Abstract

Our aim in the present paper is to introduce and study new connection between fuzzy retractions, fuzzy foldings and fuzzy deformation retracts of fuzzy open hyperboloid in fuzzy Minkowski space and fuzzy open ball in fuzzy Euclidean space. Types of fuzzy foldings and fuzzy deformation retracts of fuzzy open hyperboloid are discussed. Types of minimal fuzzy retractions of fuzzy open hyperboloid are presented. The fuzzy foldings of fuzzy open hyperboloid will be deduced. Some applications are presented.

Keywords: Fuzzy Retractions, Fuzzy Foldings, Fuzzy Deformation Retracts, Fuzzy Open Hyperboloid in Fuzzy Minkowski Space.

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1. Introduction and Definitions

A fuzzy subset \(\tilde{M}, \tilde{\mu}\) of a fuzzy manifold \((M, \mu)\) is called a fuzzy deformation retract if there exists a continuous map \(\tilde{r}: (M, \mu) \rightarrow (\tilde{M}, \tilde{\mu})\) such that \(\tilde{r}(x, \mu) = (x, \mu)\) for all \(x \in M\).

A fuzzy subset \((\tilde{M}, \tilde{\mu})\) of a fuzzy manifold \((M, \mu)\) is called a fuzzy retraction if there exists a continuous map \(\tilde{r}: (M, \mu) \rightarrow (\tilde{M}, \tilde{\mu})\) such that \(\tilde{r}(x, \mu) = (a, \mu) = (a, \mu(a))\) for all \(a \in A, \mu \in [0, 1]\) \([1, 5]\).

A fuzzy manifod is manifold which has a physical character. This character is represented by the density function \(\mu\), where \(\mu \in [0, 1]\) \([7, 8]\).

A fuzzy subset \((\tilde{A}, \tilde{\mu})\) of a fuzzy manifold \((A, \mu)\) is called a fuzzy retraction of \((\tilde{M}, \tilde{\mu})\) if there exist a continuous map \(\tilde{r}: (\tilde{M}, \tilde{\mu}) \rightarrow (\tilde{A}, \tilde{\mu})\) such that \(\tilde{r}(a, \mu(a)) = (a, \mu(a))\) for all \(a \in A, \mu \in [0, 1]\) \([1, 5]\).

A map \(\tilde{\delta}: H^n \rightarrow \tilde{H}^n\), is said to be an isometric folding of fuzzy hyperboloid in fuzzy Minkowski space into itself if for any piecewise fuzzy geodesic path \(\gamma: I \rightarrow H^n\) the induced path \(\tilde{\delta} \circ \gamma: I \rightarrow \tilde{H}^n\) is a piecewise fuzzy geodesic and of the same length as \(\gamma\), where \(J=\{0,1\}\). If \(\tilde{\delta}\) does not preserve lengths, then \(\tilde{\delta}\) is a topological folding of fuzzy hyperboloid in fuzzy Minkowski space \([8, 9, 13-19]\).

The isofuzzy folding of \(\bigcup \tilde{M_i} \subseteq H^n\) is a folding \(\tilde{\delta}: \bigcup \tilde{M_i} \rightarrow \bigcup \tilde{M_i}\) such that \(\tilde{\delta}(\tilde{M_i}) = \tilde{M}\) and any \(\tilde{M_i}\) belong to the upper hypermanifolds \(\exists \tilde{M_j}\) down \(\tilde{M}\) such that \(\mu_i = \mu_j\) for every corresponding points i.e. \(\mu(a_i) = \mu(a_i)\) \([6]\). See Figure 1.
2. Main Results

