Optimization of Distribution Generating Unit using Voltage Stability Index and Analytical Approach

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Abstract

Background/Objectives: Voltage stability has become an important issue of power system stability. In this paper Optimization of Distribution Generating Unit using Voltage Stability Index (VSI) and Analytical Approach with Forward, Backward Sweep method (FBS). Methods/Statistical analysis: In this work a novel approach has been implemented to reduce the losses in the radial distribution system. This work has been done in two phases, identification of weak nodes with voltage stability index, which are most sensitive toward voltage collapse using the Branch Injection and Branch Current (BIBC) and Branch Current and Branch Voltage (BCBV) matrix have been analyzed from the first phase of work. From second phase, optimal allocation of Distribution Generation (DG) with objective as minimizing the distribution line losses with cost attributes. It is vitally essential feature to characterize the size and location of DG to be placed with the cost of that DG installed allowing for determine cost of loss. Application/Improvements: By virtue of some intrinsically characteristics of distribution systems, as the structure is radial in nature, immensely colossal number of nodes, with an extensive range of R/X ratios, the conservative techniques diverges for the distribution system on the determination of optimum size and location of distributed generations. The projected analysis is implemented on IEEE 15, IEEE 33, and IEEE 69 bus standard test systems.

Keywords: Analytical Approach, Cost Attributes, Distribution Generation (DG), Forward Backward Sweep method (FBSM), Optimal Location

1. Introduction

In distribution systems load flow analysis received much attention because of its uniqueness as excessive range of R/X ratio, with this the distribution network are ill conditioned and normal conservative techniques like Newton Raphson (NR), Fast Decoupled Method (FDM) etc, are inefficient at solving as the range of test case increases in bus number. The main aim in this paper is to develop a new load flow technique for radial distribution test system and this proposed method involves a straight forward arithmetical expression of voltage magnitudes without trigonometric equations. This method involves KVL and KCL matrixes and analysis is worked out. Concerning to the liberalization of Electricity Systems by Electric Power Research Institute (EPRI) new methodologies for planning and operation of power system is required, dealing with issues ranging from system dynamics to longer term. One of the main changes being seen in power systems across the world is the increased proliferation of what is known as DG ranges from a few kilowatts 5KW up to 50 MW with respect to its penetration level.

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According to International Energy Agency (IEA), DG as a plant serving a consumer on-site or providing support to a distribution network, connected to the grid at distributed level voltages. Some objectives are produced by IEA and EPRI on DG in distribution systems, reduction of fossil energy consumption in electric power generation and mitigate power losses without emission of harmful carbon particles, development in nonconventional energy based DG all over the world in recent years. VSI as root, exact loss formula is used\(^6,7\) to find an optimum sizing for DG in each bus for voltage stability improvement. The standard equation for formation of distribution loss is referred form reference\(^8,9\). With the multi objective index based approach, size and allocation of multiple DG for different load models is determine and performed with Particle Swarm Optimization (PSO)\(^10\). For Radio Data System (RDS), a goal programming technique is developed on Genetic Algorithm (GA) & PSO which is presented in to calculate the DG location and size in with multi objective function. An enhanced analytical method with Loss Sensitivity Factor (LSF) as objective is proposed in reference\(^11\). The mounting penetration of DG is altering the role of distribution network, which are originally passive networks, purely for the delivering of electricity to the consumer. The increased number of this installation of DG has directed to change in the uniqueness of the network. In analytical approach method, a new work has been proposed to quickly estimated losses for identifying the best location\(^12\) the load flow is required to be performed with respect to how many number of DG installed. This method requires less computation, as this we are adopting multiple DG for getting stability with its range of constraints with cost analysis\(^12–14\).

