Abstract
In this paper, we develop an EOQ model for power demand under the condition of permissible delay in payment by considering four different situations. Mathematical formulation is derived under these four different situations. The main objective of this work is to obtain minimum total relevant cost. Next, we derive optimal solution optimal cycle time, order quantity and total relevant cost for the proposed model. The theoretical results are illustrated with numerical examples. The sensitivity analysis of the optimal solution is provided with respect to key parameters of the system. Mathematica 5.1 software is used for finding numerical results.

Keywords: Cash Discount, Demand Rate, EOQ Model, Permissible Delay, Total Relevant Cost

1. Introduction
In practice EOQ model, the demand rate is considered to be either constant or time-dependent or stock-dependent. However, in practice the demand for consumer goods may be dependents on inventory level, that is the demand rate may increase or decrease with the inventory level. Several EOQ models have established by researchers considering the varying and probabilistic demand. The varying demand was first established by Silver and Meal which is simple modification of the inventory model. Donaldon was the first researcher who developed EOQ (Economic Order Quantity) model for linearly time dependent demand. Khanna et al. established an economic order quantity model for deteriorating items having time dependent demand when delay in payments is permissible. Large number of research papers/articles done by researchers in direction like Ritchie, Mitra, Tripathi and Misra, Tripathi and Kumar. Recently, Tripathi and Panday presented an inventory model for deteriorating items with Weibull distribution time-dependent demand rate under permissible delay in payments.

Deterioration is a natural phenomenon for any type of goods. During the last few decades, large number of articles/research papers presented by researchers on inventory models for deteriorating items. Ghare and Schrader fist established an inventory model for deteriorating items. Goyal developed an EOQ model under condition of permissible delay in payments. Chang proposed an inventory model under a situation in which the supplier provides the purchaser a permissible delay in payments if the purchaser orders a large quantity. Covert and Philip extended Ghare and Schrader's model for constant deterioration rate to a two-parameter Weibull distribution. Yang et al. developed an optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. Shah and Jaswal and Aggarwal discussed the EOQ model with constant rate of deterioration. Goyal and Giri established an EOQ model on the recent trends in modeling of deteriorating inventory. Teng et al. establish an EOQ model for stock-dependent...
demand under progressive payment scheme for deteriorating items. Chung and Liao\(^{19}\) considered an inventory model for deteriorating items under the conditions of using the Discounted Cash-Flows (DCF) approach to the permissible delay in payments. Other many related articles/research papers can be found by Aggarwal and Jaggi\(^{20}\), Jamal et al.\(^{21}\), Huang\(^{22}\), Ouyang et al.\(^{23}\), Heng et al.\(^{24}\), Sarkar et al.\(^{25}\) and Misra et al.\(^{26}\). Roy\(^{27}\) developed a deterministic inventory model when the deterioration rate is time dependent, demand rate is a function of selling price and holding cost is time proportional. Liao\(^{28}\) derived a production model for a lot-size inventory model with finite production rate taking into consideration the effect of decay and the condition of permissible delay in payments, in which the restriction assumption of a permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost to the supplier and payoff the remaining balance by loan from the bank. Dye et al.\(^{29}\) developed a deterministic inventory model for deteriorating items with price-dependent demand. Liao\(^{30}\) considered the impact of the trade credit policy on the classical Economic Production Quantity (EPQ) model for an item subjected to exponential decays. Chung\(^{31}\) established an EOQ model for deteriorating items under credit linked to the ordering quantity. Several research paper published by Sharma and Singh\(^{32}\), Tayal, Singh and Sharma\(^{33}\), Shahraki et al.\(^{34}\) and Bhagoria et al.\(^{35}\) in this direction.

The rest of the paper is organized as follows. In section 2, the assumption and notations are given. In section 3, we develop the mathematical models under four different circumstances. In section 4, optimal solutions are derived. In section 5, we provide three numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out in section 6. Finally, we draw the conclusions and future research in section 7.

