form of deterministic auto regressive moving average. DFIG wind turbines are clustered based on coherency multi equivalent formed and parameters are estimated to ensure stable convergence. A review on different dynamic equivalent methods for large scale system is given in paper. Recently, in paper a steady state equivalent circuit of Doubly Fed Induction Machine (DFIM) is obtained by considering the relative position of rotor with respect to synchronously rotating reference frame and direction of rotor magnetic flux due to rotor current. Change in spatial orientation of RMF is observed when the DFIM is operating at sub-synchronous and super-synchronous mode of operation.

This paper organized in to six sections. Description about each section is as follows.

Section 2 concisely explains the modeling of Wind Turbine Generator System components. Section 3 dealt with wind power plant description under consideration. Section 4 explains dynamic equivalent formation of the wind power plant in detail. The simulation results of detailed plant model and the dynamic equivalent model shown and compared in section 5. Inference and conclusion of this paper explained in section 6.

2. Modelling of Wind Turbine Generation System Components

2.1 Wind Turbine Model

The simple aerodynamic model is used to represent the turbine. The power extracted from the wind can be calculated by the given formula:

\[ P_m = \frac{1}{2} \rho A V_{wind}^3 C_p(\lambda, \theta) \]  

(1)

The power coefficient \( C_p \) is a function of \( \lambda \) (tip speed ratio) and \( \theta \) (pitch angle).

The general functional representation of \( C_p \) is

\[ C_p(\lambda, \theta) = C_1(\frac{\lambda}{\lambda_1} - C_2\lambda - C_3\lambda^2 - C_4\lambda^3)e^{-\frac{\lambda}{\lambda_2}} \]  

(2)

\[ \lambda = \frac{\omega_r R}{V_{wind}} \]  

(4)

The tip speed ratio \( \lambda \) is be maintained at optimum value to extract optimum power from the wind turbine. This is possible only in DFIG wind turbines as wind speed varies, rotor speed can be varied to maintain \( \lambda_{opt} \).

2.2 Modelling of Doubly Fed Induction Generator

A DFIG is a slip ring induction machine where the stator and rotor-circuits are energized. The rotor is fed from an electric source namely rotor side converter. This nature of DFIG enables it to operate at variable speed. Mechanical side and electrical side of the machine is decoupled producing constant frequency when connected to grid.

Three phase stator and rotor is referred to synchronously rotating reference frame with quadrature axis (q axis) leading direct axis (d axis) by 90 degrees. The controller models shown Figure 1 to Figure 3 used in the DFIG model of WTG. Generator convention is used i.e. stator current and rotor current are considered to be positive when they are leaving and entering the machine respectively.

All the expressions are in per unit. The stator and rotor voltage equations are

\[ \frac{1}{\omega_s} \frac{d\psi_{qs}}{dt} = V_{qs} + R_s I_{qs} - \psi_{ds} \]  

(5)

\[ \frac{1}{\omega_s} \frac{d\psi_{ds}}{dt} = V_{ds} + R_s I_{ds} + \psi_{qs} \]  

(6)

\[ \frac{1}{\omega_s} \frac{d\psi_{qr}}{dt} = V_{qr} - R_s I_{qr} - \frac{(\omega_r - \omega_s)}{\omega_s}\psi_{dr} \]  

(7)

\[ \frac{1}{\omega_s} \frac{d\psi_{dr}}{dt} = V_{dr} - R_s I_{dr} + \frac{(\omega_r - \omega_s)}{\omega_s}\psi_{qr} \]  

(8)

The stator and rotor flux linkage equations are

\[ \psi_{qs} = -X_s I_{qs} + X_m I_{qs} \]  

(9)

\[ \psi_{ds} = -X_s I_{ds} + X_m I_{dr} \]  

(10)

\[ \psi_{qr} = -X_m I_{qr} + X_s I_{qs} \]  

(11)

\[ \psi_{dr} = -X_m I_{ds} + X_s I_{dr} \]  

(12)