### 2. Proposed Methodology

#### 2.1 Network Topology

This phase work is developed for two matrixes, Bus Injection to Branch-Current Matrix (BIBC) and Branch current to Bus-voltage matrix (BCBV). The procedure for RDS is as follows by the equivalent current-injection.

\[
\begin{align*}
V_m &= \text{bus voltage of } m^{th} \text{ bus} \\
S_m &= \text{injected apparent power of } m^{th} \text{ bus} \\
S_m &= P_m + jQ_m \\
I_m^k &= \left( \frac{S_m}{V_m^k} \right)^* = \left( \frac{P_m + jQ_m}{V_m^k} \right)^* - y_m (V_m^{k-1})
\end{align*}
\]

Where voltage is \( V_m^k \) and current \( I_m^k \) are at bus \( m^{th} \) at the \( k^{th} \) iteration. Consider a 15 bus network, two matrices for 15 bus Figure 1 case are shown in below matrix.

![Figure 1](image)

Therefore, the relationship between the bus current injections and branch currents can be expressed as

\[
[B] = [BIBC][I] \\
Z = \text{diag}(Z_b) \\
[BCBV] = (BIBC)^T * Z
\]

The BIBC matrix is impedance matrix i.e. incidence of proposed network. The matrix of branch currents and bus voltages as shown in Fig.1 can be obtained by above equations.

1. **Step 1**: Formulate BIBC matrix by \( Z_b \) branch impedance.
2. **Step 2**: For distribution network with \( m \)-branch and \( N \)-bus, the dimension of the BCBV matrix is \((N - 1) * m\). 
3. **Step 3**: Multiply \([BIBC]^T\) matrix with \( Z \) matrix.

#### 2.2 Voltage Stability Index

VSI gives a clear dimension of voltage collapse point in the network. The node which is nearer to zero is ready for collapse and is considered as weak node, so proper injection of power is reacquired to improve stability index. In this proposed approach, VSM has calculated for time invariant ZIP load model.

\[
SI = \text{Stability Index} \quad V_s = \text{sending bus voltage} \\
r_{nn} = \text{Branch resistance} \quad x_{nn} = \text{Branch reactance}
\]
\[ P_{m2} = \text{Total injection of Real Power from } m_2; \]
\[ Q_{m2} = \text{Total injection of Reactive Power from } m_2; \]
\[ j = \text{branch number;} \]
\[ I_m^k = \left( \frac{S_m}{\sqrt{V_m^k}} \right) = \left( \frac{P_m + jQ_m}{\sqrt{V_m^k}} \right) \]  
\[
(4)
\]
For a two bus system shown in Figure 2

\[
\begin{align*}
I_m &= \left| V_m \right| \frac{\angle \delta_m - \angle \delta_n}{R_m + jX_m} \\
\left( P_n + jQ_n \right) &= \left| V_n \right| \frac{\angle \delta_n - \angle \delta_m}{R_n + jX_n} \]
\end{align*}
\]
\[
(P_n - jQ_n) \times (R_m + jX_m) = V_n^* \left( \left| V_n \right| \angle \delta_n - \angle \delta_m \right) 
\]  
\[
(5)
\]
Above equation has a homogenous solution and independent of phase angle of the system nearer to zero, for problem computation. The roots of the \(|V(n)|\) solution of the quadratic equation gives a realistic value. From the above derivations

\[
B_{ij} = \left| V_m \right|^2 - \left( P_{m2} \ast R_{ij} - Q_{m2} \ast X_{ij} \right) \\
A_{ij} = \left( P_{m2}^2 + Q_{m2}^2 \right) \ast \left( R_{ij}^2 + X_{ij}^2 \right) \\
\begin{align*}
1. & \quad 0.707[B_{mn} - [B_{mn}^2 - 4*A_{mn}]^{1/2}]^{1/2} + 0.707[B_{mn} + [B_{mn}^2 - 4*A_{mn}]^{1/2}]^{1/2} \\
2. & \quad 0.707[B_{mn} - [B_{mn}^2 - 4*A_{mn}]^{1/2}]^{1/2} + 0.707[B_{mn} + [B_{mn}^2 - 4*A_{mn}]^{1/2}]^{1/2} \\
3. & \quad -0.707[B_{mn} + [B_{mn}^2 - 4*A_{mn}]^{1/2}]^{1/2} + 0.707[B_{mn} + [B_{mn}^2 - 4*A_{mn}]^{1/2}]^{1/2} \\
\end{align*}
\]

