Abstract

Objectives: Adaptation law designed using MIT Rule, does not guarantee the stability of the system. So to improve the overall performance of adaptive system fractional order Proportional integral derivative controller (FOPIDC) is designed and compared with the Integral order Proportional integral derivative controller (PIDC).

Methods/Statistical analysis: A controller design methodology is presented using Model Reference Adaptive System (MRAS) technique for an Inverted pendulum system using MIT Rule. The effect of change in the value of adaptation gain parameters to the response of the system has been studied. MATLAB/Simulation has been used to analyze the behavior of the system. Findings: Fractional order PID controller improves the overall transient performance better than the integral order PID controller do for MRAPIDC scheme using MIT Rule for inherently unstable system in the presence of external disturbances and band limited non-linearities. Addition to this, the time domain specifications such as rise time, settling time for different adaptation gains have been analyzed. Application/Improvements: Overall Performance of adaptive system using PID controller increased further by using fractional order PID controller.

Keywords: Adaptation Gains, Fractional Order PID Controller, Inverted Pendulum System, MIT RULE, MRAC, Proportional Integral Derivative (PID) Controller

1. Introduction

In adaptive control system, model reference adaptive system is one of the most widely used approach. Various adaptation laws can be used for the estimation of controller adjustable parameters. In this paper, adaptation laws are designed based on MIT Rule.

When the operating condition is fixed, conventional PID controllers are optimal to use where PID parameters can be tuned using robust time algorithm method etc. But when the operating condition is not fixed conventional PID controller would be under sub-optimal condition if PID parameters remains fixed. In 2001, an automatic tuning of PID controller parameters using MRAC concept and MIT rule was proposed. The proposed control scheme was applied on first order system and the effectiveness was observed in simulation results.

MRAC based PID controller using MIT rule has been presented for DC Electromotor drive. In cart inverted pendulum model one of the basic control problem is to achieve the system equilibrium at unstable point. Combination of MRAC and PID controller can be implemented for conventional inverted pendulum problem. Also, by tuning PID with genetic algorithm in modified MRAC, MRAC and modified MRAC schemes can be efficiently applied for cart-pendulum system.

The article organization is as follows: In section II, a brief introduction of MRAPIDC and MIT rule is presented. In section III, a brief introduction of fraction order system is given. In section IV, mathematical modeling of a cart-pendulum system has been derived. In section V, control algorithm using MIT rule is presented. In section VI, simulation studies have been shown, and in section VII conclusion of the article based on simulation studies is presented.

2. Model Reference Adaptive PID Controller

In model reference adaptive system (MRAS) a reference model is chosen based on our requirements.
between the output of the plant taken and the reference model is called as tracking error as shown in Figure 1. Difference between the input signal and the plant output i.e. $u_c - y_a$ is called as error signal. Tracking error through adaptation laws and error signal are then given to PID controller which adjusts the adjustable parameter of the PID controller until tracking error becomes zero.

2.1 MIT Rule
MIT rule was introduced by the researchers of MIT and was first developed for the automation systems. For the design of Model Reference Adaptive PID controller, MIT rule has been used in this paper.

Suppose $\theta$ is the adjustable controller parameters. The reference model output is taken as $y_r$ and the output of the closed loop system is taken as $y_a$ and the difference between these two i.e $y_a - y_r$ is called as tracking error $\varepsilon$.

$$\varepsilon = y_a - y_r$$ (1)

Cost function is denoted by $J$ and express in terms of tracking error as follows:

$$J(\theta) = \frac{1}{2} \varepsilon^2(\theta)$$ (2)

$\theta$, variable parameter, adjusts in such a way that the cost function is minimized, this method is called as gradient descent method.

$$\frac{d(\theta)}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \frac{\partial \varepsilon}{\partial \theta}$$ (3)

Figure 1. Model reference adaptive PID controller.

3. Fractional Order System
Fractional order systems are the generalized account of integer order systems. In 12, the theory of derivative for non-integer order was first introduced. Fractional order system could be made by following ways in MRAPIDC system:
- The use of fraction order parameter adjustment rule
- The employment of fractional order reference model in MRAC system
- Combination of fractional order parameter adjustment rule with fractional order reference model

In this paper, fractional order parameter adjustment rule is used to improve the transient performance of the system.