## 2. Assumptions and Notation

### 2.1 Notations

- \(p\): Selling price per unit.
- \(c\): The unit purchasing cost with \(p > c\).
- \(I_r\): The interest charged per dollar in stock per year by the supplier.
- \(I_e\): The interest earned per dollar per year.
- \(s\): The ordering cost per order.
- \(Q\): The order quantity.
- \(r\): The cash discount rate \(0 < r < 1\).
- \(h\): The unit holding cost per year excluding the interest charges.
- \(M_1\): The period of cash discount.
- \(M_2\): The period of permissible delay in setting account with \(M_2 > M_1\).
- \(T\): The replenishment time interval.
- \(I(T)\): The level of inventory at time \(t\), \(0 \leq t \leq T\).
- \(T_1, T_2, T_3\) and \(T_4\): The optimal replenishment time for case (1), (2), (3) and (4) respectively.
- \(Z(T)\): The total relevant cost per year.
- \(Z_1(T), Z_2(T), Z_3(T)\) and \(Z_4(T)\): The total relevant cost per year for case (1), (2), (3) and (4) respectively.
- \(Z'(T_1), Z'(T_2), Z'(T_3)\) and \(Z'(T_4)\): Optimal total relevant cost per year for case (1), (2), (3) and (4) respectively.
- \(Q'(T_1), Q'(T_2), Q'(T_3)\) and \(Q'(T_4)\) the optimal order quantity case (1), (2), (3) and (4) respectively.

### 2.2 Assumptions

- The demand rate is power inventory dependent, i.e. \(D = D[I(t)] = a[I(t)\\beta] \alpha > 0, 0 \leq \beta < 1\).
- Lead time is negligible.
- Time horizon is infinite.
- Shortages are not allowed.
- The generated sales revenue is deposited when the account is not settled in an interest bearing account. The account is settled as well as the buyer pays of units sold at the end of \(M_1\) or \(M_2\) and starts paying for the interest charges on the items in the stock.

## 3. Mathematical Formulation

According to assumption, the variation of inventory with respect to time can be represented by the following differential equation

\[
\frac{dI(t)}{dt} = -a[I(t)]^\beta, \quad a > 0, 0 < \beta < 1
\]  

(1)

The solution of (1) with the condition \(I(T) = 0\) is given by

\[
I(t) = a(T-t)^{1/(1-\beta)}, \quad \text{where } a = \{a(1-\beta)\}^{1/(1-\beta)}, 0 \leq t \leq T
\]  

(2)

Also the order quantity \(Q = I(0) = a T^{1/(1-\beta)}\)

(3)

The total demand during one cycle is \(= aT^{\beta/(1-\beta)}\)
The total relevant cost per cycle time contains of the following elements:

Cost of placing order \[ \frac{s}{T} \] (4)

Cost of purchasing \[ acT^{\beta/(1-\beta)} \] (5)

Cost of carrying inventory \[ \frac{ah(1-\beta)}{(2-\beta)}T^{\beta/(1-\beta)} \] (6)

Case (1). \( T \geq M_t \),
The customer save \( rcQ \) per cycle due to price discount, since the payment is made at time \( M_t \).

The discount per year \[ \frac{rcQ}{T} = racT^{\beta/(1-\beta)} \] (7)

The customer pays off all units ordered at time \( M_t \) to obtain the cash discount according to the assumption. Consequently, the items in stock have to financed (at the rate \( l \)) after time \( M_t \). Therefore,

The interest payable per year

\[
= \frac{ac(1-r)(1-\beta)}{(2-\beta)T}I_c(T-M_t)^{\beta/(1-\beta)}
\] (8)

Also in the period \([0, M_t]\) the customer sells the product and deposits the revenue into an account that earns interest \( I_c \) per dollar per year. Therefore,

The interest earned per year

\[
= \frac{pI_d}{T} \left[ \frac{(1-\beta)M_t(T-M_t)^{1/(1-\beta)} - (1-\beta)^{1/(1-\beta)}(T-M_t)^{1/(1-\beta)} - (1-\beta)^{2/(1-\beta)}}{(2-\beta)} \right]
\] (9)

