Cooperative Processing of Measurements in Pseudorange Radio-technical Finding System

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Abstract

Background/Objectives: The relevance of the current research is determined by the intensive development of multilateration systems which are widely used in time-positioning support systems. The authors propose to use the cooperative processing of coordinate information. Methods: In the present article a procedure of obtaining the ranging position lines with regard to each station is studied using a theory of redundant measurements which enables to implement cooperative processing. It is shown that using the least squares method, one can obtain the estimation of the required range-measurement parameters containing a minimum value of mean-square error. Findings: In our opinion, the new results of the study show that a multilateration system with one query station which forms a range line of position and with cooperative processing of Time Difference Of Arrival (TDOA) and Time Of Arrival (TOA), in some cases it is characterized by the improved accuracy compared to hyperbolic and elliptic systems. Measurement processing method studied in the present article enables to expand a system work area with respect to the accuracy for multilateration systems with short length base (distances between receiver stations). The offered procedures of radar data processing allow for a formation of range position lines that are indirectly obtained based on hyperbolic and elliptic lines of position. Cooperative processing of measurements in TOA and TDOA finding systems enables to improve the accuracy of the indirect range measurement. TDOA and TOA measurements do not improve the accuracy of the indirect range estimation with the offered method of measurement processing since the accuracy remains constant regardless of the number of stations. Application/Improvements: The study results obtained could be used to upgrade the existing systems of aircraft position estimation and to make reasonable technical demands on the systems under development.

Keywords: Cooperative Processing, Geometric Dilution of Precision (GDOP), Range, Range–Difference, Total Range

1. Introduction

Multilateration system of aircraft position estimation is a multistatic passive (or active-passive) radio-technical system which normally consists of one transmitter (query) station and several spaced receiver stations connected with a processing station via radio or wire lines. Multilateration systems use hyperbolic lines of position based on the measurement of time difference of arrival of a signal transmitted by the transponder-equipped aircraft and vehicles.

Aircraft position estimation accuracy improvement techniques investigation remains a relevant scientific-and-engineering as well as a practical problem in radio-location on the solution of which the efficiency of the air traffic control and surveillance systems depends directly. The importance of this problem is studied in [1,2] with respect to multilateration systems of different types. Multilateration system is described in the article [3] where the use of Kalman filter for non-maneuvering target is proved. In the research work [4] a multistatic system used for estimating the position of the aircraft on an airport surface is studied and the results of a simulation-statistical modeling of GDOP in the conditions of a direct vision and absence of the latter are given. The method of integration of the coordinate information from two bistatic...
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systems in a multistatic surveillance system is offered in 3. The results of the experimental use for a cooperative processing of data from several bistatic radars in a sophisticated multistatic surveillance system are shown. The use of multistatic radio-location systems for estimation of the aircraft position is studied in 4. As one can see from the present research work, the use of positioning in 3D coordinate system might result in degeneration of GDOP and consequently, in comparatively low positioning accuracy. The accuracy of the algorithms used in multistatic radio-location systems can be significantly improved by using target pressure altitude.

It is emphasized in 7 that due to a special geometrical factor in wide area multistatic radio-location systems, position estimation of a target being out of polygon, formed by the ground position of the system receiver stations, is of a special interest. For the purpose of simplifying the operation, the measurements of time difference of arrival are not considered and only the time of arrival is used. Field tests were performed using the developed algorithms. The above tests demonstrated the advantages of the presented algorithm offered. A practical method for compensation of measurement errors, which occur in multistatic radio-location systems, is offered and described in the article 8. Two methods of estimation of the time measurement of signal arrival using the measurements of a previous step are offered. Kalman filter method is offered as well at which the previous measurements are processed and due to this, these measurements have less value compared to the new measurements when making the estimate. A metaheuristic, non-deterministic optimization algorithm 9 is proposed for calculation of dilution of precision in multilateration systems. In 10 Cramer-Rao estimation is analyzed with respect to multistatic radio-location systems at using the measurements of different types which are non-linearly connected with target motion parameters. The time-difference measurements of signal arrival are analyzed, i.e. range measurements, direction finding considering Doppler frequency shift, measurements of frequency difference of signal arrivals and measurements of other types including angular measurements and phase difference measurements and/or other non-power parameters in their various combinations.