4. Inverted Pendulum
The cart-pendulum system motion is describe by the following non-linear equations:

$$M_0 + m_0 \ddot{x} + m_0 l_0 \ddot{\theta} \cos \theta + m_0 l_0 \dot{\theta}^2 \sin \theta = F_a$$ (4)

$$m_0 g l_0 \sin \theta - m_0 l_0 \dot{\theta}^2 - m_0 x \dot{l}_0 \cos \theta = I \ddot{\theta}$$ (5)

Where, $I = \frac{m_0 l_0^2}{3}$

A cart-pendulum system has been taken in this paper. A pendulum is attached to a motor driven cart. The objective of the proposed controller is to keep the pendulum in an unstable equilibrium point i.e in the upright vertical position. For keeping the pole in upright position, the controller will apply appropriate force to the cart in order to move cart in the horizontal direction. The cart-pendulum system is shown in Figure 2.

The angle between the actual position and the desired position of pole is denoted by $\theta$, $x$ is the cart displacement and $F_a$ is the control force acting on the cart

The parameters values chosen for the plant are given in Table 1.

After substituting the considered parameters of plant in (4) & (5), the transfer function of plant comes out to be

$$\frac{\theta}{F_a} = \frac{y_a}{u} = \frac{-1.33}{s^2 - 19.62}$$ (6)

Table 1. Plant Parameters value chosen

<table>
<thead>
<tr>
<th>Plant Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0$ – cartmass</td>
<td>1 Kg</td>
</tr>
<tr>
<td>$m_0$ – pendulummass</td>
<td>0.5 Kg</td>
</tr>
<tr>
<td>$g_0$ – gravity</td>
<td>9.81m/s^2</td>
</tr>
<tr>
<td>$l_0$ – lengthofpole</td>
<td>0.5m</td>
</tr>
</tbody>
</table>
5. Design of Control Algorithm for MRAPIDC

5.1 Reference Model
In MRAPIDC, first step is to select the reference model depending on the requirement. After that design of control algorithm is done in order to update the adjustable parameters of the controller. Suppose, the requirements for the system are: to operate system with rise time less than 1.5 seconds and overshoot less than 0.001%. For this system, value of $\omega_n$ is 9 and 1 respectively. Hence, the transfer function of reference model can be obtained as

$$\frac{y_r}{u_c} = \frac{w_n^2}{s^2 + 2\xi w_n + w_n^2}$$

(7)

$$\frac{y_r}{u_c} = \frac{9}{s^2 + 6s + 9}$$

(8)

5.2 MRAPIDC using MIT Rule
For derivation purpose, the generalized second order plant governed by equation (9) is considered:

$$G_a(s) = \frac{y_a}{u} = \frac{b}{s^2 + a_1s + a_2}$$

Let the transfer function of reference model be given by equation (10):

$$\frac{y_r}{u_c} = \frac{b_r}{s^2 + a_1s + a_2}$$

(10)

For MRAPIDC, as shown in Figure 1, the control law can be written as follows:

$$u = k_p e + k_i \int e \, dt + k_d \dot{e}$$

Where,

$$e = u_c - y_u$$

(12)

Differentiating (12) with respect to $t$,

$$\dot{e} = -\dot{y}_u$$

(13)

Using (11) & (13), the control law can be obtained as equation (14)

$$u = k_p e + k_i \int e \, dt - k_d y_u$$

(14)

By applying laplace transform, equation (14) can be written as,

$$u = k_p (u - y_u) + \frac{k_i}{s} (u_c - y_u) - k_d y_p s$$

(15)

Using equations (9) & (15), equation (16) can be obtained:

$$\frac{y_a(s)}{u_c(s)} = \frac{bk_p s + bk_i}{s^3 + (a_1 + b_k_1) s^2 + (a_2 + b_k_0) s + b_k_i}$$

(16)

By subtracting equation (10) from equation (16) the tracking error can be obtained as:

$$\varepsilon = y_a(s) - y_r(s)$$

$$\varepsilon = \frac{bk_p s + bk_i}{s^3 + (a_1 + b_k_1) s^2 + (a_2 + b_k_0) s + b_k_i} (u_c(s) - y_r(s))$$

(17)

According to MIT Rule, the cost function $J$ has to be minimized. Cost function is a function of adjustable parameter of the controller, as shown in Figure 2, which in MRAPIDC is $\gamma$. Therefore, following adaptation laws can be obtained:

$$\frac{dk_p}{dt} = -\gamma_p \varepsilon e \frac{bs}{s^3 + (a_1 + b_k_1) s^2 + (a_2 + b_k_0) s + b_k_i}$$