Consider a five dimensional Minkowski space \( M^5 \) which metric \( ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dw^2 \), and embed a hyperboloid given by \(-u^2 + x^2 + y^2 + z^2 + w^2 = a^2 \) [32]. In what follows, we would like to introduce and evaluate the fuzzy geodesic of fuzzy hyperboloid \( \tilde{H}^4 \) in fuzzy Minkowski space \( \tilde{M}^5 \). The fuzzy metric is defined as

\[
\begin{align*}
\alpha^2 \cosh^2 \left( \frac{t(\eta)}{\alpha} \right) &+ \sin^2 \chi(\eta) \sin^2 \vartheta(\eta)\phi^2(\eta) = 0, \\
\frac{d}{ds} \left[ \alpha^2 \cosh^2 \left( \frac{t(\eta)}{\alpha} \right) + \sin^2 \chi(\eta) \sin^2 \vartheta(\eta)\phi^2(\eta) \right] &= 0, \\
\frac{d}{ds} \left[ \alpha^2 \cosh^2 \left( \frac{t(\eta)}{\alpha} \right) \sin^2 \chi(\eta) \sin^2 \vartheta(\eta)\phi^2(\eta) \right] &= 0, \\
\frac{d}{ds} \left[ \alpha^2 \cosh^2 \left( \frac{t(\eta)}{\alpha} \right) \sin^2 \chi(\eta) \sin^2 \vartheta(\eta)\phi^2(\eta) \right] &= 0.
\end{align*}
\]

Then the Lagrangian equations are

\[
\begin{align*}
\frac{d}{ds} \left[ -t'^2(\eta) + \alpha^2 \cosh \left( \frac{t(\eta)}{\alpha} \right) \sinh \left( \frac{t'(\eta)}{\alpha} \right) \right] &= 0, \\
\left( \chi'^2(\eta) + \sin^2 \chi(\eta) \vartheta^2(\eta) \phi^2(\eta) \right) &= 0, \\
\left( \vartheta'^2(\eta) + \sin^2 \vartheta(\eta)\phi^2(\eta) \right) &= 0, \\
\left( \phi'^2(\eta) + \sin^2 \phi(\eta) \right) &= 0,
\end{align*}
\]

From equation (7) we have \( \alpha^2 \cosh^2 \left( \frac{t(\eta)}{\alpha} \right) \sin^2 \chi(\eta) \sin^2 \vartheta(\eta)\phi^2(\eta) = \beta \), if \( \beta = 0 \), we get \( \phi(\eta) = 0 \), which implies to \( \phi(\eta) = 0 \). Then from equation (2) we obtain the following coordinates of the geodesic \( \tilde{H}^4 \) defined as

\[
\begin{align*}
\tilde{u} &= \alpha \sinh \left( \frac{t(\eta)}{\alpha} \right), \\
\tilde{w} &= \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \cos \chi(\eta), \\
\tilde{x} &= \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \chi(\eta) \cos \vartheta(\eta),
\end{align*}
\]

**Figure 1.** The isofuzzy folding of \( \tilde{H}^4 \) [5].
\[ \tilde{y} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \chi(\eta) \sin \theta(\eta), \quad \tilde{z} = 0. \]

If \( \phi(\eta) = \frac{\pi}{2} \), we have the following coordinates of the geodesic hyperboloid \( \tilde{H}^2 \) given by
\[ \tilde{u} = \alpha \sinh \left( \frac{t(\eta)}{\alpha} \right), \quad \tilde{w} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \cos \chi(\eta), \]
\[ \tilde{x} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \chi(\eta) \cos \theta(\eta), \quad \tilde{y} = 0, \]
\[ \tilde{z} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \chi(\eta) \sin \theta(\eta). \]

Now, if \( \sin^2 \theta(\eta) = 0 \), we obtain the fuzzy geodesic hyperboloid \( \tilde{H}^2 \) in fuzzy hyperboloid \( H^4 \) which represented by
\[ \tilde{u} = \alpha \sin \left( \frac{t(\eta)}{\alpha} \right), \quad \tilde{w} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \cos \chi(\eta), \quad \tilde{x} = 0, \]
\[ \tilde{y} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \chi(\eta) \cos \phi(\eta), \]
\[ \tilde{z} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \chi(\eta) \sin \phi(\eta). \]

If \( \theta(\eta) = \frac{\pi}{2} \), we have the following coordinates
\[ \tilde{u} = \alpha \sin \left( \frac{t(\eta)}{\alpha} \right), \quad \tilde{w} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \cos \chi(\eta), \quad \tilde{x} = \tilde{y} = \tilde{z} = 0. \]

then, \( -\tilde{u}^2 + \tilde{w}^2 = \alpha^2 \), which is the fuzzy geodesic \( \tilde{H}^1 \) in \( \tilde{H}^4 \).