The article 11 is dedicated to the analysis of a passive multistatic system which contains one transmitter station and N of receiver stations. The receiver stations detect a signal from airborne transponder of an aircraft and perform the measurement of the time of reception of such signal retransmitted by airborne transponder. The transmitter station is combined with one of the receiver stations, which enables to measure a slant range to a target. Normally in such systems, it is possible to obtain the time intervals being directly proportional to the measurements of two types: one slant range \( \hat{R}_{1} \) and 0.5N (N-1) of range difference \( \hat{R}_{i,j} \) or slant range and N-1 of total range \( \hat{R}_{1}, \hat{R}_{2}, \ldots, \hat{R}_{N-1} \).

The physics of the aforementioned measurements obtained independently from each other enables to create a system of linear algebraic equations 1 + 0.5N(N-1) with respect to N of unknown variables of range estima-

2. Results

Figure 1 illustrates geometry of a multistatic radar system which contains one transmitter station and N of receiver stations. The receiver stations detect a signal from airborne transponder of an aircraft and perform the measurement of the time of reception of such signal retransmitted by airborne transponder. The transmitter station is combined with one of the receiver stations, which enables to measure a slant range to a target.
tion or time intervals corresponding to it and in case of using a combination of range and range-difference methods, a system of equations will be as follows:

\[
\begin{align*}
\hat{t}_1 &= 2 \cdot t_1, \\
\Delta t_2 &= t_1 - t_2 + t_3 - t_4 + \ldots + t_{N-1} + t_N, \\
\Delta t_3 &= t_1 - t_2 + t_3 - t_4 + \ldots + t_{N-2} + t_{N-1} + t_N, \\
\Delta t_4 &= t_1 - t_2 + t_3 - t_4 + \ldots + t_{N-3} + t_{N-2} + t_{N-1} + t_N, \\
\Delta t_5 &= t_1 - t_2 + t_3 - t_4 + \ldots + t_{N-4} + t_{N-3} + t_{N-2} + t_{N-1} + t_N. \\
\end{align*}
\]

\( (1) \)

In matrix notation, the equation system (1) will be as follows:

\[
\begin{bmatrix}
\hat{t}_1 \\
\Delta t_2 \\
\Delta t_3 \\
\Delta t_4 \\
\Delta t_5
\end{bmatrix}
= \begin{bmatrix}
2 \\
1 \\
1 \\
1 \\
1
\end{bmatrix}
\begin{bmatrix}
t_1 \\
t_2 - t_3 \\
t_3 - t_4 \\
t_4 - t_5 \\
t_5
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
\]

With respect to five-station system, matrix \( A \) will be as follows:

\[
A^T = \begin{bmatrix}
2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & -1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0
0 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0
0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0
0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & -1 & 0
\end{bmatrix}
\]

\( (3) \)

\[
\tilde{S} = \begin{bmatrix}
\tilde{t}_1 \\
\tilde{t}_2 \\
\vdots \\
\tilde{t}_5
\end{bmatrix}
- matrix (row-vector) of unknown variables of range estimation with number of dimensions \( 1 \times N \).
\]

Solving the vector-matrix equation (2) using the known least squares method \( [14] \), we will obtain as follows:

\[
\tilde{S} = \left[ (A^T \Lambda W^{-1} A)^{-1} \right] A^T \Lambda W^{-1} \hat{H},
\]

\( (4) \)

\[
W = \begin{bmatrix}
1 & 0 & 0 & 0 & 0
0 & 2 & 1 & 1 & 0
0 & 1 & 1 & 1 & 1
0 & 1 & 1 & 2 & 0
\end{bmatrix}
\]

\( (5) \)

Matrix of primary measurements accuracy with number of dimensions \( [1 + 0.5N(N-1)] \times [1 + 0.5N(N-1)] \) the main diagonal of which contains the error variance of TDOA and TOA measurement and the secondary diagonals reflect possible correlation relationships between them.

\[
\sigma_t^2 - variance of estimating the time difference of signal arrivals to the stations;
\]

\[
\Lambda = \begin{bmatrix}
0 & 0 & 0 & 0 & 0
0 & j & 0 & 0 & 0
0 & 0 & \ddots & \ddots & \ddots
0 & 0 & 0 & \ddots & \ddots
0 & 0 & \cdots & \cdots & \cdots
\end{bmatrix}
\]

\( (6) \)

Figure 1. Geometry of multistatic radar system task.

