Enhancing GPS Receiver Tracking Loop Performance in Multipath Environment using an Adaptive Filter Algorithm

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Abstract
Accurate tracking of the GPS signal in a base band correlator is critical for the positioning accuracy. However, the performance of the received signal is degraded by the presence of a multipath signal. This paper proposes the tracking of the GPS signal in the presence of the multipath using a RAKE receiver with multiple fingers. Each finger in the RAKE receiver is modelled to track the direct and respective multipath signals using the Least Mean Square (LMS) algorithm. The LMS algorithm provides an estimate of the multipath signal, and the RAKE combiner suitably combines the navigation data from the individual finger with the estimated multipath value and provides the final navigation data. A discussion on the use of the LMS for the carrier and code tracking is also presented.

Keywords: Multiple Finger Combiner, LMS, Multipath, RAKE Receiver

1. Introduction
Multipath in a GPS receiver is the result of reflections or diffractions of the satellite signals from objects such as ground, water surfaces, vehicles, hills, trees and nearby buildings. The reflected signal takes more time to reach the receiver than that of the direct signal (line-of-sight signal). The GPS receiver cannot distinguish between the direct and the reflected signals. Hence, the receiver’s tracking loops can align the locally generated code and carrier to the composite signal of the direct and the reflected signals instead of the direct signal alone causing the multipath error¹. The multipath signals which are coming far behind can easily be eliminated using the auto correlation and cross correlation properties of the GPS signal. However, the multipath signal following the direct signal at close proximity are serious in nature, as they cannot be completely eliminated by the correlation properties and some suitable form of multipath mitigation has to be implemented in the receiver. When the multipath signal arrives with one chip gap, the correlation function of the signal can differentiate it from the direct signal. When the multipath delay is different from the correlator design value (early - late spacing value) then some form of mitigating the multipath is needed for the proper tracking of the GPS signal. In GPS applications, the desired signal is only the direct path signal; all other signals distort the C/A code and L2C code modulations and carrier phase observations of the desired signal¹. The reflected signals shift the correlation peak and corrupt the theoretically symmetric receiver correlation envelope¹. These phenomena cause the ranging measurements error in both the stationary and mobile GPS receivers. The multipath performance is affected by the strength, delay and phase of the reflected signals. Since, the characteristics of the direct and reflected paths are not known, it makes the modelling of the multipath error more difficult. The multipath error is difficult to

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remove\textsuperscript{4}, since the pseudo range measurement is derived from the code tracking DLL and pseudo range errors due to multipath are nonlinear functions of multipath amplitude delay, phase and phase error. Further the multipath is not spatially correlated and the multipath error is not zero-mean.

The multipath tracking error can be reduced by the following methods\textsuperscript{5}:

1. Minimising the multipath power by selecting the optimum radiation pattern of the receiving antenna.
2. Changing the polarisation and location for the receiver antenna.
3. Optimise the estimation techniques of the receiver with respect to the multipath fading channel.

Specular multipath arise due to smooth surface like standing water and are discrete in nature and the diffuse multipath arise due to scatterers that are diffuse in nature and form source of diffraction\textsuperscript{6}. To get accurate positioning results, it is necessary to minimise the magnitude of multipath disturbances that are available in the GPS observations. The LMS algorithm\textsuperscript{7} is a popular algorithm for estimation of multipath in such a scenario.

One of the methods to mitigate the multipath effects in GPS receivers is the use of the RAKE receivers. RAKE receiver employs some form of estimation algorithms to estimate the presence of the signal. RAKE receiver has two or more number of fingers depending on the number of multipath signals that are expected for estimation in the specific application. In each of the fingers of the RAKE receiver, the estimation algorithm is designed to estimate presence of the respective multipath. Hence, the RAKE receivers designed with multiple fingers are capable of mitigating the multipath effects. The RAKE receiver in spread spectrum applications is nothing but a bank of correlators. Each correlator has two inputs, which are the received GPS input signal and the spreading sequence with the required offset. In each of the correlators, the multipath can be estimated using the LMS algorithm. The use of RAKE structure for the frequency tracking using FLL (Frequency Locked Loop) is discussed in\textsuperscript{8}. The multipath mitigation using adaptive filters for a static receiver like a reference receiver of a DGPS (Differential GPS) is presented in\textsuperscript{8,10} proposes a multipath mitigation technique for tracking GPS signal in a typical indoor application based on Deconvolution.