If \( \chi(\eta) = \frac{\pi}{2} \), we have the following coordinates of the geodesic \( \tilde{H}^1 \) in \( \tilde{H}^4 \) defined as
\[ \tilde{u} = \alpha \sin \left( \frac{t(\eta)}{\alpha} \right), \quad \tilde{w} = 0, \quad \tilde{x} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \cos \theta(\eta), \]
\[ \tilde{y} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \theta(\eta) \cos \phi(\eta), \]
\[ \tilde{z} = \alpha \cosh \left( \frac{t(\eta)}{\alpha} \right) \sin \theta(\eta) \sin \phi(\eta). \]

If \( t(\eta) = 0 \), we have the following coordinates
\[ \tilde{u} = 0, \quad \tilde{w} = \alpha \cos \chi(\eta), \quad \tilde{x} = \alpha \sin \chi(\eta) \cos \theta(\eta), \]
\[ \tilde{y} = \alpha \sin \chi(\eta) \sin \theta(\eta) \cos \phi(\eta), \]
\[ \tilde{z} = \alpha \sin \chi(\eta) \sin \theta(\eta) \sin \phi(\eta). \]

then, \( \tilde{x}^2 + \tilde{y}^2 + \tilde{w}^2 = \alpha^2 \), is the fuzzy geodesic sphere \( \tilde{S}^1 \) in \( \tilde{H}^4 \).

From the above discussion, we obtain the following theorem (See Figures 2, 3, 4).

**Theorem 1:** The fuzzy geodesics of a fuzzy hyperboloid give fuzzy hyperboloid, fuzzy spheres, fuzzy one-dimensional space.

**Theorem 2:** Let \( \tilde{H}^n(\eta) \subseteq \tilde{M}^{n+1}(\eta) \) be a fuzzy hyperboloid in a fuzzy Minkowski space time which is homeomorphical to \( \tilde{D}^{n+1}(\eta) \subseteq \tilde{R}^n(\eta) \) and \( \tilde{f}^n(\eta): \tilde{H}^n(\eta) \rightarrow \tilde{H}^n(\eta) \) be a fuzzy folding, then there is an induced fuzzy folding \( \tilde{\beta}(\eta): \tilde{D}^{n+1}(\eta) \rightarrow \tilde{D}^{n+1}(\eta) \) such that the following diagram is commutative
\[ \tilde{H}^n(\eta) \xrightarrow{\tilde{f}^n(\eta)} \tilde{H}^n(\eta) \]
\[ \tilde{\beta}(\eta) \downarrow \quad \downarrow \tilde{\beta}(\eta) \]
\[ \tilde{D}^{n+1}(\eta) \xrightarrow{\tilde{f}^n(\eta)} \tilde{D}^{n+1}(\eta) \]

**Proof:** Let \( \tilde{f}(\eta): \tilde{H}^n(\eta) \rightarrow \tilde{H}^n(\eta) \) be a fuzzy folding. Then under the central projection \( \tilde{\beta}(\eta): \tilde{\beta}(\eta): \tilde{H}^n(\eta) \rightarrow \tilde{D}^{n+1}(\eta) \) there is an induced fuzzy folding \( \tilde{f}(\eta): \tilde{D}^{n+1}(\eta) \rightarrow \tilde{D}^{n+1}(\eta) \) such that \( \tilde{\beta}(\eta) \circ \tilde{f}(\eta) = \tilde{\beta}(\eta) \circ \tilde{f}(\eta) \).

![Figure 2](image2.png) Types of fuzzy retractions of fuzzy hyperboloid.

![Figure 3](image3.png) Depicted fuzzy retractions of fuzzy hyperboloid embedded in three-dimensional Minkowski space.