In matrix notation, the equation system (1) will be as follows:

\[
\hat{H} = A \tilde{S},
\]

\( (2) \)

\[
\hat{H}^T = \begin{bmatrix}
\hat{t}_1, \Delta \hat{t}_2, \Delta \hat{t}_3, \ldots, \Delta \hat{t}_{N,0.5N(N-1)}
\end{bmatrix} - matrix (row vector) of primary measurements of time difference of signal arrivals to the stations with number of dimensions \( 1 \times [1 + 0.5N(N-1)] \).
\]

\( A \) - matrix of coefficients at unknowns with number of dimensions \( N \times [1 + 0.5N(N-1)] \) the values of which are equal to one if the relevant unknowns take place in this equation and equal to zero in the contrary case.


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navigational satellite system data or in some other way, we will obtain \( t_{ij} = t_i + t_j \). In the second case knowing the distances between stations, a TDOA should be measured between a signal along the base line during time \( t_{ij} \) and a signal transmitted by airborne transponder of an aircraft \( \Delta t = t_{ij} - (t_i + t_j) \) from which \( t_{ij} = t_i + t_j - \Delta t \).

The system of linear algebraic equations for TDOA and TOA finding system will be represented as follows:

\[
\begin{bmatrix}
\hat{t}_i \\
\hat{t}_{i2} \\
\hat{t}_{i3} \\
\hat{t}_{i4} \\
\hat{t}_{i5} \\
\vdots \\
\hat{t}_{i(N-1)}
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
\vdots \\
1
\end{bmatrix} \cdot \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
t_{i} \\
t_{i2} \\
t_{i3} \\
t_{i4} \\
t_{i5} \\
\vdots \\
t_{(N-1)}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}.
\] (7)

Then taking into consideration (3)-(6), we will obtain the expressions for slant range with regard to five-station range- and range-difference finding system:

\[
\hat{R}_1 = 0.5c\hat{t}_1,
\]

\[
\hat{R}_2 = 0.5c(2\hat{t}_{12} - \hat{t}_i),
\]

\[
\hat{R}_3 = 0.5c(2\hat{t}_{13} - \hat{t}_i),
\]

\[
\hat{R}_4 = 0.5c(2\hat{t}_{14} - \hat{t}_i),
\]

\[
\hat{R}_5 = 0.5c(2\hat{t}_{15} - \hat{t}_i),
\] (10)

where, the range estimation covariance error matrix of unknown variables has the following expression:  

\[
K_x = \text{diag}(A^T A)^{-1} \sigma_r^2.
\] (11)

Then taking into account (11) and (3), we will obtain the expressions for variances of range estimation with respect to each station for range-and range-difference method of a cooperative processing:

\[
K_x = \text{diag} \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
4 & 4 & 4 & 4 & 4 \\
1 & 13 & 13 & 13 & 13 \\
4 & 20 & 20 & 20 & 20 \\
1 & 13 & 13 & 21 & 13 \\
4 & 20 & 20 & 20 & 20 \\
1 & 13 & 13 & 13 & 21 \\
4 & 20 & 20 & 20 & 20
\end{bmatrix} \sigma_r^2.
\] (12)
And for TDOA and TOA methods of a cooperative processing:

\[
K_S = \text{diag}\begin{bmatrix}
1 & -1 & -1 & -1 \\
4 & 4 & 4 & 4 \\
-1 & 1 & 1 & 1 \\
4 & 4 & 4 & 4 \\
-1 & 1 & -1 & 1 \\
4 & 4 & 4 & 4 \\
-1 & 1 & 1 & 1 \\
4 & 4 & 4 & 4 \\
\end{bmatrix}
\]

\[
\sigma_s^2 = 2 - \frac{1}{4} - \frac{1}{4} - \frac{1}{4} - \frac{1}{4}
\]  \hspace{1cm} (13)