Total relevant cost \( Z_1(T) \) per year is given by

\[
Z_1(T) = \frac{s}{T} + racT^{\beta/(1-\beta)} + \frac{ah(1-\beta)}{(2-\beta)}T^{\beta/(1-\beta)}
+ \frac{ac(1-r)(1-\beta)}{(2-\beta)T}I_c(T-M_t)^{\beta/(1-\beta)}
- \frac{pI_d}{T} \left[ \frac{(1-\beta)M_t(T-M_t)^{1/(1-\beta)} - (1-\beta)^{1/(1-\beta)}(T-M_t)^{1/(1-\beta)} - (1-\beta)^{2/(1-\beta)}}{(2-\beta)} \right]
\] (10)

Case (2). \( T < M_t \),
In this situation the customer sells \( a[I(t)]^{\beta} \), \( T \) units in total at time \( T \) and has \( c(1-r)a[I(t)]^{\beta/T} \) to pay the supplier in full at time \( M_t \). Consequently there is no interest payable, while the cash discount is the same as that case (1) discussed above. However,

The interest earned per year

\[
= \frac{pI_d}{T} \left[ \int_0^T a[I(t)]^{\beta} \cdot dt + (M_t - T) \frac{a[I(t)]^{\beta}}{(2-\beta)} \right]
= \frac{aa^\beta}{T} pI_d(1-\beta)T^{\beta/(1-\beta)} \left\{ (1-\beta)T \left(1 - (T-M_t) \right) \right\}
\] (11)

Now the total relevant cost per year \( Z_2(T) \) per year is given by

\[
Z_2(T) = \frac{s}{T} + racT^{\beta/(1-\beta)} + \frac{ah(1-\beta)}{(2-\beta)}T^{\beta/(1-\beta)}
- \frac{aa^\beta}{T} pI_d(1-\beta)T^{\beta/(1-\beta)} \left\{ (1-\beta)T \left(1 - (T-M_t) \right) \right\}
\] (12)

Case (3). \( T < M_t \),
In this situation, there is no discount i.e. \( r = 0 \), (since the payment is made at time \( M_t \)), Now

The interest payable per year

\[
= \frac{ac(1-\beta)}{(2-\beta)T}I_c(T-M_t)^{\beta/(1-\beta)}
\] (13)

The interest earned per year

\[
= \frac{pI_d}{T} \left[ \frac{(1-\beta)M_t(T-M_t)^{1/(1-\beta)} - (1-\beta)^{1/(1-\beta)}(T-M_t)^{1/(1-\beta)} - (1-\beta)^{2/(1-\beta)}}{(2-\beta)} \right]
\] (14)

Thus , the total relevant cost \( Z_3(T) \) per year is given by

\[
Z_3(T) = \frac{s}{T} + racT^{\beta/(1-\beta)} + \frac{ah(1-\beta)}{(2-\beta)}T^{\beta/(1-\beta)}
+ \frac{ac(1-\beta)}{(2-\beta)T}I_c(T-M_t)^{\beta/(1-\beta)}
- \frac{aa^\beta}{T} pI_d(1-\beta)T^{\beta/(1-\beta)}
\]
4. Determination of Optimal Solution

To determine optimal solution, differentiating (10), (12), (15) and (17) with respect to \( T \) two times, we get

\[
\frac{d^2 Z_4(T)}{dT^2} = \frac{2s}{T^3} + \frac{ract}{T^{(2-\beta)/\beta}} + \frac{ah}{T^{(2-\beta)/\beta}} \left\{ \frac{(1-\beta)^2}{T^{1-(\beta-1)/\beta}} - \frac{1}{T} \frac{(T-M_1)^{2-(\beta-1)/\beta}}{T} - \frac{1}{T^2} \frac{(T-M_2)^{2-(\beta-1)/\beta}}{T} \right\}
\]

\[
\frac{d Z_4(T)}{dT} = \frac{s}{T^2} + \frac{ract}{T^{(2-\beta)/\beta}} + \frac{ah}{T^{(2-\beta)/\beta}} \left\{ \frac{(1-\beta)^2}{T^{1-(\beta-1)/\beta}} - \frac{1}{T} \frac{(T-M_1)^{2-(\beta-1)/\beta}}{T} - \frac{1}{T^2} \frac{(T-M_2)^{2-(\beta-1)/\beta}}{T} \right\}
\]

\[
Z_4(T) = \frac{s}{T^2} + \frac{ract}{T^{(2-\beta)/\beta}} + \frac{ah}{T^{(2-\beta)/\beta}} \left\{ \frac{(1-\beta)^2}{T^{1-(\beta-1)/\beta}} - \frac{1}{T} \frac{(T-M_1)^{2-(\beta-1)/\beta}}{T} - \frac{1}{T^2} \frac{(T-M_2)^{2-(\beta-1)/\beta}}{T} \right\}
\]

