Tuning methods of multi-loop controller for a multivariable process
V. Alamelumangai and S.P. Natarajan
Electronics and Instrumentation Engineering, Annamalai University, Chidambaram-608002, India
almjai@yahoo.com

Abstract
All chemical processes in process industries usually have two or more controlled outputs requiring two or more manipulated variables generally called Multi Input Multi Output (MIMO) process. In this paper an attempt is made to review the practical difficulty in applying control strategies for a multivariable process. The control strategies considered here are multi-loop PID control, multi-loop IMC-PID control and multi loop H∞-PID control. These strategies are reviewed by applying them to the quadruple tank process which is the bench mark multivariable process.

Keywords: MIMO process, multi-loop controller, IMC control, H∞ control, quadruple tank process

Introduction
In the past years, even though modern control theory has been developed significantly. Controllers designed using modern control theory is usually very complex. PID controller is a versatile device to control a wide variety of processes and still widely used in the process industries due to its easy implementation.

Traditionally, PID tuning methods are based on a plant model. In many process industries the process dynamics are poorly known. More recently, considerable effort has been devoted to the development of methods that connect the identification properties with controller design to produce more reliable control systems. There exists numerous PID tuning procedures that if appropriately used can yield successful tuning.

In SISO systems, the primary objective is to maintain only one variable nearer to its set point, though several measured variables involved (ex: cascade and feed forward control). By contrast, multivariable control involves the objective of maintaining several controlled variables at independent set points.

The quadruple tank process taken to study the tuning techniques of the PID controller is a multivariable process, which has a multivariable zero. The location of the multivariable zero can be changed either to left or to right half of the s-plane simply by changing a valve position. Control problem is simple for the minimum phase system, compared with non-minimum phase system. Multivariable control system could be with a decentralized (Henrik & Rantzer, 1999; Henrik, 2000) or centralized controller. The straight forward extension of controller tuning techniques used in SISO system (Astrom & Haggluned, 1998) can be used to design a decentralized controller for the multivariable process.

The decentralized PI controller based on manual tuning is reported in (Henrik, 2000). Multi-loop controller for a minimum phase system and decentralized control of sequentially minimum phase system are presented in (Henrik & Rantzer, 1999; Henrik, 2000). The decentralized controller using co-efficient diagram method is presented in (Arjin et al., 2004) while the QFT methodology is used to provide robust control in (Mahdi et al., 2006). The multivariable IMC controller is designed in (Edward et al., 2000). Number of controllers was designed and the results were published using the bench mark quadruple tank process as mentioned above. But practical difficulty in implementing the control system in a real process is not reported till date. This paper aims to identify the model of the quadruple tank process and to design controller such that the controller should give closed loop stability, good set point tracking and disturbance rejection with minimum settling time along with the non-oscillatory controller output.

The organization of the paper includes motivation and description of the process; theoretical background of the tuning techniques; the simulation & experimental results and discussion and finally the concluding remarks.

Process description
A schematic diagram of the process is shown in Fig. 1. The target is to control the level in the lower two tanks with two pumps. The process inputs are $v_1$ and $v_2$ (input voltages to the pumps) and the outputs are $y_1$ and $y_2$ (voltages from level measurement devices).

Mass balances and Bernoulli’s law yield,
\[
\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1k_1}{A_1}v_1 \quad (1)
\]
\[
\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2k_2}{A_2}v_2 \quad (2)
\]
\[
\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{1-\gamma_2}{A_3}k_2v_2 \quad (3)
\]
\[
\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{1-\gamma_1}{A_4}k_1v_1 \quad (4)
\]

Where, $A_i$ cross-section area of tank $i$; $a_i$ cross-section area of the outlet hole; $h_i$ water level ($i$ varies from 1 to 4).
The voltage applied to Pump $i$ is $v_i$ and the corresponding flow is $k_iv_i$. The parameters $\gamma_1, \gamma_2 \in (0,1)$ are determined from how the valves are set prior to an experiment. The flow to tank 1 is $\gamma_1 k_1 v_1$ and the flow to tank 4 is $(1-\gamma_1) k_1 v_1$ and similarly for tank 2 and tank 3. The acceleration of gravity is denoted by $g$. The measured level signals are $k_1 h_1$ and $k_2 h_2$. The parameter values of the laboratory process (Henrik, 2000) are given in Table 1.