2. GPS Multipath and its Mitigation

2.1 Multipath

The input signal at the receiver in a multipath scenario can be represented as sum of the direct and reflected signals:

\[ s(t) = s_d(t) + s_m(t) \]  

The direct signal can be represented as

\[ s_d(t) = A c(t) \sin \omega_0 t \]  

where \( A \) is the amplitude, \( c(t) \) is the modulated code, \( t \) is the time delay and \( \omega_0 \) is the frequency of the direct (line of sight) GPS signal.

The multipath can be represented as added signal of all the reflected signals (1 to L)

\[ s_m(t) = \sum_{l=1}^{L} A c(t - \tau_l) \sin(\omega_0 t + \theta_l) \]  

The same can be modified and represented as a consolidated multipath signal

\[ s_m(t) = \omega_m A c(t - \delta_m^*) \sin(\omega_0 t + \theta_m^*) \]  

The received GPS signal at the receiver can be represented as the direct and the multipath signals:

\[ s(t) = s_d(t) + s_m(t) \]  

\[ s(t) = A c(t) \sin \omega_0 t + \omega_m A c(t - \delta_m^*) \sin(\omega_0 t + \theta_m^*) + n(t) \]  

where

\( \omega_m^* A, c(t - \delta_m) and \theta_m^* \) - Amplitude, delay and phase of the multipath signal

\( n(t) \) - zero-mean Gaussian noise \textbf{8b}

The conventional methods do not provide the flexibility for implementation in SDR platform and mitigation solution for dynamically updating the model. Hence, for modelling the multipath, LMS algorithm is adopted here. The LMS algorithm can provide the update of the error signal which can be used for dynamic updates.

2.2 Multipath Adaptation using LMS Algorithm

The LMS algorithm\textsuperscript{7} can be effectively utilized for the adaptation of the tracking loop in the multipath environment. The LMS algorithm is capable of modeling the presence of the multipath signal and provides the estimates
that can be used by the multipath mitigating algorithm. The inputs and outputs of the LMS algorithm for adapting to the various situations are summarised below:

Input:
- Tap-weight vector, \( W(n) \)
- Input vector, \( x(n) \)
- Desired output, \( d(n) \)

Output:
- Filter output, \( y(n) \)
- Tap-weight vector update, \( W(n + 1) \)

The LMS filter output with \( N \) taps can be represented as

\[
y(n) = \sum_{i=0}^{N-1} w_i(n)x(n - i) \quad (7)
\]

Equation (7) can be modified to represent as

\[
y(n) = W^T(n)x(n) \quad (8)
\]

The error estimation is given by

\[
e(n) = d(n) - y(n) \quad (9)
\]

The adapted tap weights’ update is given by

\[
W(n + 1) = W(n) + 2\mu e(n)x(n) \quad (10)
\]

2.3 RAKE Receiver for Multipath Mitigation

A simplified block diagram of the RAKE receiver with elements to detect the \( L \) strongest multipath components is shown in Figure 1. The input signal \( R(t) \) is multiplied by the sine and cosine components of the local oscillator to get the \( I(X_l(t)) \) and \( Q(X_q(t)) \) components to process as a Costas PLL loop. The I and Q components are filtered using a low pass filter and fed to the RAKE fingers for further processing. Each of the RAKE fingers represented in the diagram is a correlator used in the conventional tracking applications. The RAKE fingers are used to detect the multipath components in the received signal and the number of fingers depends on the number of the multipath signals expected due to reflection. All the fingers in the receiver get the reference input signal for the proper tracking output estimation with the finite delay as per the expected delay in the multipath signal.

The outputs from various fingers can be combined using multiple algorithms to obtain improved results. From the Figure 1, the RAKE receiver for GNSS applications is essentially a bank of correlators. Each correlator has two input signals the received signal and a spreading sequence with offset. Mathematically, the correlator can be expressed as

\[
y(nT_s) = \int_{(n-1)T_s}^{nT_s} r(t)c(t - t_\ell)dt \quad (11)
\]

where \( T_s \) is the symbol duration, \( r(t) \) is the received signal, \( c(t) \) is the spreading sequence and \( t_\ell \) is the channel propagation delay. In a multipath channel, received signal \( r(t) \) is a combination of multiple replicas of the transmitted signal whereas each has propagated through different path to arrive the receiver’s antenna. Each path introduces its own path loss (gain) and delay. The received signal can be written as

\[
r(t) = \sum_{\ell=0}^{L-1} g_\ell x(t - t_\ell) + n(t) \quad (12)
\]

where \( L \) is the number of paths, \( g_\ell \) is the \( \ell \)th path gain, \( t_\ell \) is the delay and \( n(t) \) is the channel background noise.