![Figure 4](image4.png) Types of fuzzy retractions of one-dimensional fuzzy hyperboloid embedded in two-dimensional Minkowski space.
\textbf{Theorem 3:} Let $H^* (\eta) \subseteq \tilde{M}^{n+1} (\eta)$ and let $\tilde{f} (\eta): H^* (\eta) \subseteq \tilde{M}^{n+1} (\eta) \rightarrow \tilde{H}^2 (\eta) \subseteq \tilde{M}^1 (\eta)$ be a fuzzy retraction of $H^* (\eta) \subseteq \tilde{M}^{n+1} (\eta)$ onto $\tilde{H}^2 (\eta) \subseteq \tilde{M}^1 (\eta)$. Then $M \subset \tilde{D}^1 (\eta) \subseteq \tilde{R}^2 (\eta)$ is equal to the maximum folding of $H^* (\eta)$, $\tilde{f} (H^* (\eta)) = \tilde{H}^* (\eta)$.

\textbf{Proof:} Let $H^* (\eta) \subseteq \tilde{M}^{n+1} (\eta)$ be a fuzzy hyperboloid in fuzzy Minkowski space time $\tilde{M}^{n+1} (\eta)$ and the fuzzy retraction of $H^* (\eta) \subseteq \tilde{M}^{n+1} (\eta)$ is given by $\tilde{r}_k (\eta)(\tilde{H}^k (\eta)): \tilde{H}^{k-1} (\eta) \subseteq \tilde{M}^{k+1} (\eta)$, $k = n-1$, $n-2$, ... 4, 3, 2. Under the central projection $\tilde{p} (\eta)$, where $\tilde{p} (\eta): H^* (\eta) \subseteq \tilde{M}^{n+1} (\eta) \rightarrow \tilde{D}^{k-1} (\eta) \subseteq \tilde{R}^k (\eta)$, there exists an induced retraction of $\tilde{D}^{k-1} (\eta) \subseteq \tilde{R}^k (\eta)$ given by $\tilde{r}_k (\tilde{D}^k (\eta)): \tilde{D}^{k-1} (\eta) \subseteq \tilde{R}^k (\eta)$, $k = n-1, n-2, ... 4, 3, 2$. Now consider the fuzzy folding $\tilde{f} (\eta): H^* (\eta) \rightarrow \tilde{H}^* (\eta)$ such that $\tilde{f} (H^* (\eta)) = H^* (\eta)$. Then under the central projection induces fuzzy folding $\tilde{f}_n (\eta): \tilde{D}^{n-1} (\eta) \rightarrow \tilde{D}^{n-1} (\eta)$, $\tilde{f}_n (\eta)(\tilde{D}^{n-1} (\eta)) = \tilde{D}^{n-1} (\eta)$), then the fuzzy retraction induces fuzzy folding $\tilde{f}_n (\eta)(\tilde{H}^2 (\eta)) = \tilde{H}^2 (\eta)$ where $\tilde{f}_n (\eta)(\tilde{H}^2 (\eta)) = \tilde{H}^2 (\eta)$. Hence $\tilde{f}_n (\eta)(\tilde{H}^2 (\eta) = \tilde{H}^2 (\eta)$.

\textbf{Theorem 4:} The maximum fuzzy folding of $\tilde{H}^2 (\eta) \subseteq \tilde{M}^2 (\eta)$ into itself is equal to the fuzzy circle $\tilde{C}^1 (\eta) \subseteq \tilde{R}^2 (\eta)$ or $\tilde{H}^1 (\eta) \subseteq \tilde{M}^1 (\eta)$.

\textbf{Proof:} Let $\tilde{H}^2 (\eta) \subseteq \tilde{M}^2 (\eta)$ be a fuzzy hyperboloid in fuzzy Minkowski space time $\tilde{M}^2 (\eta)$, and $\tilde{f} (\eta): \tilde{H}^2 (\eta) \rightarrow \tilde{H}^2 (\eta)$. Since $\tilde{f} (\tilde{H}^2 (\eta) \subseteq \tilde{M}^1 (\eta)) = \tilde{C}^1 (\eta) \subseteq \tilde{R}^2 (\eta)$. This retraction of $\tilde{H}^2 (\eta) \subseteq \tilde{M}^2 (\eta)$ is a fuzzy circle $\tilde{C}^1 (\eta) \subseteq \tilde{R}^2 (\eta)$, then $\max \tilde{f} (\tilde{H}^2 (\eta) = \tilde{C}^1 (\eta) = \tilde{H}^1 (\eta) \subseteq \tilde{M}^1 (\eta)$. Also max $\tilde{f} (\tilde{H}^2 (\eta) \subseteq \tilde{M}^1 (\eta) = \tilde{f} (\tilde{H}^2 (\eta) = \tilde{H}^1 (\eta) \subseteq \tilde{M}^1 (\eta)$.