The transfer function model of the process can be analytically found around the operating condition as shown in the reference (Henrik, 2000). The parameter values of the laboratory process are shown in Table 1. The nominal operating conditions of the laboratory quadruple tank process (Real Time) are given in Table 2.

The transfer function model of the process is found to be

$$ G_p(s) = \frac{2.6}{(1+62s)} + \frac{1.5}{(1+23s)(1+62s)} + \frac{1.4}{(1+30s)(1+90s)} + \frac{2.8}{(1+90s)} $$

The quadruple tank process is fabricated with reference to (Henrik, 2000). The schematic diagram of the process is shown in Fig. 2. The differential pressure transmitters are used to sense the level in tank 1 & 2. Rotameters are used to measure the flow into the tanks. The height of each tank is 36 cms. As the QTP is a two by two process, there are two inputs and two outputs and hence there are two different ways to form the control loops. To select any one way of forming control loop Relative Gain Array (RGA) is used. RGA provides steady state measure of interactions for decentralized control. Based on RGA analysis the variables are paired as 1-1and2-2 and hence the system is closed-loop stable.

The parameter values of the quadruple tank process are given in Table 3. The transfer function of the process is found to be

$$ G_p(s) = \frac{3.06e^{-50s}}{(450s+1)} + \frac{2.353e^{-9s}}{(516s+1)} + \frac{1.6e^{-62s}}{(487.5s+1)} + \frac{4.4e^{-21.5s}}{(553.5s+1)} $$

There are two basic multivariable control techniques. The first is a straightforward extension of control techniques used in SISO process to MIMO process as shown in Fig. 3. This is termed as ‘multi-loop’ control and has been applied with success for many decades. Multiple single loop controller, have basically PID control algorithm.

The important features of this approach are:

(i) The single-loop controllers are completely independent algorithms that do not communicate directly among themselves.

(ii) The manipulation made by one controller can influence other controller variables, that is, there can be interaction through the process among the individual control loops. The advantage of multi-loop control is the use of single algorithm, which is especially important when the control calculations are implemented with analog computing equipments. The second advantage is the ease of understanding by plant operating personnel, which results from the simplicity of control structure. The multi-loop approach is generally preferred for its simplicity when it provides good performance.

The second main category is coordinated or centralized control, in which a single control algorithm uses all measurements, to calculate all manipulated variable simultaneously as shown in Fig. 4. The term centralized denotes a control algorithm that uses all (process output and input) measurement simultaneously to determine the values of all manipulated variables.

**Decentralized PID controller**

Decentralized PID control is one of the most common control schemes for interacting MIMO plants in the chemical process industries. The main reason for this is its relatively simple structure, which is easy to understand and to implement. The most common design procedure for tuning a PID controller is Ziegler-Nichols Method. In MIMO systems the tuning problem is complicated because of the interaction between loops. A change in a single parameter affects, in general all other loops as well. Decentralized controllers can reduce the interaction to a great extent. Hence “n×n” multivariable system requires “n” multi-loop controllers (Stephanopoulos, 1994). The general control configuration of a MIMO process with two inputs and two outputs is shown in Fig. 5. The control system could be designed using multi-loop controller. The multi-loop controller design procedure is a straightforward extension of SISO controller design methods.

An open loop Ziegler Nichols formulae are used to calculate PI and PID controller parameters.

**IMC-PID controller**

The basic structure of internal model controller is shown in Fig. 6. The control system shown in dotted lines.
Fig. 1. Schematic diagram of the quadruple tank process

Fig. 2. Quadruple tank process front panel diagram

Fig. 3. The quadruple tank process under multi-loop control

Fig. 4. The quadruple tank process under centralized control

Fig. 5. Block diagram of process with two controlled outputs and two manipulations

Fig. 6. Block diagram of Internal Model Control system
includes the two blocks labeled controller and model. The control system has at its input the set point and process output (measurement) and its output the manipulated variable (process input).