The Tenth order model of DFIG is given by
\[ \frac{d\phi}{dt} = \frac{1}{L_s} \left[ I_a - I_{ref} - \frac{R_s}{L_s} I_a \right] \]
\[ \frac{d\theta}{dt} = \frac{1}{\lambda_s} \left[ \frac{1}{L_s} I_a - \frac{R_s}{L_s} I_a - \frac{1}{L_s} I_a \right] \]  

\[ \frac{d\phi}{dt} = \frac{1}{L_s} \left[ I_a - I_{ref} - \frac{R_s}{L_s} I_a \right] \]
\[ \frac{d\theta}{dt} = \frac{1}{\lambda_s} \left[ \frac{1}{L_s} I_a - \frac{R_s}{L_s} I_a - \frac{1}{L_s} I_a \right] \]  

\[ \frac{dx_1}{dt} = K_{1f}(P_{ref} - P_{gen}) \]
\[ \frac{dx_2}{dt} = K_{12}(K_{p1}P_{ref} - P_{gen}) + x_1 - I_{qr} \]
\[ \frac{dx_3}{dt} = K_{13}(Q_{ref} - Q_{gen}) \]
\[ \frac{dx_4}{dt} = K_{14}(K_{p3}Q_{ref} - Q_{gen}) + x_3 - I_{dr} \]
\[ \frac{dx_5}{dt} = K_{15}(\omega_r - \omega_{ref}) \]
\[ \frac{dx_6}{dt} = x_5 - x_6 - \theta + K_{p}(\omega_r - \omega_{ref}) \]
\[ \frac{d\theta}{dt} = x_6 \]

\[ V_{wind}(t) = V_a + V_r(t) + V_g(t) + V_l(t) \]

Gust and ramp component are combined and defined as a variable wind component (it is also called gust). Therefore, an average value and a variable component are used to represent the wind speed profile at every WTG.

### 3. Detailed Wind Power Plant

The wind power plant considered in this paper consists of 20 DFIG units connected to utility system at the point of common coupling. The total installed capacity of the wind power plant is 20 MW. Identical DFIG units are considered. The WPP network of 24 bus system is given in Appendix Figure 4. DFIG wind generators are arranged in 5 rows. Non-uniform distribution of wind speed is considered. The generators located in first row receive higher wind speed and the wind speed decreases slightly as we move across the rows.

Electrical and mechanical parameters of the wind power plant are given in paper. The single line diagram of the entire wind power plant as shown in Appendix Figure 4 and the single equivalent is given in Figure 5.

### 4. Procedure for Dynamic Equivalent Formation

For forming single equivalent model, the following simple three step procedure is adopted.

**Figure 1.** Rotor speed controller.

**Figure 2.** Reactive power controller.

**Figure 3.** Pitch angle controller.

**Figure 4.** The Single line diagram of the detail wind power plant.

**Figure 5.** Reduced equivalent system.
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- Equivalent mechanical parameters are obtained for equivalent WTGS.
- Equivalent electrical and controller parameters are obtained for equivalent WTGS.
- Two network reduction methodologies are used.

### 4.1 Equivalent Mechanical Parameters

The total mechanical power is given by

\[ P_{me} = \sum_{i=1}^{ng} \sum_{j=1}^{2} \frac{1}{2} \rho A_{wi} C_{p} \left( \lambda_{wi}^{i}, \theta_{wi}^{i} \right) v_{wind,i}^{3} \]  \hspace{1cm} (24)

Where \( ng \) is the number of WTGs in the wind power plant. The blade length of equivalent wind turbine is same as that of the individual wind turbine.

Then the equivalent wind power becomes

\[ P_{me}^e = \frac{1}{2} \rho A_{wi} C_{p} \left( \lambda_{e}, \theta_{e} \right) \sum_{i=1}^{ng} v_{wind,i}^{3} \]  \hspace{1cm} (25)

The equivalent wind speed is given by

\[ V_{wind}^e = \sqrt{\frac{\sum_{i=1}^{ng} v_{wind,i}^{3}}{ng}} \]  \hspace{1cm} (26)

Thus the equivalent angular speed, \( \omega_{e} \), is defined in the same speed range as the angular speed of the individual turbines.

\[ T_{me}^e = n_{g} \left[ \frac{1}{2} \rho R_{f}^{2} \frac{\lambda_{e}}{\lambda_{r}} \frac{\theta_{e}}{\theta_{r}} \frac{v_{wind}^{3}}{w_{r}^{3}} \right] \]  \hspace{1cm} (27)

\[ T_{me}^e = B_{e} \left[ w_{r}^{3} \frac{\lambda_{e}}{\lambda_{r}} \frac{\theta_{e}}{\theta_{r}} \frac{v_{wind}^{3}}{w_{r}^{3}} \right] \]  \hspace{1cm} (28)

Where \( B_{e} \) is the torque parameter for calculating either mechanical torque or power of a WTG.