Now the total relevant cost per year \( Z_4(T) \) per year is given by

\[
Z_4(T) = \frac{s}{T^2} + \frac{ract}{T^{(2-\beta)/\beta}} + \frac{ah}{T^{(2-\beta)/\beta}} \left\{ \frac{(1-\beta)^2}{T^{1-(\beta-1)/\beta}} - \frac{1}{T} \frac{(T-M_1)^{2-(\beta-1)/\beta}}{T} - \frac{1}{T^2} \frac{(T-M_2)^{2-(\beta-1)/\beta}}{T} \right\}
\]
\[-\frac{2pI_daa^\theta}{T^3}\left[-(1-\beta)M_1(T - M_2)\right]^{\gamma / (1-\beta)}
\]
\[-\frac{(1-\beta)^2}{(2-\beta)}\left\{\left[(T - M_1)^{(2-\beta)/ (\beta - 1)} - T^{(2-\beta)/ (\beta - 1)}\right]\right\}\]

\[
\frac{d^2Z_1(T)}{dT^2} = \frac{2s}{T^3} + \frac{r \alpha (2-\beta - 1)T}{(1-\beta)^2} + \frac{ah\beta T}{(2-\beta)(1-\beta)} + \frac{pI_daa^\theta}{T} \left[\beta(T - M_2)T^{(2-\beta)/ (\beta - 1)}\right] (1-\beta)
\]
\[+ T^{\beta / (\beta - 1)}\right\] - \frac{2pI_daa^\theta}{T^3} \left[(1-\beta)^2(T - M_2)T^{(2-\beta)/ (\beta - 1)}\right] (2-\beta)

The necessary and sufficient condition for finding minimum value of $Z(T)$ is given by:

\[
\frac{dZ_1(T)}{dT} = 0, \text{ and } \frac{d^2Z_1(T)}{dT^2} > 0, \quad (i = 1, 2, 3, 4)
\]

\[
s = \frac{r \alpha (2-\beta - 1)T}{(1-\beta)^2} + \frac{ah\beta T}{(2-\beta)(1-\beta)}
\]

\[
- \frac{M_1(T - M_2)^{(\beta - 1) / (\beta)}}{T} - (1-\beta)\left\{(1-\beta)^2(T - M_2)^{(2-\beta)/ (\beta - 1)}\right\}
\]

\[
\frac{d^2Z_3(T)}{dT^2} = \frac{2s}{T^3} + \frac{r \alpha (2-\beta - 1)T}{(1-\beta)^2} + \frac{ah\beta T}{(2-\beta)(1-\beta)}
\]

\[
+ acl\left\{\left[(T - M_2)^{\beta / (\beta - 1)} - 2(T - M_2)^{\gamma / (\beta - 1)}\right] T^2\right\}
\]

\[
\left[-\frac{\beta M_2(T - M_2)^{(2-\beta)/ (\beta - 1)}}{(1-\beta)} - (T - M_2)^{\gamma / (\beta - 1)} + T^{\beta / (\beta - 1)}\right]
\]

\[
\left[\frac{2pI_daa^\theta}{T^2}\left[-M_2(T - M_2)^{\beta / (\beta - 1)} - (1-\beta)\left(T - M_2\right)^{\gamma / (\beta - 1)}\right] + (1-\beta)T^{\beta / (\beta - 1)}\right]
\]

\[
\frac{d^2Z_3(T)}{dT^2} + \frac{2s}{T^3} + \frac{r \alpha (2-\beta - 1)T}{(1-\beta)^2} + \frac{ah\beta T}{(2-\beta)(1-\beta)}
\]

\[
+ acl\left\{\left[(T - M_2)^{\beta / (\beta - 1)} - 2(T - M_2)^{\gamma / (\beta - 1)}\right] T^2\right\}
\]