From the above equations fourth is a feasible solution as first three are nearly equal to zero and not feasible solutions. So tha the stability index is

\[
|V_m|^2 \left( 4[I_{mn} R_{mn} + Q_{m2} X_{mn}] \frac{2}{\sqrt{|V_m|^2 - 4[(P_m X_m^2 + Q_m R_m)]^2}} \right) \]  
\[
(6)
\]
By Eq.6 Voltage Stability Margin (VSM) has been determined for each bus and the bus with minimum VSM is resolved.

### 3. Mathematical Modeling for DG in Analytical Approach

According to real and reactive power flow and terminal characteristics of DG, DG is mainly classified into four types. Assuming \( \phi = (\text{sign}) \times \tan(\cos^2(\text{DG}_{pl})) \)

**Sign** = +1: DG injects reactive power;  
**Sign** = −1: DG consumes reactive power;  
**P** = Real power loss;  
**Q** = Reactive power loss

The active and reactive power injected at bus \( m \), where the DG located, are given by (7) and (8)

\[
P_m = P_{DGm} - P_{dn} \quad (7)
\]
\[
Q_m = Q_{DGm} - Q_{dn} = P_m = \xi * P_{DGm} - Q_{dn} \quad (8)
\]

As loss are derived below \( m=1..N \).  
\[
V_m = \sum_{n=1}^{N} Z_{mn} I_n + \sum_{n=1}^{N} S_{mn} I_n + \sum_{n=1}^{N} Z_{mn} I_n + \sum_{n=1}^{N} S_{mn} I_n
\]
and

Where \( \theta_m = \delta_m - \phi_m \) so that \( \phi_m = \tan(\frac{Q_m}{P_m}) \)

Considering real term form the above eqn  
\[
P_m = \sum_{n=1}^{N} |I_n| \left( \cos \theta_m \cos \phi_m + \sin \theta_m \sin \phi_m \right) \]
\[
Q_m = \sum_{n=1}^{N} |I_n| \sin(\theta_m - \phi_m) \]

From the expansion of trigonometric cosine function, therefore

\[
P_m = \sum_{n=1}^{N} \left[ \delta_m (P_m P_n + Q_m Q_n) + \beta_m (Q_m P_n - P_m Q_n) \right] \]
\[
Q_m = \sum_{n=1}^{N} \left[ \delta_m (P_m P_n + Q_m Q_n) + \xi_m (Q_m P_n - P_m Q_n) \right] \]
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\[ a_{nm} = R_{nm} \left( \cos(\delta_n - \delta_m) \right) \quad \beta_{nm} = R_{nm} \left( \sin(\delta_n - \delta_m) \right) \]

The total active power loss of the system from Eq. 9 is given by

\[ \frac{dP}{dP_{Doa}} = \sum_{m=1}^{n} \left[ a_{nm} (P_m - Q_m) - \beta_{nm} (Q_m - P_m) \right] \]

\[ X_m = \sum_{n=1}^{m} (a_{mn} Q_n - \beta_{mn} P_n) \quad Y_m = \sum_{n=1}^{m} (a_{mn} P_n - \beta_{mn} Q_n) \]

\[ P_{DGm} = \frac{\delta_{nm} (P_{m}\delta Q_{nm}) - \beta_{nm} (\delta Q_{nm}) - X_m^\delta - Y_m^\delta}{\delta_{nm}} \]

Type 1 DG: Only active power injection by DG with power factor as Unity, so \( DG_{pf} = 1; \zeta = 0 \).

\[ P_{DGm} = \frac{\delta_{nm} (P_{m}\delta Q_{nm}) - \delta_{nm} (\delta Q_{nm}) - X_m^\delta - Y_m^\delta}{\delta_{nm}} \]

