Abstract

In this paper, PID controller with ant colony optimization is developed for the conical tank. Ant Colony Optimization (ACO) is a recently developed meta-heuristic approach for solving hard combinational optimization problems. Each individual ant can find a solution or at least part of a solution to the optimization problem on its own, but only when many ants work together they can find the optimal solution. Since the optimal solution can only be found through the global cooperation of all the ants in a colony, it is an emergent result of such cooperation. The results of PID controller with ant colony optimization provide a remarkable improvement in tracking a given set point, when compared with the Ziegler Nichols closed loop tuning and feedforward plus feedback controller tuning methods.

Keywords: Ant Colony Optimization, Conical Tank, Performance Measures, Pid Tuning

1. Introduction

Practically every system in the universe is non-linear. Controlling and tuning of non-linear system is very difficult. Linearizing the system reduces the accuracy in real time. Thus the system has to be modeled perfectly. In this paper, the modeling is done for a conical tank which is a non-linear system and the level control is achieved using PID control tuning based on ant colony optimization. In most of the process industries, conical tank is implemented since there is less deposition of high viscous fluids at its base. Controller is the most vital part of any industries and care should be taken while selecting and designing a controller in order to have good set point tracking, disturbance rejection. The major issues faced during a controller design are inherent non-linearity of the systems, unmeasured parameters and some constraints. ACO traces the shortest paths similar to the ants searching for its food. With the use of ACO in the PID control tuning, the performance of the system is improved to a great extent.

1.1 Conical Tank

A conical tank consists of reservoir, water pump, and current to pressure converter, control valve, rotameter, compressor, Differential Pressure Transmitter (DPT), ADAM module, and a personal computer which acts as a controller. The input side flow rate is adjusted by the stem position of the valve which can be given from the personal computer. The current range for adjusting the valve position is 4-20mA, which is converted to 3-15 psi. The level of the fluid is measured using Differential Pressure Transmitter (DPT) which is calibrated for 0-75 cm and is converted to an output current range of 4-20 mA which in turn is converted to 1-5V range, which is given to the controller through Analog to Digital Converter (ADC) of ADAM module. The ADAM module has 4 slots for four converter cards. In our process, 2 slots are used, one containing Analog to Digital Converter (ADC) card and the other containing the Digital to Analog Converter (DAC) card. There are 8 analog input channels with a range of 4-20 mA in ADC card and DAC card has 4 analog output

*Author for correspondence
channels with a range of -10 V to +10V containing both positive and negative terminals. The mathematical modeling of the conical tank is described.

The area of the tank be \( A \),

\[ A = \pi r^2 \]  

(1)

\[ \tan \theta = \frac{r}{h} = \frac{R}{H} \]  

(2)

\[ r = R \times \frac{h}{H} \]  

(3)

By mass balance equation,

Rate of inflow – Rate of outflow = Accumulation in the tank

\[ F_{in} - F_{out} = A \frac{dh}{dt} \]  

(4)

\[ F_{out} = k\sqrt{h} \]  

(5)

where \( k \) is the discharge coefficient,

\[ F_{in} - k\sqrt{h} = A \frac{dh}{dt} \]  

(6)

\[ \frac{dh}{dt} = \frac{F_{in} - k\sqrt{h}}{A} \]  

(7)

On substitution, we get

\[ F_{in} - F_{out} = \frac{1}{3} \left[ A \frac{dh}{dt} + h^2 \frac{2\pi r^3 \, dh}{H \, dt} \right] \]  

(8)

The experimental setup of the conical tank is shown in Figure 1.

### 1.2 Modeling of Conical Tank

The system identification is one of the important steps in the process modeling\(^5\). A step change is given for modeling a system. Since conical tank is a non-linear system, it is divided into number of regions. Then transfer function for each region is modeled accordingly. We assume a proper model for the given system so that the estimated parameter almost matches with the real time parameters. The model for each region is obtained using the response curves obtained in Figure 2 to Figure 7.

The model for 0-15 mm is

\[ G(s) = \frac{3.12 \, e^{-1.84s}}{4.13s + 1} \]  

(9)

The model for the level 15-27 mm is

\[ G(s) = \frac{2.8 \, e^{-1.92s}}{9.1s + 1} \]  

(10)

**Figure 1.** Experimental set-up of conical tank.

The model for the level 27-36 mm is

\[ G(s) = \frac{2.3 \, e^{-2.1s}}{13.2s + 1} \]  

(11)

The model for the level 36-45 mm is

\[ G(s) = \frac{1.8 \, e^{-2.3s}}{15.8s + 1} \]  

(12)

The model for the level 45-57 mm is

\[ G(s) = \frac{1.6 \, e^{-2.5s}}{17.7s + 1} \]  

(13)

The model for the level 57-66 mm is

\[ G(s) = \frac{1.2 \, e^{-2.8s}}{19.8s + 1} \]  

(14)

### 1.3 ANT Colony Optimization Algorithm

ACO algorithm is based on the behavior of ants. When an ant travels in a path, it leaves away a substance called pheromone. The other ant which travels along the path sees that it travels in the path where the traces of pheromones are more. This path is again marked with their own pheromones. The pheromone gets evaporated over time\(^9\). The ACO can be used for a continuous time systems where it is divided into regions. The fitness for every region is checked and accordingly they are designed. These regions serve as the local nodes for the artificial ants\(^10\). Other optimizing algorithm becomes difficult when large number
of variables is used. ACO is specifically designed for different variables. The global search can be attained by two methods. One is random walk and the other is diffusion. In random walk the ants find out new paths and update the domain with these new paths.