\textbf{Theorem 5:} The minimum fuzzy folding of $\tilde{H}^2 (\eta) \subseteq \tilde{M}^2 (\eta)$ induces minimum fuzzy folding of retraction of $\tilde{H}^2 (\eta) = \tilde{M}^3 (\eta)$ equal to $\tilde{E}^1 (\eta) \subseteq \tilde{R}^3 (\eta)$.

\textbf{Proof:} Assume $\tilde{f}_n (\eta): \tilde{H}^2 (\eta) \subseteq \tilde{M}^2 (\eta) \rightarrow \tilde{H}^2 (\eta) \subseteq \tilde{M}^1 (\eta)$ be a fuzzy folding from $\tilde{H}^2 (\eta)$ into $\tilde{H}^2 (\eta)$ such that $\tilde{f}_n (H^2 (\eta)) = \tilde{H}^2 (\eta)$. Also let $\tilde{f}_n (\eta): \tilde{f}_n (\tilde{H}^2 (\eta)) \rightarrow \tilde{f}_n (\tilde{H}^2 (\eta))$.

Since $\tilde{f}_n (\tilde{H}^2 (\eta) \subseteq \tilde{M}^1 (\eta)) = \tilde{C}^1 (\eta) \subseteq \tilde{R}^2 (\eta)$, then the fuzzy folding $\tilde{f}_n (\tilde{H}^2 (\eta)) = \tilde{H}^2 (\eta)$ induces the fuzzy folding of retraction given by $\tilde{f}_n (\tilde{H}^2 (\eta)) \rightarrow \tilde{f}_n (\tilde{C}^1 (\eta))$ such that $\tilde{f}_n (\tilde{H}^2 (\eta)) = \tilde{H}^2 (\eta)$. Also let
**Proof:** Let \( \tilde{H}^n(\eta) \subseteq \tilde{M}^{n+1}(\eta) \) be a fuzzy hyperboloid in a fuzzy Minkowski space time \( \tilde{M}^{n+1}(\eta) \) with retractions \( \tilde{H}^{n-1}(\eta), \tilde{H}^{n-2}(\eta), ..., \tilde{H}^{n}(\eta), \tilde{H}^{n+1}(\eta) \). Then we get the following chain.

\[
\tilde{f}_n(\eta) \rightarrow \tilde{f}_{n-1}(\eta) \rightarrow \tilde{f}_{n-2}(\eta) \rightarrow \tilde{f}_{n-3}(\eta) \rightarrow \tilde{f}_{n-4}(\eta) \rightarrow \tilde{f}_{n-5}(\eta) \rightarrow \tilde{f}_{n-6}(\eta) \rightarrow \tilde{f}_{n-7}(\eta) \rightarrow \tilde{f}_{n-8}(\eta) \rightarrow \tilde{f}_{n-9}(\eta) \rightarrow \tilde{f}_{n-10}(\eta) \rightarrow \tilde{f}_{n-11}(\eta) \rightarrow \tilde{f}_{n-12}(\eta) \rightarrow \tilde{f}_{n-13}(\eta) \rightarrow \tilde{f}_{n-14}(\eta) \rightarrow \tilde{f}_{n-15}(\eta) \rightarrow \tilde{f}_{n-16}(\eta) \rightarrow \tilde{f}_{n-17}(\eta) \rightarrow \tilde{f}_{n-18}(\eta) \rightarrow \tilde{f}_{n-19}(\eta) \rightarrow \tilde{f}_{n-20}(\eta) \rightarrow \tilde{f}_{n-21}(\eta) 
\]

This is the general commutative diagram such that \( \tilde{f}_{n+1}(\eta) \circ \tilde{f}_n(\eta) = \tilde{f}_n(\eta) \circ \tilde{f}_i(\eta) \).