Type 2 DG: Only reactive power injection by DG with PF as Zero. So \( DG_{pf} = 0; \zeta = \infty \).

\[ Q_{DGm} = \frac{\delta_{nm} (Q_{m}\delta Q_{nm}) + \delta_{nm} (\delta Q_{nm}) - Y_m^\delta}{\delta_{nm}} \]

Type 3: Active (P) and Reactive (Q) power injection mutually and the PF ranges between \( 0 < DG_{pf} < 1; \zeta = \text{constant sign}=+1 \), the DG optimal size at \( m \)-th bus is given by \( P_{DGm} \).

\[ P_{DGm} = \frac{\delta_{nm} (P_{m}\delta Q_{nm}) - \beta_{nm} (\delta Q_{nm}) + X_m^\delta + Y_m^\delta}{\delta_{nm}} \]

Type 4: Mutual Active (P) and Reactive (Q) power injection and the PF ranges between \( 0 < DG_{pf} < 1; \zeta = \text{constant sign}=-1 \), the DG optimal size at \( m \)-th bus is given by \( P_{DGm} \).

\[ P_{DGm} = \frac{\alpha_{nm} (P_{m}\delta Q_{nm}) - \beta_{nm} (\delta Q_{nm}) + X_m^\delta + Y_m^\delta}{\delta_{nm}} \]

4. Optimal Location and Size

From the analytical expressions proposed above, the optimal sizes at diverse locations the losses are calculated. The position with minimum losses is considered as the optimal location at which the DG should be added. As loss reduction is improved with \( \alpha \) and \( \beta \) coefficients from the base case to DG allocation.

5. Cost analysis

The economics attributes of energy loss, P and Q power of DG considered based on:

Energy Loss (EL): The cost of energy loss on annual basis is given by

\[ EL_{cost} = (\text{Total real power loss}) \times (E \times T) \times Rs \]

Where \( E = 4.63 \text{ Rs/kWh} \); \( T = 8760 \text{ ours/year} \)

Cost attributes of DG is selected as per the data:

\[ C(P_{DG}) = a \times P + b \times P_{DG} + c \text{ Rs/ kWh} \]

\[ C(Q_{DG}) = \frac{\text{Cost}(S_m) - \text{Cost}(S_m - Q)}{K} \times \text{Rs/h} \]

where \( a = 0.25; b = 20; c = 0 \).

6. Algorithm for Optimal Allocation and Size DG

Minimization of active power loss with main objective having two constraints that to restrain the voltage limits within \( 1 \pm 0.05 \) pu and power balance.

Step 1: Run the load flow with FRS method for finding the voltages and VSI with base case.

Step 2: With the proposed equations above, find DG size and place on every node except for reference bus.

Step 3: Calculate the \( P_{loss} \) from step2, at every bus.

Step 4: Having the system with minimum power loss (\( P_{lossmin} \)) is chosen as optimal place and allocate the DG at that place.

Step 5: Check whether the appropriate bus voltages and VSI are within the acceptable range with power balance.

Step 6: If the above step 5 is not obeyed, then omit distribution generator DG from that bus and return to Step 4. The same algorithm is to be followed for all the classifications of DG which are classified above.
7. Computational Results

7.1 Bus system

With analytical approach, Table 1 shows the simulation results for proposed network. The table contains location at which the DG allocated and the optimal size of DG in MVA, power factor of DG unit and cost attributes for different busses. It is noticed that bus 7 has minimum voltage of 0.9794 pu having the VSI of 0.9241 pu at 7th bus only.

7.2 Bus Test System

For proposed 33 bus network Table 1 show the voltage and stability index in tabular form. Form that bus 18 has minimum voltage of 0.9546 pu having the VSI of 0.8323 pu at 18th bus only. The voltage and stability index is plotted in Figure 4 where minimum voltage point and index point is highlighted. In 33-bus test system, from Table 1, bus 6 found to be the best location with minimum power loss. The optimal power factor for this system obtained is 0.8225 lagging and corresponding optimal size is 3.0135 MVA has a cost for real power is 49,774 Rs/hr (2488.78 kW) according to cost analysis. The total losses with optimal location, optimal size, having best power factor, are 68.495 kW with real power loss cost as 30.38 lakh Rs/year.