(18)

$$\frac{dk_i}{dt} = -\gamma_i \varepsilon e \frac{b}{s^3 + (a_1 + b_k_1) s^2 + (a_2 + b_k_0) s + b_k_i}$$

(19)

$$\frac{dk_0}{dt} = \gamma_d \varepsilon e \frac{bs^2}{s^3 + (a_1 + b_k_1) s^2 + (a_2 + b_k_0) s + b_k_i} \cdot y_p$$

(20)

According to MRAC theory, equations (18),(19) and (20) cannot be used directly for the tuning of controller parameters. Thus, the adaptation laws are required to be...
approximated in terms of reference model. This can be achieved by making assumption that reference model and cart-pendulum model are approximately equal. By comparing the denominator of equations (10) & (16), the following adaptation laws:

\[
\frac{dk_p}{dt} = -\gamma_p e \cdot \frac{b_i s}{s^2 + a_{r1} s + a_{r2}} \cdot e \quad (21)
\]

\[
\frac{dk_i}{dt} = -\gamma_i e \cdot \frac{b_i}{s^2 + a_{r1} s + a_{r2}} \cdot e \quad (22)
\]

\[
\frac{dk_d}{dt} = \gamma_d e \cdot \frac{b_i s^2}{s^2 + a_{r1} s + a_{r2}} \cdot \gamma_d \quad (23)
\]

### 6. Result and Discussion

In this section, the effect of introducing backlash nonlinearities with dead bandwidth and disturbance at the input of the cart-pendulum system, when incorporated with MRAPIDC and MRAFOPIDC schemes, have been analyzed. The simulation results obtained from the implementation of MRAPIDC and MRAFOPIDC schemes are compared. Two cases for different sets of adaptation gains have been considered to analyze the adaptability and controlling ability of the MRAPIDC and MRAFOPIDC for the inverted pendulum system, when subjected with external disturbances and band limited non-linearities. Figure 3–5 shows the step response for the considered plant, when incorporated with MRAPIDC and MRAFOPIDC schemes. Table 2–4 shows the time domain analysis of MRAPIDC and MRAFOPIDC scheme for different sets of adaptation gains.

#### Table 2. Time domain analysis of MRAPIDC and MRAFOPIDC for \( \gamma_p = 10000, \gamma_i = 8000, \gamma_d = 70000 \)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Rise time</th>
<th>Settling time</th>
<th>Overshoot</th>
<th>Undershoot</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>1.1193</td>
<td>1.9446</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MRAPIDC</td>
<td>1.5824</td>
<td>14.6357</td>
<td>45.3349</td>
<td>9.0487</td>
<td>0.0606</td>
</tr>
<tr>
<td>MRAFOPIDC</td>
<td>1.5667</td>
<td>6.5170</td>
<td>60.2882</td>
<td>0</td>
<td>0.0249</td>
</tr>
</tbody>
</table>

#### Table 3. Time domain analysis of MRAPIDC and MRAFOPIDC for \( \gamma_p = 100000, \gamma_i = 19000, \gamma_d = 350000 \)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Rise time</th>
<th>Settling time</th>
<th>Overshoot</th>
<th>Undershoot</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>1.1193</td>
<td>1.9446</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MRAPIDC</td>
<td>1.9757</td>
<td>10.8413</td>
<td>28.1380</td>
<td>0</td>
<td>0.0102</td>
</tr>
<tr>
<td>MRAFOPIDC</td>
<td>1.0111</td>
<td>1.5622</td>
<td>1.9418</td>
<td>0</td>
<td>3.2674e-04</td>
</tr>
</tbody>
</table>

(A) For \( \gamma_p = 10000, \gamma_i = 8000, \gamma_d = 70000 \)

(B) For \( \gamma_p = 100000, \gamma_i = 19000, \gamma_d = 350000 \)
All the computer simulation models are run for 30 seconds with fixed step size. It is to be noted that the above results have been achieved by taking integration order of 1.5, 0.2, 0.8 for proportional, integral and derivative part respectively in MRAFOPIDC scheme, which is by default 1, 1, 1 in case of MRAPIDC. From simulation studies, it can be concluded that for inherently unstable system, in the presence of external disturbances and band limited non-linearities, the fractional order PID controller improves the overall transient performance better than the integral order PID controller for MRAPIDC scheme.

8. References