\[
-\frac{\beta M_2(T - M_2)^{(2-\beta)/ (\beta - 1)}}{(1-\beta)} - (T - M_2)^{\gamma / (\beta - 1)} + T^{\beta / (\beta - 1)}\right]
\]

\[
\left[\frac{2pI_daa^\theta}{T^2}\left[-M_2(T - M_2)^{\beta / (\beta - 1)} - (1-\beta)\left(T - M_2\right)^{\gamma / (\beta - 1)}\right] + (1-\beta)T^{\beta / (\beta - 1)}\right]\]

\[
\frac{d^2Z_3(T)}{dT^2} = \frac{2s}{T^3} + \frac{r \alpha (2-\beta - 1)T}{(1-\beta)^2} + \frac{ah\beta T}{(2-\beta)(1-\beta)}
\]

\[
+ acl\left\{\left[(T - M_2)^{\beta / (\beta - 1)} - 2(T - M_2)^{\gamma / (\beta - 1)}\right] T^2\right\}
\]

\[
-\frac{\beta M_2(T - M_2)^{(2-\beta)/ (\beta - 1)}}{(1-\beta)} - (T - M_2)^{\gamma / (\beta - 1)} + T^{\beta / (\beta - 1)}\right]
\]

\[
\left[\frac{2pI_daa^\theta}{T^2}\left[-M_2(T - M_2)^{\beta / (\beta - 1)} - (1-\beta)\left(T - M_2\right)^{\gamma / (\beta - 1)}\right] + (1-\beta)T^{\beta / (\beta - 1)}\right]\]
EOQ Model with Inventory Level Dependent Demand Rate under Permissible Delay in Payments with Cash Discount

$$+ \frac{p}{T^2} \left[ \left(1 - \beta \right)M_2 \left(T - M_2 \right)^{\frac{1}{(1 - \rho)}} \right]$$

$$+ \frac{(1 - \beta)^2}{(2 - \beta)} \left\{ \left(T - M_2 \right)^{\frac{(2 - \rho)}{(1 - \rho)}} - T^{\frac{(2 - \rho)}{(1 - \rho)}} \right\} = 0 \quad (28)$$

Let us take the parameter values as

$$\text{Example 1 (case 1).} \quad \text{Let us take the parameter values as} \quad a = 200; \quad \beta = 0.02; \quad h = 8$/\text{unit/year}; \quad I_i = 0.10$/\text{year}; \quad I_d = 0.04$/\text{year}; \quad s = 30$/\text{order}; \quad c = 85$/\text{unit}; \quad p = 15$/\text{unit}; \quad r = 0.05; \quad M_1 = 0.020134 \text{ year}. \quad \text{Substituting these values in equation} \quad (26) \quad \text{and solving; we get optimal} \quad T = T_1 = 0.155313 \text{ years; and optimal ordered quantity} \quad Q(T) = 32.6391 \text{ units; and total relevant cost per year} \quad Z(T) = 730.568, \quad \text{which verified case 1 i.e.} \quad T \geq M_1 \text{ Where} \quad \frac{d^2Z_1}{dT^2} = 15941.0 > 0.$$  

$$\text{Example 2 (case 2).} \quad \text{Let us take the parameter values as} \quad a = 600; \quad \beta = 0.04; \quad h = 6$/\text{unit/year}; \quad I_i = 0.08$/\text{year}; \quad I_d = 0.10$/\text{order}; \quad c = 15$/\text{unit}; \quad p = 40$/\text{unit}; \quad r = 0.05; \quad M_1 = 0.151125 \text{ year}. \quad \text{Substituting these values in equation} \quad (27) \quad \text{and solving; we get optimal} \quad T = T_1 = 0.0581032 \text{ years; and optimal ordered quantity} \quad Q(T) = 38.739 \text{ units; and total relevant cost} \quad Z(T) = 507.842, \quad \text{which verified case 2 i.e.} \quad T \geq M_1 \text{ Where} \quad \frac{d^2Z_2}{dT^2} = 84840.22 > 0.$$  

