Abstract

For the gun, the hit probability is a function of many error budgets. A selected design parameter among error budgets is assumed to be a vertical gun jump. The hit probability depends on the vertical gun jump for a given range. To satisfy a given hit probability for the given range, the vertical gun jump should be confined into the interval. At first, to obtain a vertical variance at the target, the unit partial is determined using the trajectory. The unit partial means the difference between two vertical displacements at a given range under two fire angles whose difference is the unit quantity. Using the unit partial and the variance of the vertical gun jump, we may determine the hit probability using the Polya-Williams’ formula. Finally, we may obtain the allowable interval for a given jump within which the given hit probability is satisfied.

Keywords: Allowable Interval, Design Parameters, Error Budgets, Hit Probability, Optimal Design Framework, Standard Deviation, Vertical Gun Jump

1. Introduction

Hit probability and range requirement affect the selection of gun and platform components from libraries of the gun and platform components. The libraries of the gun and platform components can be varied according to the given operational combat environment scenarios. For selected gun and platform components, the sensitivity analysis for design parameters is performed. To satisfy the hit probability and range, the maximum standard deviations of the design parameters should be determined around their nominal values.

With determined standard deviations and nominal values of design parameters, the estimation of hit probability and range are carried out. If the hit probability and range estimation satisfy their initial requirements, then optimal design framework is done for design parameters. If not, gun and platform components from their libraries are reselected and then the estimation of hit probability and range can be repeated.

Some error budgets affect the hit probability and range. The error budgets for the hit probability for the gunnery are four categories such as the mechanical errors, the specification of the weapon associated with the gun, meteorological errors and the sensor errors. The external ballistics is related to hit probability since the bullet delivery is determined by the trajectory.

In this paper, the estimation of the hit probability and range are achieved. A selected design parameter among many error budgets is assumed to be a vertical gun jump in the category of the mechanical errors. The hit probability depends on the vertical gun jump for a given range. To satisfy a given hit probability for the given range, the vertical gun jump should be confined into the interval. The maximum allowable interval for the vertical gun jump for the gun is determined. In other words, the maximum
standard deviation for vertical gun jump may be determined for optimal design framework.

2. Important Equations for Trajectories and Hit Probability

2.1 The Trajectory and Related Equations

The standard differential equations 1,3,4,5 are described as:

\[
\frac{dv_x}{dx} = -C_d v(v_x - w_x)
\]
\[
\frac{dv_z}{dz} = -C_d v(v_z - w_z)
\]
\[
\frac{dv_y}{dy} = -C_d v(v_y - w_y) - g
\]

where, \(v_x, v_z, v_y\) are range, upward and cross range velocity, respectively and \(w_x, w_z, w_y\) are range, upward and cross range velocity of winds, respectively. The constant value \(C_d\) is a modified drag coefficient value and the value is assumed to be \(1.8253 \times 10^{-4}\) m\(^{-1}\). The value \(g\) is the gravitational acceleration. The value \(v\) is the scalar value of the velocity vector and is expressible as \(\sqrt{v_x^2 + v_y^2 + v_z^2}\) m/sec.

The above standard differential equations are solved by the numerical analysis. One of the numerical method is the Runge-Kutta method. The Runge-Kutta method is used in this paper.

Suppose that the range is set to be \(x = 700\)m. Assume that the muzzle velocity \(v_m\) is \(241\) m/sec and the gun firing angle is 5.0 degree. Then using the trajectory derived from the differential equations, the unit partial may be determined by \(\frac{dy}{d\phi_0} = 0.3518 \left[ \frac{m}{\text{mil}} \right]\). The unit partial \(\frac{dy}{d\phi_0}\) means the difference between the vertical displacement under the fire angle 5 degree+0.5 mili-radian and that under the fire angle 5 degree-0.5 mili-radian. Figure 1 shows two trajectories and Figure 2 shows the zoom-in view for Figure 1. As shown in Figure 2, at the target, the variance due to the error of the fire vertical angle is

\[
\sigma_y^2 = 0.3518 \cdot \sigma_v^2
\]