The effect of the parallel path is to subtract the effect of the manipulated variable from the process output. If it is assumed for the moment that the model is a perfect representation of the process, then the feed back signal is equal to the influence of the disturbances and is not affected by the action of the manipulated variables. Thus, the system is effectively open-loop and the usual stability problems associated with feedback have disappeared. The overall system is stable simply if and only if both the process and the IMC controller are stable.

Moreover, the IMC controller plays the role of a feed forward controller and can be designed as such. But the IMC controller does not suffer from the disadvantages of the feed forward controllers. It can cancel the influence of (unmeasured) disturbances because the feedback signal is equal to this influence and modifies the controller settings accordingly.

If the model does not mimic the dynamic behavior of the process perfectly then the feedback signal expresses both the influence of (unmeasured) disturbances and the effect of this model error. The model error gives rise to feedback in the true sense and leads to possible stability problems. This forces the designer to “detune” the ideal feed forward controller for “robustness”. In the design procedure PID controller block is cascaded with filter block “$T_1$”. The filter parameter “$\lambda$” is used to calculate this filter block. The filter parameter is a tuning parameter. Its value is selected based on the response specification.

Following the design procedure presented in the literature (Donald & Coughanowr, 1991), the parameters of the IMC-PID controller are derived and are given below:

**IMC-PID controller parameters**

$$K_c = \frac{2\tau + t_d}{2(\lambda + t_d)} \quad \text{Fig. 18. Response of}$$

$$T_i = \tau + \frac{t_d}{2} \quad \text{------- (6)}$$

$$T_D = \frac{\lambda t_d}{2\tau + t_d} \quad \text{------- (7)}$$

$$T_1 = \frac{\lambda t_d}{2(\lambda + t_d)} \quad \text{------- (8)}$$

Where,

$\lambda$ = Filter parameter; $\tau$ = Time constant; $t_d$ = Time delay

**H$_\infty$ PID controller**

The mathematical symbol “H$_\infty$” stands for the Hardy Space of all complex valued functions of a complex variable, which are analytic and bounded in the open right-half complex plane. For a linear (continuous-time, time-invariant) plant, the H$_\infty$ norm of the transfer matrix is the maximum of its largest singular value over all frequencies (Skogestad & Postlethwaite, 1996).

“H$_\infty$ optimal control Theory”, which addresses the issue of worst-case controller design for linear plants subject to unknown additive disturbances and plant uncertainties, including problems of disturbance attenuation, model matching and tracking.

Loop-shaping H$_\infty$ controllers could be designed for both stable and integrating processes. H$_\infty$ controllers for typical industrial processes have a PID structure interpretation. Further, if the pre-compensator is carefully chosen, the design indicator will be independent of plant constants. This fact leads to the derivation of a direct relation between the controller parameters and the plant constants, and the results can be viewed as new PID tuning rules for stable and integrating processes. The main contribution of the tuning rules is that the PID parameters can be obtained using one design parameter which reflects the trade-off between stability robustness and time domain performance of the system.

The design procedure of the loop shaping H$_\infty$ controller proceeds as below (Grimble, 1990):

• Loop shaping: Pre compensator $W_1$ and/or post compensator $W_2$ are used to shape the singular values of $G$ (plant). So that the shaped plant $\hat{G} = W_2GW_1$ has the desired open-loop shape.

• Robust stabilization: For the shaped plant $\hat{G}$, solve the following H$_\infty$ optimization problem:

$$\varepsilon_{\text{max}}^{-1} = \left\| \begin{bmatrix} (I + GK)^{-1} & (I + GK)^{-1}G \end{bmatrix} \right\|_{\infty} \quad \text{------- (9)}$$

Here $\varepsilon_{\text{max}}$ is a design indicator. A reasonable value (e.g. $> 0.2$) indicates that the loop shapes can be well approximated and the closed-loop system will have good, robust stability.

The final feedback controller is constructed as $K = W_1K'W_2$.