$$\text{Example 3 (case 3).} \quad \text{Let us take the parameter values as} \quad a = 700; \quad \beta = 0.03; \quad h = 6$/\text{unit/year}; \quad I_i = 0.08$/\text{year}; \quad I_d = 0.07$/\text{year}; \quad s = 60$/\text{order}; \quad c = 25$/\text{unit}; \quad p = 45$/\text{unit}; \quad r = 0.04; \quad M_1 = 0.09524 \text{ year}. \quad \text{Substituting these values in equation} \quad (28) \quad \text{and solving; we get optimal} \quad T = T_1 = 0.130425 \text{ years; and optimal ordered quantity} \quad Q(T) = 39.843 \quad \text{units; and total relevant cost} \quad Z(T) = 1461.44, \quad \text{per year which verified case 3 i.e.} \quad T \geq M_2 \text{ Where} \quad \frac{d^2Z_3}{dT^2} = 51085.5 > 0.$$  

$$\text{Example 4 (case 4).} \quad \text{Let us take the parameter values as} \quad a = 400; \quad \beta = 0.05; \quad h = 7$/\text{unit/year}; \quad I_i = 0.09$/\text{year}; \quad s = 25$/\text{order}; \quad c = 30$/\text{unit}; \quad p = 30$/\text{unit}; \quad r = 0.02; \quad M_1 = 0.19336 \text{ year}. \quad \text{Substituting these values in equation} \quad (29) \quad \text{and solving; we get optimal} \quad T = T_1 = 0.103255 \text{ year; and optimal ordered quantity} \quad Q(T) = 47.5963 \text{ units; and total relevant cost} \quad Z(T) = 506.259, \quad \text{which verified case 4 i.e.} \quad T \geq M_2 \text{ Where} \quad \frac{d^2Z_4}{dT^2} = 38550.534 > 0.$$  

### 6 Sensitivity Analysis

We have discussed sensitivity analysis by varying the parameters $s, I_i, I_d$ and $r$ and keeping the remaining parameters at their original values as in numerical examples 1 to 4. The corresponding changes for the replenishment cycle time, economic order quantity and total relevant cost are provided in the following Tables 1 to 4.

#### Case (1) Table 1.

<table>
<thead>
<tr>
<th>S</th>
<th>$T_1$(in years)</th>
<th>$Q(T_1)$</th>
<th>$Z(T_1)$</th>
<th>$\frac{d^2Z_1}{dT^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.167812</td>
<td>35.3216</td>
<td>761.515</td>
<td>14756.0</td>
</tr>
<tr>
<td>40</td>
<td>0.17944</td>
<td>37.8207</td>
<td>790.313</td>
<td>13802.9</td>
</tr>
<tr>
<td>45</td>
<td>0.190357</td>
<td>40.1701</td>
<td>817.355</td>
<td>13014.6</td>
</tr>
<tr>
<td>50</td>
<td>0.200678</td>
<td>42.3938</td>
<td>842.928</td>
<td>12348.6</td>
</tr>
</tbody>
</table>

#### Table 1(b). The sensitivity analysis of $I_i$ keeping all the parameters same as in Example 1

<table>
<thead>
<tr>
<th>$I_i$</th>
<th>$T_1$(in years)</th>
<th>$Q(T_1)$</th>
<th>$Z(T_1)$</th>
<th>$\frac{d^2Z_1}{dT^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>0.153194</td>
<td>32.1848</td>
<td>734.588</td>
<td>16625.1</td>
</tr>
<tr>
<td>0.12</td>
<td>0.151162</td>
<td>31.7492</td>
<td>738.538</td>
<td>17318.3</td>
</tr>
<tr>
<td>0.13</td>
<td>0.149212</td>
<td>31.3314</td>
<td>742.418</td>
<td>18020.2</td>
</tr>
<tr>
<td>0.14</td>
<td>0.147338</td>
<td>30.9299</td>
<td>746.233</td>
<td>18731.0</td>
</tr>
</tbody>
</table>