The mutation in genetic algorithm is given by

\[ \Delta(T, R) = R (1 - r^{(1 - T)b}) \]  

Where, \( T = \frac{current\ number\ of\ iteration}{Total\ iteration} \)

b is the degree of non-linearity, r is the random number and R is the maximum step size.

The search horizon of mutation is reduced by increasing the number of iterations. When reducing the search horizon, the chances are high for locating the maximum concentration area. It is achieved by trial diffusion. This is similar to crossover in genetic algorithm, where 2 parents are selected randomly. The resulting child can have characteristics from first parent or from the second parent or it have a combination of both. The distribution of characteristics from the first and second parent to the child can be given with a weighting factor. The ant distribution for local search and global search is shown in Figure 8.
When weighting factor lies in the range of 0.5 and 0.75,
\[ x_{\text{Child}} = x_{\text{Parent1}} + (-\alpha) x_{\text{Parent2}} \]  
(16)

When weighting factor lies in the range of 0.75 and 1,
\[ x_{\text{Child}} = x_{\text{Parent1}} \]  
(17)

When weighting factor lies in the range of 0.5 and 0.75,
\[ x_{\text{Child}} = x_{\text{Parent2}} \]  
(18)

The number of ants using a path is updated locally. The regions with superior pheromones are only updated. The selection of region \( j \) by the local ants is given by,
\[ P_{j}(t) = \frac{\tau_{j}(t)}{\sum_{k} \tau_{k}(t)} \]  
(19)

Once the selection of destination is done, the ants follow the shortest path. If the fitness of the region is found to be improved the position vector is incremented. The increase in fitness and deposition of pheromones are directly proportional. Then the modified pheromone value is given by,
\[ \text{pheromone}_{\text{new}} = \frac{P_{\text{new}} - P_{\text{old}}}{P_{\text{old}}} + \text{pheromone}_{\text{old}} \]  
(20)

The flow diagram of ant colony optimization is shown in Figure 9. Specify the initial parameters after creation of new population. These parameters make the system attain global minimization. Once the initialization is done, 90% of the ants is sent to crossover and mutation. Then, remaining 10% ants are sent for trial diffusion. The path with less traces of pheromone is considered to be the weakest path, is updated with its value. The stronger pheromone path is also updated. If the updated path is a new function, the path is included in the functional table. If the path already exists, the value is incremented.

### 1.4 PID Control

PID control is most widely used technique in process industries. It is beneficial because of its simplicity and robust nature. For designing a PID controller, there are three control parameters which have to be tuned manually or by some tuning techniques. Among different tuning techniques Ziegler Nichols method is the most widespread method which takes into the account the plant gain at which the system becomes unstable. The general form of PID control is given below,
\[ G_c(s) = K_c \frac{\tau_s}{\tau_s^2 + \tau_s + 1} \]  
(21)
For the optimization of PID control parameters, we formulate an objective function which includes different transient response indices and integral absolute error. By optimizing IAE we can obtain better load disturbance rejection also, improved transient response is obtained. ACO is a newly developed optimization technique which is simple, can have a flexible objective function and it gives a global optimization solution in comparison with existing optimization algorithms. The block diagram of ACO based PID tuning is illustrated in Figure 10.

1.5 Objective Function

For tuning the PID controller, the controller parameters \(k_p, k_i, k_d\) are to be manipulated. The performance index of the transient response must be kept under tolerance value.

\[
IAE = \int_0^t \left| e(t) \right| dt
\]  

(22)

The equation (22) gives the integral absolute error, which is used for performance evaluation. The four objectives are included to formulate the cost function. Overshoot is the difference between the maximum response obtained to the steady state value. The overshoot should always be within lower tolerance range. Rise time is the time taken by the system to reach 90% of its final value. It should be minimum for good performance. Settling time is the time to reach final value. Integral Absolute Error (IAE) is the error function included to the cost function.

The objective function for the optimization algorithm is as follows,

\[
f = f_{mo} + f_n + f_t + f_{ua}
\]  

(23)

\(f_{mo} = \) maximum response – steady state response

\(f_n = t_r\)

\(f_{ua}\) is given in the equation

2. Tuning of PID Using ACO

In order to achieve an optimal value of the control parameters \(k_p, k_i, k_d\), the control parameters are represented as nodes. Each node denotes a value of control parameter. Here some \(m\) ants are randomly selected from a desired space, and they are distributed to respective nodes. The matrix giving pheromone intensity is initialized. Initialize the loop counter and generation counter to zero. Since every ant must visit all the \(n\) nodes, a routing table saves list of ants visiting time at \(t\). At start, initialize the node counter to one ant counter from 1 to \(m\). Note the node numbers from where each of \(m\) ants start their travel on the routing table and repeat to complete the routing list. Whenever the ant travels from one node to another, enter the second node into the routing list. Calculate the cost function for each ant for its travel and find which one is having the optimized cost function. Calculate the pheromone matrix. Each time the whole process repeats, the intensity matrix has to be updated. Finally, optimal controller parameters are obtained.

3. Simulation Result

The various models for the conical tank are simulated and the results are shown in Figure 11 to Figure 14, when the input is a step signal of amplitude 10 units.

The simulation has been done for different models, and it is observed that PID tuning with ant colony optimization yields better results compared to Ziegler Nichols technique. Also, the peak overshoot is high in Ziegler Nichols technique. The performance of ACO-based PID tuning and Ziegler Nichols technique are shown in Figure 15 and Figure 16.

4. Conclusion

Tuning of PID controller with ACO results in optimum values of \(k_p, k_i, k_d\) when compared with Ziegler Nichols technique for non-linear processes like conical tank. The desired response was achieved by optimizing the value of the PID tuning parameters. ACO provides good dis-
turbance rejection as well as set point tracking than the conventional PID tuning.

5. References