One advantage of the approach is that the H$_\infty$ optimization problem (Equ. 9) can be solved explicitly with out iteration. Suppose a minimal state-space realization for the shaped plant is

$$A = Ax + Bu$$

$$y = Cx$$

Then we have $\varepsilon_{\text{max}} = (1 + \lambda_{\text{max}}(VX))^{-1/2}$, where $X$ and $Y$ are the unique semipositive stabilizing solutions to the following algebraic Riccati equations:

$$ATX + XA - XBBD^TX + CT(C)^{T} = 0 \quad \text{------- (10)}$$

$$AY + YA^T - YC^TY + BB^T = 0$$

An optimal H$_\infty$ controller can be constructed using the following generalized state-space description:

$$\dot{x} = (Q(A + BD^TX) + \varepsilon_{\text{max}}^{-1}V^T(C)x) + \varepsilon_{\text{max}}^{-1}Vu \quad \text{------- (12)}$$
Most stable processes encountered in industry can be described as first order plus deadtime (FOPDT) models:

$$p(s) = \frac{k}{Ts + 1} e^{-\tau s} \quad (14)$$

An interesting observation is that when we choose a PI precompensator:

$$W_1 = \frac{kr}{\lambda T} \left(1 + \frac{1}{Ts}\right) \quad (15)$$

(where $\lambda$ is a design parameter), the optimum $\varepsilon_{\text{max}}$ in Eqn. 9 depends only on $\lambda$ and is independent of $\tau$. This property guarantees that direct relations between the parameters of the $H_\infty$ controller and the deadtime $\tau$ can be obtained, and a parameterization of the $H_\infty$ optimal controller for this type of process can be found.

Now the transfer function of the shaped plant is $(\lambda/\tau s)e^{-\tau s}$. To make use of the state-space solution, the delay is approximated by

$$\frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s} \quad (16)$$

Thus a minimal state-space realization for the shaped plant is:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -2/\tau \end{bmatrix}, \quad B = \frac{\lambda/\tau}{\lambda/\tau}, \quad C = \begin{bmatrix} 1 & -2 \end{bmatrix} \quad (17)$$

It is easy to verify that now the unique stabilizing solutions to Eqns. 10 and 11 are

$$X = \begin{bmatrix} \tau(1 + \lambda)/\tau & -\tau \\ \lambda/\tau & \tau \end{bmatrix}, \quad Y = \begin{bmatrix} \frac{\lambda(1 + \lambda)}{2\tau} & \frac{\lambda^2}{4\tau} \\ \frac{\lambda^2}{2\tau} & \frac{\lambda^2}{4\tau} \end{bmatrix} \quad (18)$$

respectively. Thus

$$YX = \begin{bmatrix} 1 + 2\lambda + \lambda^2/2 & -(\lambda + \lambda^2/2) \\ \lambda/2 + \lambda^2/4 & -\lambda^2/4 \end{bmatrix} \quad (19)$$

is independent of $\tau$, and so will be $\varepsilon_{\text{max}}$.

Since the state-space realization of the shaped plant is of order 2, it is easy to verify its $H_\infty$ optimal controller given by Eqn. 12 has the following structure:

$$R(s) = \frac{K_c (1 + \tau s/T_1)}{1 - \tau s/T_2} \quad (20)$$

Where $K_c$, $T_1$, $T_2$ depend only on $\lambda$. We note that the controller cancels the pole of the shaped plant (in pade approximation form), thus $T_1$ always equals 2. The terms $\varepsilon_{\text{max}}$, $K_c$, $T_2$ as function of $\lambda$ can be obtained using a least-squares fit technique:

$$\varepsilon_{\text{max}} = \left(0.572\lambda + 1.472\right)^{-1}$$

$$K_c = 0.633\lambda + 1.154$$

$$T_1 = 2$$

$$T_2 = 5.314\lambda + 0.951$$

Thus the final $H_\infty$ controller for the FOPDT model has the following structure:

$$K(s) = \frac{K_c (1 + \tau s/T_2)}{1 + \tau s/T_1} \frac{\lambda T}{kr} \left(1 + \frac{1}{Ts}\right) \quad (22)$$

This can be transformed into the following practical PID structure:

$$K(s) = \frac{K_p (1 + \frac{1}{T_1 s} + T_1 s)}{1 + \frac{1}{T_d s}} \quad (23)$$