Ideally the system will be using \( L \) correlators to process the received signal to fully exploit the multipath gain. This bank of correlators is otherwise called as \( L \)-finger RAKE receiver whose output can be written as

\[
z(nT_s) = \sum_{\ell=0}^{L-1} g_\ell y_\ell(nT_s) \quad (13)
\]

The delay estimates \( t_\ell \) of the RAKE receiver should be as close to the path delays \( t_\ell \) as possible. Autocorrelation of the spreading sequence is practically zero if the offset parameter is larger than one chip duration, which means that if any channel delay estimate is equal or larger than one chip duration, the associated correlator output contains only noise. Each correlator output is weighed by coefficient \( g_\ell \), these weighted outputs are summed together by the combining process. There are basically two choices for the coefficients: equal gain combining (all \( g_\ell \) equal to 1) and maximum ratio combining (\( g_\ell \) equal to complex...
conjugate of \(x\). The \(p^{th}\) finger discrete-time correlator can be expressed as

\[
y[n] = \sum_{t=0}^{N-1} r[n - (N - m_l)]e[n - N]
\]  

where \(N\) is the spreading ratio, \(m_l\) an integer that has discretised \(p^{th}\) path delay (assuming path delays are multiple integer of chip duration).

The basic configuration of the \(k^{th}\) \((k = 1, 2 \ldots L)\) finger for detecting the \(k^{th}\) delayed version of the transmitted signal is given in Figure 2.

The remaining fingers configuration looks the same as given Figure 2, except that the appropriate time delay is given for the PRN code.

### 2.4 RAKE Finger Combiner

The first method of combining the finger outputs is equal gain combining. Here the gains of all the finger paths are set to 1. Irrespective of the dominance of the reflection parameter level in the finger, the gain is uniformly set to all the fingers and their estimates.

\[
m(t) = m_1(t) + m_2(t) + \ldots + m_L(t)
\]

where

\[
m_1(t), m_2(t), \ldots m_L(t) \quad \text{- Navigation message from fingers 1, 2 \ldots L}
\]

The next method is the maximum ratio combining. Here the weight age is assigned to the individual fingers path as per the dominance of the reflection parameter in the overall signal. The gain in the individual path of the finger is set to be in accordance with the complex conjugate of the amplitude of the reflected signal for the corresponding finger.

\[
m(t) = m_1(t)w_1 + m_2(t)w_2 + \ldots + m_L(t)w_L
\]

where

\[
m_1(t), m_2(t), \ldots m_L(t) \quad \text{- Navigation message from fingers 1, 2 \ldots L}
\]

\[
w_1, w_2, \ldots w_L \quad \text{- Weights from the fingers 1, 2 \ldots L}
\]

The value of the weights from the fingers depends on the presence of the multipath components available in the input signal. This means that some form compensation has been done for the received signals depending on the level of multipath and the time delay of the reflected signal with respect to the direct signal. The final data \(m(t)\) will be multipath mitigated message, which is free from the influence of multipath signals.

### 3. Simulation and Results

#### 3.1 LMS Implementation

The LMS algorithm is used initially to analyse the performance of the code tracking loop and the carrier tracking loop separately before using into the combined tracking function. After the observations of the performance of the individual code and carrier tracking loops, to the LMS algorithm, the full tracking architecture with the LMS algorithm will be developed. First the performance study is carried out for the code tracking loop. The implementation architecture of the LMS algorithm for code loop tracking is represented in Figure 3.

The code correction analysis for multipath mitigation is implemented using Matlab. The PRN code was generated and fed to the multipath channel. The multipath affected signal from the channel is fed to the LMS algorithm as the input signal \(x(n)\) and also fed to the code tracking loop. A sample of the generated PRN code is also fed to the LMS algorithm as the desired signal \(d(n)\). The LMS adapted error signal \(e(n)\) which provides an estimate of the multipath is added with the error signal from the code discriminator \(e_c(n)\) and fed to the code generator for providing the required correction.
The plots representing various stages of the code tracking loop that is capable of multipath adaptation using the LMS algorithm are given in Figure 4. Various plots that are shown in Figure 4 are: 1) the PRN code as the desired signal, 2) the multipath signal, 3) the PRN code with the multipath signal, 4) the LMS output and 5) the LMS error signal. From the Figure 4, it is observed that the LMS algorithm is capable of following the variations in the multipath input signal by comparing the desired signal. This indicates that the multipath variation in the code can be modeled by the use of LMS algorithm. A threshold can be set to limit the LMS output amplitude, as the timing of the crossover of the signal which determines the pseudo range calculation is more important than the amplitude.