The equivalent generator inertia constant \( H_{e} \) is given by

\[ H_{e} = \sum_{i=1}^{ng} H_{i} \]  \hspace{1cm} (29)

The equivalent generator parameters in p.u (100 MVA Base) are given in Table 1.

### Table 1. WTGS parameters of simple equivalent system

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values in p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator reactance((X_s))</td>
<td>0.1777</td>
</tr>
<tr>
<td>Magnetizing reactance((X_m))</td>
<td>0.1754</td>
</tr>
<tr>
<td>Stator resistance((R_s))</td>
<td>0.0050</td>
</tr>
<tr>
<td>Rotor resistance((R_r))</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

### 4.2 Equivalent electrical and controller Parameters

Voltage magnitude of equivalent is same as that of the individual wind turbine. Current magnitude is \( ng \) times larger.

\[ T_{g} = \sum_{i=1}^{ng} T_{gi} = V_{di}^{*} \sum_{i=1}^{ng} I_{di} + E_{qg}^{'} \sum_{i=1}^{ng} I_{qi} \]  \hspace{1cm} (30)

\[ T_{g} = E_{qg}^{'} I_{dq} + E_{qg}^{'} I_{qs} \]  \hspace{1cm} (31)

where

\[ I_{dq} = \sum_{i=1}^{ng} I_{di} \]  \hspace{1cm} (32)

The total power injected to rotor in the equivalent model is

\[ T_{r} = V_{qr}^{*} \sum_{i=1}^{ng} I_{qr} + V_{dr}^{*} \sum_{i=1}^{ng} I_{dr} \]  \hspace{1cm} (33)

\[ T_{r} = V_{qr}^{*} I_{qr} + V_{dr}^{*} I_{dr} \]  \hspace{1cm} (34)

All other electrical parameters are scaled as follows.

\[ X_{m} = \frac{X_{m}}{ng} \]  \hspace{1cm} (35)

\[ R_{e} = \frac{R_{e}}{ng} and R_{e}^{'} = \frac{R_{e}^{''}}{ng} \]

The active and reactive power reference for the equivalent controller

\[ P_{ref}^{e} = \sum_{k=1}^{n_{g}} P_{ref,k} = C \sum_{k=1}^{n_{g}} \omega_{e}^{3} = C' \omega_{e}^{3} \]  \hspace{1cm} (36)

\[ Q_{ref}^{e} = \sum_{k=1}^{n_{g}} Q_{ref,k} = n_{g} C_{ref}^{e} \]  \hspace{1cm} (37)

The following parameters are magnified by number of generators \((ng)\) times

\[ P_{ref}^{e} = \sum_{g} P_{ref,g}^{e} = \sum_{g} K_{pg} \omega_{e}^{3} \]  \hspace{1cm} (38)

\[ Q_{ref}^{e} = \sum_{g} Q_{ref,g}^{e} = \sum_{g} K_{qg} \omega_{e}^{3} \]  \hspace{1cm} (39)

All other parameters remain equal to those used for the individual turbines.
4.3 Network Reduction Methodology
The Network consisting of 23 buses is reduced to 3 buses by network reduction. The equivalent impedance (R + j X) is connected to bus 2 and the single equivalent generator is connected to bus 3. Two Network reduction methodology is proposed in this paper.

4.4 Method 1-Simple method
The concept developed in the paper is based on the conservation of real power consumed and reactive power consumed/generated by the collector systems. Since Identical DFIG’s are used, the current which is injected is the same.

\[ Z_s = \sum_{m=1}^{n} m^2 Z_m \]  

Where \( Z_m \) represents the individual series impedances. Similarly, the same equation can be used for the entire power plant. It is computed by using the total losses in the collector system. This method is clearly explained in paper 12.

\[ \sum_{k=1}^{1} \sum_{m=1}^{n} m^2 Z_m \]  

4.5 Method 2-Kron’s Reduction
Kron’s Reduction is used for Network reduction of entire power plant. The procedure for the network reduction is given as follows

