A Simple Proof on Coloring of Dominated Special Graphs

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Abstract

In this simple survey paper we prove a simple result for a coloring on dominated Special graphs.

1. Introduction

Definition 1
A proper coloring of a graph $G = (V(G), E(G))$ is an assignment of colors to the vertices of the graph, such that any two adjacent vertices have different colors.

Definition 2
A dominator coloring of a graph $G$ is a proper coloring in which each vertex of the graph dominates every vertex of some color class.

Definition 3
A $k$-coloring of $G$ is a coloring that uses at most $k$ colors. The chromatic number of $G$ is $\beta(G) = \min\{k / G$ has a proper $k$-coloring$\}$. A coloring of $G$ can also be thought of as a partition of $V(G)$ into color classes $V_1, V_2, \ldots, V_q$, and a proper coloring of $G$ is then a coloring in which each $V_i, 1 \leq i \leq q$ is an independent set of $G$, i.e., for each $i$, the subgraph of $G$ induced by $V_i$ contains no edges.

Definition 4
A dominating set $S$ is a subset of the vertices in a graph such that every vertex in the graph either belongs to $S$ or has a neighbor in $S$. The domination number is the order of a minimum dominating set.

2. Theorem

For the path $P_n$ on $n$ vertices, $\beta_{cpl}(P_n) = 2j$, $\beta_{cpl}(P_{nj+1}) = \beta_{cpl}(P_{nj+2}) = 2j + 1$, $\beta_{cpl}(P_{nj+3}) = \beta_{cpl}(P_{nj+4}) = \beta_{cpl}(P_{nj+5}) = \beta_{cpl}(P_{nj+6}) = 2j + 2$, and $\beta_{cpl}(P_{nj+7}) = 2j + 3$.

Proof
Let $n = 8j + r$ where $0 \leq r \leq 7$. Clearly $\beta_{cpl}(P_1) = 1$, and $\beta_{cpl}(P_2) = 1$. If $3 \leq r \leq 6$ we can choose the two vertices in any color class of order two or any two on adjacent singleton color classes to get $\beta(P_i; S) \geq 2$ for any coupled proper coloring, and it is easy to find a particular $S$ for which $\beta(P_i; S) = 2$, so $\beta_{cpl}(P_n) = 2$. For $P_7$, the proper coloring $(1, 2, 4, 3, 2, 3, 1, 4, 5, 6, 8, 7, 6, 7, 5, 8, \ldots, 4_{j-3}, 4_{j-2}, 4_j, 4_{j-1}, 4_{j-2}, 4_{j-1}, 4_j, \ldots, 4_{j-3}, 4_{j-2}, 4_{j-1})$ shows that $\beta_{cpl}(P_7) \geq 3$. Let $S$ be any coupled proper coloring of $P_7$. If $S$ has one singleton color class and three pairs, let the singleton be $v_j$, then at least one of the colored pairs has neither vertex adjacent to $v_j$, so $\beta(P_7; S) \geq 3$. Suppose $S$ has at least three singleton color classes $\{v_i\}, \{v_j\}, \text{and} \{v_k\}$ with $i < j < k$.

If $i \leq j - 2$ and $k \geq j + 2$, then $\beta(P_7; S) \geq 3$. If, for example, $i = j - 1$ then there are at least three vertices whose colors are not those of $v_j$, $v_j$, and $v_{j+1}$, so one can use a color pair or two singletons from these three vertices along with $v_j$ to see that $\beta(P_7; S) \geq 3$.

Hence, $\beta_{cpl}(P_7) = 3$. For $n = 8j + r$ with $j \geq 1$, $8j$ consecutive vertices will be colored as $L: (1, 2, 4, 3, 2, 4_{j-1}, 4_{j-2}, 4_{j-3}, 4_{j-2}, 4_{j-1}, 4_j, \ldots, 4_{j-3}, 4_{j-2}, 4_{j-1}, 4_j, 4_{j+1}, 4_{j+2}, 4_{j+3}, 4_{j+2}, 4_{j+1}, 4_j)$. For each group of four colors, at most one pair can be used in any independent set. Use $L$ for $P_{8j+1}$, $(4_{j+1}, L)$ for $P_{8j+2}$, $(4_{j+1}, 4_{j+2}, L)$ for $P_{8j+3}$, $(4_{j+1}, 4_{j+2}, 4_{j+3}, L)$ for $P_{8j+4}$, $(4_{j+1}, 4_{j+2}, 4_{j+3}, 4_{j+4}, L)$ for $P_{8j+5}$, $(4_{j+1}, 4_{j+2}, 4_{j+3}, 4_{j+4}, 4_{j+5}, L)$ for $P_{8j+6}$, and $(4_{j+1}, 4_{j+2}, 4_{j+3}, 4_{j+4}, 4_{j+5}, 4_{j+6})$.
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To conclude, it is noted that along with further study of colored-domination and colored independence,
We have a colored-problem associated with essentially every graph parameter. 196 Colored Problems in Graphs

3. References


\[ \begin{align*}
4_{j+4}, 4_{j+1}, 4_{j+3}, 4_{j+2}, L \) for \( P_{8j+7} \) to see that \( \beta_{cpl}(P_{8j}) \leq 2j, \beta_{cpl}(P_{8j+1}) \leq 2j+1, \ldots, \beta_{cpl}(P_{8j+7}) \leq 2j+3. \\

Let \( S \) be any proper coupled coloring of \( P_n \) with \( n \geq 8. \) We can find a sufficiently large \( S \)-independent set as follows. Start with \( S = \emptyset \) and repeat the following until fewer than eight vertices remain. Choose a vertex \( v \) of degree at most one. Put \( v \) in \( S \) and if there is another vertex \( v' \) with \( \{v, v'\} \in S, \) then also put \( v' \) in \( S. \) Delete \( v, v', \) and any vertex in \( N(v) \in N(v') \) or of the same color as a vertex in \( N(v) \in N(v'). \) If \( \{v, v'\} \in S, \) then two vertices are added to \( S \) and at most eight deleted. If \( \{v\} \in S, \) then one vertex is placed into \( S \) and at most three are deleted. At the first point where fewer than eight vertices remain, at least one-fourth of the deleted vertices are in \( S, \) that is \( |S| \geq 2j. \) From what remains we can add the required number of vertices to \( S \) in the same manner as we did for \( P_1, P_2, \ldots, P_j. \) Hence, \( \beta_{cpl}(P_{8j}) \geq 2j, \beta_{cpl}(P_{8j+1}) \geq 2j+1, \ldots, \beta_{cpl}(P_{8j+7}) \geq 2j+3, \) and the proof is complete.