The carrier tracking architecture of the LMS adaptation is given in Figure 5. The input signal to the LMS is the GPS base band signal that is affected by the multipath and noise in the channel. The desired signal is the locally generated GPS base band signal with the prompt code added to the carrier. The output from the LMS filter is the navigation data error signal and the weights of the filter.

In the LMS algorithm, the user has control over two variables, the filter order (M) and the step size μ. The step size is determined as the trade-off between the time to converge and the range of allowable error value of the LMS filter. The filter order is determined by how quickly the weights need to converge to the zero. To investigate the performance of the LMS filter in the multipath environment, the simulation of the GPS signal was carried with a sample size of 50000 samples. Here, the filter order is selected at an optimum value of 7 (M = 7). The step size is selected based on the performance of the LMS filter in capturing the variation in the input signal with respect to the desired input signal. The LMS filter performance with M = 7 and μ = 0.004 is shown in Figure 6 and Figure 7.

**Figure 4.** Code tracking plots with LMS adaptation.

**Figure 5.** Architecture of carrier tracking with LMS.

**Figure 6.** LMS plot for the first 500 samples M = 7, μ = 0.004.

**Figure 7.** LMS plot for the last 500 samples M = 7, μ = 0.004.
respectively. Figure 6 shows the performance for the first 500 samples and Figure 7 shows for the last 500 samples respectively. Various plots that are shown in Figure 6 and 7 are: 1) the multipath affected input signal having the carrier and code, 2) the PRN prompt code, 3) the locally generated carrier and code signal as desired signal, 4) the LMS output and 5) the LMS error signal.

On analyzing the plots LMS output and the LMS error in Figure 6, it is observed that the LMS error signal settles within an amplitude of 1 after processing nearly 100 samples, consequently the LMS output is capable of following the desired signal after the 100 samples. Similarly, the LMS output and LMS error plot of Figure 7, indicates few ripples in the error signal reaching an amplitude closer to 1, leading to the fluctuations in the LMS output. To investigate the possible improvement in the performance of the LMS filter with low value of \( \mu \), the same simulation exercise was carried out with \( \mu = 0.0005 \). This indicates that the LMS is able to estimate the presence of multipath.

From the Figure 6 to Figure 9, it observed that as expected during the discussion on the effect of \( \mu \), the error is less when the lower value of \( \mu \) (0.0005) is used and LMS output is closer to the desired signal and it takes longer for the LMS output to settle. The error amplitude of the LMS filter is observed to be less both in the beginning and at the end of the samples when \( \mu \) value is 0.0005. From the LMS output of the two plots, it is observed that the capability of the LMS filter in following the desired signal is better in the case where the \( \mu \) value is 0.0005. This indicates that the LMS is able to estimate the presence of multipath when the step size is smaller and is capable of modeling the multipath. The convergence of the weights and the error signal are shown in Figure 10 and Figure 11 respectively.

From Figure 10 and Figure 11, it is observed that the weights in the taps of the LMS filter are converging to zero, indicating that the LMS algorithm with the step size is smaller and is capable of modeling the multipath.

![Figure 8](Image)

**Figure 8.** LMS plot for the first 500 samples \( M = 7 \), \( \mu = 0.0005 \).

![Figure 9](Image)

**Figure 9.** LMS plot for the last 500 samples \( M = 7 \), \( \mu = 0.0005 \).

![Figure 10](Image)

**Figure 10.** Weights plot of the LMS \( M = 7 \), \( \mu = 0.0005 \).
From Figure 10 and Figure 11, it is observed that the weights in the taps of the LMS and LMS error signal are converging to zero, indicating that the LMS algorithm with the selected filter order and $\mu$ value can be used in the estimation of the multipath.

### 3.2 RAKE Receiver Implementation

The architecture for implementing the RAKE receiver for tracking the GPS signal with the multipath mitigation capability is shown in Figure 12. The tracking loop architecture represented in Figure 12 has three fingers for the estimation and mitigation of the multipath. As discussed earlier, the number of fingers is decided by the number of multipath signals to be estimated in the algorithm.

In the RAKE fingers the GPS base band signal which is fed to the tracking loop for the tracking the GPS signal is also fed as the input signal for the LMS algorithm. The local signals generated from the tracking loop are fed as

![Diagram](image)

Figure 11. Error signal plot of the LMS $M=7$, $\mu=0.0005$.