#### Table 1(c). The sensitivity analysis of $r$ keeping all the parameters same as in Example 1

<table>
<thead>
<tr>
<th>$r$</th>
<th>$T_1$(in years)</th>
<th>$Q(T_1)$</th>
<th>$Z(T_1)$</th>
<th>$\frac{d^2Z_1}{dT^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.154935</td>
<td>32.5581</td>
<td>803.692</td>
<td>15995.3</td>
</tr>
<tr>
<td>0.07</td>
<td>0.154556</td>
<td>32.4768</td>
<td>876.814</td>
<td>16050.4</td>
</tr>
<tr>
<td>0.08</td>
<td>0.154177</td>
<td>32.3956</td>
<td>949.934</td>
<td>16106.0</td>
</tr>
<tr>
<td>0.09</td>
<td>0.153798</td>
<td>32.3143</td>
<td>1023.05</td>
<td>16162.2</td>
</tr>
</tbody>
</table>
Table 1(d). The sensitivity analysis of \( I_d \) keeping all the parameters same as in Example 1

<table>
<thead>
<tr>
<th>( I_d )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.155296</td>
<td>32.6355</td>
<td>730.526</td>
<td>15942.9</td>
</tr>
<tr>
<td>0.06</td>
<td>0.15528</td>
<td>32.6321</td>
<td>730.484</td>
<td>15944.5</td>
</tr>
<tr>
<td>0.07</td>
<td>0.155263</td>
<td>32.6284</td>
<td>730.442</td>
<td>15946.3</td>
</tr>
<tr>
<td>0.08</td>
<td>0.155247</td>
<td>32.625</td>
<td>730.4</td>
<td>15947.9</td>
</tr>
</tbody>
</table>

Table 2(a). The sensitivity analysis of \( s \) keeping all the parameters same as in Example 2

<table>
<thead>
<tr>
<th>( S )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.0609397</td>
<td>40.711</td>
<td>524.643</td>
<td>80240.321</td>
</tr>
<tr>
<td>12</td>
<td>0.0636475</td>
<td>42.597</td>
<td>540.696</td>
<td>76213.738</td>
</tr>
<tr>
<td>13</td>
<td>0.0662424</td>
<td>44.4076</td>
<td>556.093</td>
<td>72644.021</td>
</tr>
<tr>
<td>14</td>
<td>0.0687374</td>
<td>46.1512</td>
<td>570.91</td>
<td>69444.066</td>
</tr>
</tbody>
</table>

Table 2(b). The sensitivity analysis of \( I_d \) keeping all the parameters same as in Example 2

<table>
<thead>
<tr>
<th>( I_d )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>0.0570322</td>
<td>37.9955</td>
<td>475.383</td>
<td>89089.675</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0560287</td>
<td>37.2993</td>
<td>442.808</td>
<td>93414.486</td>
</tr>
<tr>
<td>0.11</td>
<td>0.0550858</td>
<td>36.6457</td>
<td>410.125</td>
<td>97811.844</td>
</tr>
<tr>
<td>0.12</td>
<td>0.0541976</td>
<td>36.0304</td>
<td>377.34</td>
<td>102278.889</td>
</tr>
</tbody>
</table>

Table 3(a). The sensitivity analysis of \( s \) keeping all the parameters same as in Example 3

<table>
<thead>
<tr>
<th>( s )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.136063</td>
<td>106.268</td>
<td>1498.97</td>
<td>48868.5</td>
</tr>
<tr>
<td>70</td>
<td>0.141486</td>
<td>110.637</td>
<td>1535.0</td>
<td>46933.9</td>
</tr>
<tr>
<td>75</td>
<td>0.146713</td>
<td>114.853</td>
<td>1569.69</td>
<td>45223.9</td>
</tr>
<tr>
<td>80</td>
<td>0.151764</td>
<td>118.932</td>
<td>1603.2</td>
<td>43694.9</td>
</tr>
</tbody>
</table>

Table 3(b). The sensitivity analysis of \( I_s \) keeping all the parameters same as in Example 3

<table>
<thead>
<tr>
<th>( I_s )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>0.129589</td>
<td>101.059</td>
<td>1462.3</td>
<td>52934.6</td>
</tr>
<tr>
<td>0.10</td>
<td>0.128794</td>
<td>100.42</td>
<td>1463.12</td>
<td>54766.3</td>
</tr>
<tr>
<td>0.11</td>
<td>0.128037</td>
<td>99.8115</td>
<td>1463.91</td>
<td>56613.4</td>
</tr>
<tr>
<td>0.12</td>
<td>0.127316</td>
<td>99.2321</td>
<td>1464.67</td>
<td>58463.3</td>
</tr>
</tbody>
</table>