7.3 Bus Test System

For proposed 69 bus network Table 1 show the voltage and stability index in tabular form. Form that bus 27th has minimum voltage of 0.9694 pu having the VSI of 0.8834 pu at 27th bus only. From Figure 5 it is noticed that minimum power loss appears at bus 61. The test case needs both active and reactive power injection will reduce losses. The optimal power factor for this system obtained is 0.82 lagging and corresponding optimal size is 2.2228 MVA has a cost for real power is 36,232 Rs/hr (1811.51 kW). The total losses with optimal location, optimal size, having best power factor is 23.67 kW with real power loss cost as 9.60 lakh Rs/year.

Figure 3

The voltage and stability index is plotted in Figure 3 where minimum voltage point and index point is highlighted. From Figure 3 it is noticed that minimum power loss appears at bus 3. The optimal power factor for this considered system is 0.72 lagging and corresponding optimal size is 1.4206 MVA has a cost for real power is 20,457 Rs/hr (1022.9 kW) Rs. The total losses with optimal location, optimal size, having best power factor, are 5.7881 kW with a loss cost 6.40 lakh Rs/year.
Table 1. Review of results for analytical approach

<table>
<thead>
<tr>
<th>Test system</th>
<th>Power factor</th>
<th>VSI without DG</th>
<th>Min Voltage without DG</th>
<th>Load demand with real power loss without DG</th>
<th>Active power loss cost Rs/year without DG</th>
<th>DG value in MVA</th>
<th>Voltage Stability index</th>
<th>Min Voltage</th>
<th>Load demand with Real power loss</th>
<th>Active DG cost Rs/hr</th>
<th>Active Ploss cost Rs/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 bus</td>
<td>0.72</td>
<td>0.800464@13</td>
<td>0.9445@13</td>
<td>1.2264M@61.795 kW</td>
<td>25.037 lakh</td>
<td>1.4205 MVA@3</td>
<td>0.9241@7</td>
<td>0.9791@7</td>
<td><a href="mailto:1.2264M@15.7881kW">1.2264M@15.7881kW</a></td>
<td>20,457</td>
<td>6.40lakh</td>
</tr>
<tr>
<td>33 bus</td>
<td>0.825</td>
<td>0.6675@18</td>
<td>0.90367@18</td>
<td>3.715MW@211.264 kW</td>
<td>85.698 lakh</td>
<td>3.0135 MVA@6</td>
<td>0.8323@18</td>
<td>0.9546@18</td>
<td>3.715MW@68.495 kW</td>
<td>49,774</td>
<td>30,38,221</td>
</tr>
<tr>
<td>69 bus</td>
<td>0.82</td>
<td>0.69102@65</td>
<td>0.9113@65</td>
<td>3.9718MW@219.54 kW</td>
<td>89.238 lakh</td>
<td>2.2228 MVA@61</td>
<td>0.8831@27</td>
<td>0.9694@27</td>
<td><a href="mailto:3.9718MW@23.67kW">3.9718MW@23.67kW</a></td>
<td>36,232</td>
<td>9,60,429</td>
</tr>
</tbody>
</table>

8. Conclusion

This paper work about VSI(VSI) and optimal sizing and placement of DG by an analytical approach with cost attributes in primary distribution systems. With FBS method proposed analytical expressions are evaluated for finding optimal size and allocation are tested and verified on three test distribution cases. Results showed that the location, size, and operating power factor of distributed generator are crucial factors in reducing losses with their cost attribute. It can be concluded that VSI and voltage magnitude has been increased and there is a extensive range of decrease in real power loss after placing DG.

9. References

3. World Final Electricity Consumption.
5. Electric Power Research Institute