2.2 Hit Probability

If the hit probability on a rectangular area is described by

\[
\text{pssh} = \frac{1}{2\pi\sigma_x\sigma_y} \int_{-(x_h+y_k)}^{x_h+y_k} \exp \left( \frac{-((x-h)^2)}{2\sigma_x^2} \right) \exp \left( \frac{-((y-k)^2)}{2\sigma_y^2} \right) dx \] 

where, \(\sigma_x\) and \(\sigma_y\) are the standard deviations of the horizontal and vertical distributions of impact points and \(h\) and \(k\) denote the horizontal and vertical centers of hitting patterns. The width and the height of the rectangular target are \(2a\) and \(2b\), respectively. Assume that the aiming point is the same as the center of the hitting pattern, then we obtain \(h-k=0\)

The area of the target is given by \(-a \times a, -b \times b\). The hit probability can be expressed as:

\[
P_h = \frac{1}{\pi} \int_{-a}^{a} \int_{-b}^{b} \frac{a}{\sqrt{2\pi} x e^{-x^2}} dx \int_{0}^{\infty} \frac{b}{\sqrt{2\pi} y e^{-y^2}} dy
\]

Figure 1. Two trajectories with different gun jump angle.

\[
\frac{dy}{d\phi_0} = 0.3518 \left[ \frac{m}{\text{miliradian}} \right]
\]
where the error function
\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \]
is introduced. Then, we obtain that the hit probability is represented by
\[ p_h = \text{erf} \left( \frac{a}{\sqrt{2}\sigma_x} \right) \text{erf} \left( \frac{b}{\sqrt{2}\sigma_y} \right) \]
If the width of the target is infinitely long, then
\[ p_h = \text{erf} \left( \frac{b}{\sqrt{2}\sigma_y} \right) \]
Using the approximate identity \( \text{erf}(x) \equiv \sqrt{1 - \text{erf}^2 \left( \frac{4x^2}{\pi} \right)} \), we obtain the Polya-Williams’ approximation
\[ p_h = \sqrt{1 - \text{erf} \left( \frac{2b^2}{\pi\sigma_y^2} \right)} \]
Under a given hit probability \( p_h \) the maximal allowance of the standard deviation of the vertical gun jump is either
\[ \sigma_{y*} = \frac{b}{\sqrt{2}\text{erf}^{-1}(p_h)} \quad \text{or} \quad \sigma_{y*} = \frac{\sqrt{-2}}{\pi\ln(1-(p_h^2))} \]
By using
\[ \sigma_{y*}^2 = 0.3518^2 \sigma_{yo}^2 \]
we obtain
\[ \sigma_{y*} = \frac{b}{0.3518\sqrt{2}\text{erf}^{-1}(p_h)} \quad \text{or} \quad \sigma_{y*} = \frac{\sqrt{-2}}{0.3518\sqrt{\pi\ln(1-(p_h^2))}} \]

3. Results

The initial gun jump angle \( \phi \) is assumed to be 5 degrees. The value of target height \( h \) is given by 2m. In Table 1, data for height of target and firing elevation angle are tabulated. To satisfy the minimum hit probability (50%) for the range 700m, it turns out that the maximum allowable standard deviation for gun oscillation is equal to 8.4287 mil (mili-radian). The approximate maximum value of the gun jump error is 8.4570 mil (mili-radian). Figure 3 shows a maximum allowable standard deviation of the gun jump under the given each hit probability.

4. Discussion

For the gun, the hit probability is a function of many error budgets. A selected design parameter among error budgets is assumed to be a vertical gun jump. Then hit probability depends on the vertical gun jump for a given range. To satisfy a given hit probability for the given range, the vertical gun jump should be confined into the interval. By an experiment, we have determined the maximum standard deviation of the vertical gun jump to satisfy the given hit probability.

5. Acknowledgement

This study was supported by the Research Program (the specialized research center for future ground system analysis (Contract No. 311165-911074201)) funded by the Agency for Defense Development in South Korea.

<table>
<thead>
<tr>
<th>Table 1. The parameters of the target and firing elevation angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of target (m3)</td>
</tr>
<tr>
<td>Initial firing elevation angle (deg)</td>
</tr>
</tbody>
</table>

![Figure 3. Hit probability as a function of standard deviation of maximum allowable gun jump.]
6. References