Table 3(c). The sensitivity analysis of \( I_d \) keeping all the parameters same as in Example 3

<table>
<thead>
<tr>
<th>( I_s )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.128695</td>
<td>100.34</td>
<td>1449.06</td>
<td>51938.0</td>
</tr>
<tr>
<td>0.09</td>
<td>0.126952</td>
<td>98.9397</td>
<td>1436.52</td>
<td>52844.7</td>
</tr>
<tr>
<td>0.10</td>
<td>0.125196</td>
<td>97.5291</td>
<td>1423.82</td>
<td>53814.2</td>
</tr>
<tr>
<td>0.11</td>
<td>0.123428</td>
<td>96.1095</td>
<td>1410.95</td>
<td>54855.8</td>
</tr>
</tbody>
</table>

Table 4(a). The sensitivity analysis of \( s \) keeping all the parameters same as in Example 4

<table>
<thead>
<tr>
<th>( s )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.112882</td>
<td>52.2787</td>
<td>552.524</td>
<td>34477.712</td>
</tr>
<tr>
<td>35</td>
<td>0.121717</td>
<td>56.5944</td>
<td>595.149</td>
<td>31120.707</td>
</tr>
<tr>
<td>40</td>
<td>0.129925</td>
<td>60.6187</td>
<td>634.887</td>
<td>28199.86</td>
</tr>
<tr>
<td>45</td>
<td>0.137622</td>
<td>64.4047</td>
<td>672.264</td>
<td>25526.348</td>
</tr>
</tbody>
</table>

Table 4(b). The sensitivity analysis of \( I_d \) keeping all the parameters same as in Example 4

<table>
<thead>
<tr>
<th>( I_d )</th>
<th>( T_s ) (in years)</th>
<th>( Q(T_s) )</th>
<th>( Z(T_s) )</th>
<th>( \frac{d^2 Z_s}{dT^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.101969</td>
<td>46.9725</td>
<td>486.803</td>
<td>39624.993</td>
</tr>
<tr>
<td>0.12</td>
<td>0.0995579</td>
<td>45.8041</td>
<td>447.664</td>
<td>41831.99</td>
</tr>
<tr>
<td>0.13</td>
<td>0.0984248</td>
<td>45.2555</td>
<td>427.988</td>
<td>42963.81</td>
</tr>
<tr>
<td>0.14</td>
<td>0.0973357</td>
<td>44.7286</td>
<td>408.246</td>
<td>44113.576</td>
</tr>
</tbody>
</table>

All the above observations from Tables 1–4, can be explained as follows:

- If the ordering cost \( s \) increases, slight decrease in economic order quantity (except case 2)
- And increase in total relevant cost \( Z(T) \). That is, change in \( s \) leads slight negative change in \( Q \) and total relevant cost \( Z(T) \).
- If the interest charge \( I_s \) per dollar increases, the economic order quantity slightly changes and slight increase in total relevant cost \( Z(T) \). That is, change in \( I_s \) positive change in \( Q \) and slight positive change in total relevant cost \( Z(T) \).
- If the cost discount rate \( r \) increases, the economic order quantity slightly decreases and the total relevant cost increases. That is, change in \( r \) leads slight negative in \( Q \) and increase in total relevant cost \( Z(T) \).
If the interest earned $I_d$ per dollar increases, the economic order quantity slightly decreases and the total relevant cost decreases except case 1.

7. Conclusion and Future Research

In this paper we have developed an inventory model under trade credit with power inventory dependent demand, considering four different cases. We have proved several managerial phenomenons.

- The higher value of $s$, $I_1$, $I_2$, and $r$ caused higher value of the total relevant cost except case 1 (for $I_1$), case 2 (for $I_2$), case 3 (for $I_3$) and case 4 (for $I_4$). We have also provided some examples to validate the proposed model and its optimal solution. Finally sensitivity analysis of the system is also discussed.

- The model proposed in this manuscript can be extended for several ways. For instance, we may extend the model for time dependent deterioration as well as two parameters Weibull distribution deterioration. We may also extend the model for adding freight charges and others.

8. References