Formulae (12) and (13) were obtained at the assumption of equality to zero of the off-diagonal matrix elements (5) and (8). One can show that for two, three, four, six and seven-station systems, a multiplier before variance of indirect measurements is equal to 9/4, 19/12, 41/36, 11/12, and 23/28 correspondently, i.e. the involvement of a bigger number of values improves the estimation accuracy. Considering that off-diagonal matrix elements (5) might have a different correlation coefficient, the off-diagonal elements in (12) will not be identical however, their difference will be insignificant. Taking into consideration the correlation in matrix (8) does not change the coefficient values at variances but results in increasing the off-diagonal elements in (13).

We will compare the following systems of target location: TDOA and TOA and range ones taking into consideration the obtained pseudorange measurements of TDOA and TOA types. Target position in space is determined by its rectangular coordinates \(X_s, Y_s, H_s\) (Figure 1) and knowing the station coordinates \(x_i, y_i, h_i\), it can be found using a range measurement method by solving the simultaneous linear algebraic equations of the following type (at \(i = 1, N, N>3\)):

\[
R = \sqrt{(X_s - x_i)^2 + (Y_s - y_i)^2 + (H_s - h_i)^2}.
\]  \hspace{1cm} (14)

For TDOA and TOA finding systems, target position is determined by solving the following simultaneous equations:

\[
r_s = \sqrt{(X_s - x_i)^2 + (Y_s - y_i)^2 + (H_s - h_i)^2} - \sqrt{(X_s - x_j)^2 + (Y_s - y_j)^2 + (H_s - h_j)^2}.
\]  \hspace{1cm} (15)

\[
r_s = \sqrt{(X_s - x_i)^2 + (Y_s - y_i)^2 + (H_s - h_i)^2} + \sqrt{(X_s - x_j)^2 + (Y_s - y_j)^2 + (H_s - h_j)^2}.
\]  \hspace{1cm} (16)

Equations (14) - (16) can be solved with respect to \(X_s, Y_s, H_s\) using the methods known, and target position is determined by the formula \(\sigma = \sqrt{\text{trace}(D^T W^{-1} D)}\) where \(D\) – a partial derivative matrix (matrix of direction cosines).

Figure 2 shows the calculation results of mean square deviation of target location for the case \(\sigma_R = 50, d=20\ km\) and of estimation of the different horizontal range \(R_o\) from the origin of coordinates at target flight level of 5 km. This enables to analyze the accuracy of target location using long length base \((R_o>>L)\) and short length base \((R_o<<L)\) systems, \(L\) – distance between transmitter and receiver stations. Mean square deviations are numerated as follows: 1 – at TDOA method of processing of four measurements; 2 – at TDOA method of processing of ten measurements; 3 – at a cooperative processing of range and TDOA measurements; 4 – at TOA method of processing; 5 – at a cooperative processing of range and TOA measurements.

**Figure 2.** Mean square deviations of target location using different methods.

### 3. Discussion

Unlike conventional approaches to mathematical description of multilateration systems operation in this article it is offered to use the cooperative processing of coordinate information. The essence of this approach lies in the processing of one ranging and several range-differential (total-range) measurements to obtain estimates of distances with respect to each of the positions. This allows in some cases significantly expanding the working area of the system in terms of accuracy.

### 4. Conclusion

1. The analyzed procedures of radar data processing allow for the reduction of hyperbolic and elliptic lines of position to range ones obtained indirectly.
2. The use of a partial cooperative processing of measurements in multistatic radio-technical systems expands a work area of a system with respect to the accuracy for long length base systems. Accuracy gain for short length base systems is insignificant.

3. At using a cooperative processing of measurements in range-and total range finding system, the accuracy of range estimation using the indirect method significantly improves as long as the number of stations increases. At using the range and total range measurements, the accuracy of the indirect range estimation remains constant regardless of the number of stations.

4. Total-range method is characterized by high accuracy of target location however the arrangement of measurements in such system still remains an open question since re-reflection from the ground surface might affect the accuracy of estimating the reception time of a signal along the base line.

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6. **References**