- Admittance Matrix \([Y]\) for the given wind power plant is obtained. It includes bus1 to bus 23. The dimension of the Y matrix is 23*23.
- Kron’s reduction is applied to retain bus 3 to get equivalent impedance.
- The reduction formula is given by

\[ R_s + jX_s = \frac{1}{[Y_{RR}^{-1} - [Y_{RE}^{-1}][Y_{EE}^{-1}][Y_{ER}^{-1}]]} \]  

Where \( Y_{RE} \) dimension is 1*1 which corresponds to retained bus after Kron’s reduction. \( Y_{EE} \) Dimension is 1*E sub matrix of \([Y]\) where E is the number of buses eliminated after Kron’s reduction. \( Y_{RE} \) transpose of \( Y_{ER} \). Dimension is E*E.

5. Dynamic Simulation

### Comparison Results

The detailed, existing and proposed equivalent systems are simulated using DIgSILENT PowerFactory software13. The state variables are initialized by power flow analysis. Balanced, positive sequence, AC power flow method is used for initialization. The voltage dependency of loads is considered. The network is represented as balanced and positive sequence. The step size for the electromechanical transients is 0.001 sec.

A three phase fault is applied at bus 2 at 1 sec for both the detailed system and equivalent system. The fault is cleared at 1.1 sec. The dynamic response of wind Power plant under a grid disturbance is plotted. The variation of voltage, real power and reactive power and current, when fault occurs at bus 2 are also discussed.

It is observed from the Figures 6 to Figure 10, the dynamic response of detailed system and equivalent systems match accurately. But the Figure 10 the proposed equivalent is not following the speed of detailed system because proposed equivalent single DFIG machine. To get a better dynamic response of proposed system, the combined machine and turbine inertia has to be optimized. Whenever a 3 phase fault occurs at bus 2, the voltage fall reach the wind turbines, Figure 6 real power generated reduces but mechanical power remains the same hence the machine accelerates. When the voltage recovers, as the speed of rotation is higher, active power tends to be slightly higher. The proposed equivalent system proved more accurate than the existing system.

Figure 6. Real power at PCC.

Figure 7. Reactive power at PCC.
The study is extended to line to line and single phase to ground faults. The real power (P), reactive power (Q) and voltage (V) of proposed, existing system are compared with detailed system as shown in Appendix Table 2. This clearly shows that proposed system is much closer to detail system.

It is inferred from Table 3 (given in Appendix) that time required to simulate equivalent system is very less compared to the detailed system. Processor – Intel® core™ i3 -4005U CPU @ 1.70 GHz is used for this simulation.

<table>
<thead>
<tr>
<th>Simulation Test cases</th>
<th>Detail system</th>
<th>Existing Equivalent</th>
<th>Proposed Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three phase to ground fault</td>
<td>216 sec</td>
<td>8.34 sec</td>
<td>8.34 sec</td>
</tr>
<tr>
<td>Line to Line fault</td>
<td>213 sec</td>
<td>9.52 sec</td>
<td>9.51 sec</td>
</tr>
<tr>
<td>Single phase to ground fault</td>
<td>242 sec</td>
<td>10.06 sec</td>
<td>10.04 sec</td>
</tr>
</tbody>
</table>

Table 2. Comparison between Detail system, Existing system and Proposed system

<table>
<thead>
<tr>
<th>Fault Cases at bus 2</th>
<th>%error(detail vs existing)</th>
<th>% error (detail vs existing)</th>
<th>P,Q,V values at PCC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P(MW)</td>
<td>Q(Mvar)</td>
</tr>
<tr>
<td>Three phase fault</td>
<td></td>
<td>0.1309</td>
<td>2.78</td>
</tr>
<tr>
<td>Line-Line fault</td>
<td></td>
<td>0.0811</td>
<td>0.6745</td>
</tr>
<tr>
<td>Single-phase fault</td>
<td></td>
<td>0.189</td>
<td>0.673</td>
</tr>
</tbody>
</table>

Figure 8. Voltage at PCC.

Figure 9. Current at PCC.

Figure 10. DFIG rotor speed.
6. Conclusion

A method of forming dynamic equivalent of entire WPP is proposed in this paper. The transient stability performance of detailed power plant and the equivalent system is almost the same which validates our equivalence approach. Simulation time required to perform transient stability analysis is very less for proposed equivalent system than the detailed WPP system.

7. References