Now, under the central projection map \( \tilde{P}(\eta) \), there exists a fuzzy open disk \( \tilde{D}^{n-1}(\eta)(p_r, r) \subseteq \tilde{R}^n(\eta) \) with retractions, \( \tilde{D}^{n-2}(\eta)(p_r, r_1), \tilde{D}^{n-3}(\eta)(p_r, r_2), ..., \tilde{D}^1(\eta)(p_r, r_{n-1}), \tilde{D}^0(\eta)(p_r, r_n) \) \( (p_r, r_n) \), \( \tilde{D}^0(\eta)(p_r, r_n) \) where \( r_{n-1} < r_{n-2} < r_{n-3} < ... < r_i < r_2 < r_1 \). Also we get the following chain.

\[
\tilde{R}_n(\eta) \rightarrow \tilde{R}_{n-1}(\eta) \rightarrow \tilde{R}_{n-2}(\eta) \rightarrow \tilde{R}_{n-3}(\eta) \rightarrow \tilde{R}_{n-4}(\eta) \rightarrow \tilde{R}_{n-5}(\eta) \rightarrow \tilde{R}_{n-6}(\eta) \rightarrow \tilde{R}_{n-7}(\eta) \rightarrow \tilde{R}_{n-8}(\eta) \rightarrow \tilde{R}_{n-9}(\eta) \rightarrow \tilde{R}_{n-10}(\eta) \rightarrow \tilde{R}_{n-11}(\eta) \rightarrow \tilde{R}_{n-12}(\eta) \rightarrow \tilde{R}_{n-13}(\eta) \rightarrow \tilde{R}_{n-14}(\eta) \rightarrow \tilde{R}_{n-15}(\eta) \rightarrow \tilde{R}_{n-16}(\eta) \rightarrow \tilde{R}_{n-17}(\eta) \rightarrow \tilde{R}_{n-18}(\eta) \rightarrow \tilde{R}_{n-19}(\eta) \rightarrow \tilde{R}_{n-20}(\eta) \rightarrow \tilde{R}_{n-21}(\eta) 
\]

This is the general commutative diagram such that \( \tilde{g}_{i+1}(\eta) \circ \tilde{R}_i(\eta) = \tilde{R}_i(\eta) \circ \tilde{g}_i(\eta) \).

### 3. Applications

3.1 The stream function of the acoustic gravity tripo
der of vortices is generalized to permit a study of the Earth’s atmosphere under complex meteorological conditions, characterized by sheared horizontal flows and parabolic density and pressure profiles [7]. See Figure 7.

3.2 Consider the flow of the fluid inside a tube [3]. If we represent the velocity of the fluid as a membership degree \( \mu \in [0,1] \), then \( \mu = 1 \), in the mid of the medium where the velocity of the fluid takes a maximum and is symmetric round this line but at the edge of the tube the velocity of the fluid vanishes, i.e., \( \mu = 0 \).

3.3 The Ritz variational method [6] during the calculation of the ground state energy in a fuzzy framework. Consider a Hamilton \( H \), and an arbitrary square integrable function \( \Psi \), so that \( \langle \Psi / \Psi \rangle = 1 \). Considering \( \Psi \) as a fuzzy function and the ranking system as defined in [12], similar to [6] it can be shown that \( \langle \psi / H / \psi \rangle \) is a fuzzy upper bound on \( E_0 \) (ground-state energy). Now \( \langle \psi / H / \psi \rangle \) should be minimizing the distance between \( E_0 \) and \( \langle \psi / H / \psi \rangle \). This can be done by minimizing distance between \( E_0 \) and \( \langle \psi / H / \psi \rangle \). The rest of the discussion is the same as that provided in [6].

### 4. Conclusion

In the present paper, we obtain and study some types of fuzzy retractions of fuzzy hyperboloid in a fuzzy Minkowski space. Also, we deduced some types of fuzzy deformation retract of fuzzy hyperboloid. The relations between the fuzzy folding and the fuzzy deformation retracts of fuzzy hyperboloid are obtained. Some applications are presented.

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6. References