![Diagram](image)

Figure 12. Architecture of the tracking loop with RAKE receiver.

The desired signals to the LMS filter. This local signal in the individual finger has the base band RF with the required PRN code (prompt code of the carrier tracking loop). The LMS performs the adapting function and provides the error signal, which contains the estimate information of the level of multipath suffered by the input signal. The lowest value of the error signal indicates that the severity of multipath is less. The error signal from the LMS algorithm is fed to the carrier discriminator function in the tracking loop for error correction. The navigation data from the tracking loop of the corresponding finger is fed to the RAKE finger combiner. The operation in the remaining finger remains the same except that the local signal fed to the LMS algorithm as the desired signal has different delay to estimate the multipath. The local signal is time shifted in the respective finger to the required sample time duration up to which the multipath signal are expected. In the present implementation the local signal in the subsequent fingers (2 and 3) are shifted by one and two additional sample duration respectively to estimate the multipath. The RAKE combiner implemented in the present architecture is set as equal gain combiner, where the gain for the navigation data from each finger is multiplied by one.

The RAKE receiver for the mitigation of the multipath signals explained above is implemented in Matlab with two fingers. A data set having the GPS signal in the base band that is affected due to multipath signal is used to verify the performance. The signal was used by a conventional tracking algorithm to identify the performance of the conventional tracking methods. The output from such a tracking loop without the multipath mitigation that uses the dataset mentioned above is presented in Figure 13.

From the Figure 13, it can be observed that the peak of the amplitude is varying during the integration update of the tracking loop in the present data set. During heavy
multipath influence the tracking with the conventional tracking will be further worse. Sometimes the multipath signal influence will be so severe that the zero crossing of the data will be degraded leading to the pseudo range errors.

The same data set used by RAKE receiver using the LMS for mitigating the multipath signals. The outputs at various stages of the tracking loop with the RAKE receiver configuration is given in Figure 14 and Figure 15. Figure 14 shows the tracking output of the first and second fingers in the RAKE receiver. The difference in amplitude in the two figures indicates that the mode 1 in the individual fingers of the RAKE receiver is responding to the multipath estimate of the respective fingers.

Figure 15 shows the RAKE combiner output of the receiver that combines the outputs from the individual fingers. Since, the equal gain combiner is used in this implementation, the RAKE combiner output is the resultant addition of the output from the individual fingers. From the Figure 13 and Figure 15, it can be observed that the RAKE receiver configuration could adapt to multipath

![Figure 14. Tracking plot from finger No 1 and 2 of the RAKE receiver.](image)

![Figure 15. Tracking plot of the RAKE combiner.](image)

and sufficient level of multipath mitigation is provided by the implemented architecture. The LMS error estimate a measure of the multipath estimation of the two Fingers is shown in Figure 16. From the figure, it can be observed that initially the error signal magnitude is large and subsequently converging towards the value close to zero indicating that the algorithm is estimating the multipath effectively.

For mitigating severe multipath from more reflected signals, the requirement of the number of fingers has to be suitably amended during the design of the same.

4. Conclusion

Multipath effects play a major role in the tracking of GPS signals. The multipath signals result in reduction in the amplitude of the detected navigation data from the GPS tracking loop. In some cases the timing of the detected signal is severely degraded which result in errors in the calculation of the pseudo ranges. This problem is more pronounced in the urban environment as the high rise buildings can cause the reflection of the signal close to the receiver leading to the multipath signals adding to the direct signal.

The mitigation of multipath signal is essential for the proper tracking of the GPS signal. In this paper, the multipath mitigation technique by using the RAKE receiver concept was investigated and implemented for tracking the GPS signal. The RAKE receiver uses the LMS algorithm for the estimation of the multipath in the individual fingers. The results obtained through the implementation of the same using Matlab shows the capability of the RAKE receiver with two fingers in mitigating multipath signals. More fingers can be added to the RAKE receiver, if the number of reflected signals is higher. The results
presented during the analysis and investigation of the LMS algorithm implementation proves the suitability of the LMS algorithm in the estimation of the multipath effects. The selection of the step size and the filter order in the LMS algorithm is critical in the estimation capability of the LMS algorithm. In this paper the step size of 0.005 ( = 0.005) and filter order 7 (M = 7) was selected based on analysis and iterative methods to have optimum settling time, peak error value and the capability to track continuously by the LMS algorithm. Here the low value of step size selected for reducing the LMS error rather even though it increases the settling time in the beginning. Similarly, the selection of the number of fingers for the RAKE receiver is also important in managing the multipath mitigation.

5. References

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