Where

$$K_p = \frac{MK_c (1 + \tau s/T_2)}{kr T_1} = \frac{0.265\lambda + 0.807}{k} \left(\frac{T}{\tau} + 0.5\right)$$

$$T_1 = T + \frac{\tau}{T_2} = T + \frac{\tau}{2}$$

$$T_d = T_1 T + \tau = 2T + \tau$$

$$T_f = T_2 = 5.314\lambda + 0.951$$

Where,

$\lambda$ = Design parameter; $T$ = Time constant; $\tau$ = Dead time

**Simulation and experimental result**

The minimum phase characteristics of the quadruple tank process are identified as transfer function model using reaction curve method. The process maintained operating conditions initially by giving 50% input to the pump 1 and 2. The 10% change in pump 1 applied by keeping pump 2 input constant and the level in tank 1 and 2 are recorded with the sampling time of 0.1 secs. The same procedure is repeated by changing input to pump 2 keeping pump 1 input constant. The open loop response of the process for change in input 1 is shown in Fig. 7. The experimentally identified transfer function model is

$$G_p(s) = \begin{bmatrix} 3.06e^{-50s} & 2.353e^{-9s} \\ (450s + 1)(516s + 1) & 1.6e^{-62.55} & 4.4e^{-21.5s} \\ (487.5s + 1)(553.5s + 1) \end{bmatrix}$$

This transfer function model is used to design the controllers. The non-linear model is simulated using SIMULINK software with PID, IMC-PID and H$_\infty$-PID.
Table 5. Performance evaluation of the controllers both in simulation and experiment

<table>
<thead>
<tr>
<th>Control Strategies</th>
<th>Simulation results using nonlinear model</th>
<th>Experimental results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ISE</td>
<td>Settling time</td>
</tr>
<tr>
<td>Loop1</td>
<td>Loop2</td>
<td>Loop1</td>
</tr>
<tr>
<td>PID</td>
<td>65</td>
<td>72</td>
</tr>
<tr>
<td>IMC PID</td>
<td>95</td>
<td>81</td>
</tr>
<tr>
<td>H∞ PID</td>
<td>73</td>
<td>112</td>
</tr>
</tbody>
</table>

**Fig. 7.** Open-loop response of the process with 10% change applied to tank 1

**Fig. 8.** Servo response of the process with PID controller (Simulation)

**Fig. 9.** Response of PID controller (Simulation)

**Fig. 10.** Servo and regulatory response of the process with IMC-PID controller (Simulation)

**Fig. 11.** Response of IMC-PID controller (Simulation)

**Fig. 12.** Servo and regulatory response of the process with PID controller (Simulation)
“Controller for multivariable process”

Alamelumangai & Natarajan

controllers. The servo and regulatory responses of the process with PID controller is shown in Fig. 8. The corresponding controller response is shown in Fig. 9. The servo and regulatory responses of the process with IMC-PID controller is shown in Fig. 10. The corresponding controller response is shown in Fig.11. The servo and regulatory responses of the process with H$_\infty$-PID controller is shown in Fig. 12. The corresponding controller response is shown in Fig.13. Simulink oriented dSpace control system is used for the real-time control of the quadruple tank process with sampling time of 0.1 sec. The speed of the processor used is 250 MHz and the conversion time of the ADC is 2 micro seconds. The servo and regulatory responses of the real process with PID controller is shown in Fig.14. The servo and regulatory responses of the real process with IMC-PID controller is shown in Fig.15 and the corresponding controller response is shown in Fig. 16. The servo and regulatory responses of the real process with H$_\infty$-PID controller is shown in Fig.17 and the corresponding controller response is shown in Fig. 18.

Conclusion
In this paper minimum phase characteristics of the quadruple tank process is identified as a transfer function model. The PID, IMC-PID & H$_\infty$-PID controllers are designed and the performance of the controller is evaluated both in simulation and experiment. The ISE, settling time and response of the controller are taken as performance indices. Even though IMC-PID and H$_\infty$-PID controllers are comparable with each other, the response of the IMC-PID controller is less oscillatory than the response of the H$_\infty$-PID controller. PID controller response is more oscillatory than the other two controllers. As the final control element of this process is a variable speed pump, care should be given to select the advanced control strategies, because they used to make the controller to take more effort, that may some time cause the damage to final control element. Considering this practical difficulty in implementing the control strategies on a real process, this paper reviews the performance of PID, IMC-PID and H$_\infty$-PID controllers. Therefore IMC-PID controller is the practically suitable controller among the three controllers for the quadruple tank process.

References