A Novel Investigation on BER Measurement of SC-FDMA System with Combined Tomlinson-Harashima Precoding and Reed Solomon Coding

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Abstract
Objective: Designing energy efficient mobile communication system is the main challenge for the LTE researchers due to increasing of users and their demands in accessing broadband services on mobile phones. High data rate, reduced Bit Error Rate (BER) and low Peak to Average Power Ratio (PAPR) are the requirement of the current telecommunication technology in both uplink and downlink. In this paper, various modulations techniques (QPSK, BPSK and QAM) and pre-coding schemes (DPC, THP, and RS) are proposed to analyze the uplink characteristics. Methods/Analysis: This paper investigates Single Carrier Frequency Division Multiple Access (SC-FDMA) and performs its link level models with Tomlinson-Harashima Pre-coder (THP) and Reed Solomon coding (RS) in separate and integrative manner which is proposed. Findings: In order to prove the hypothetical results, various simulations performed for BER of SC-FDMA system. Results proved that the integration of THP and RS coder in the SC-FDMA transmission, improves the system performance in case of BER. The proposed techniques can be implemented in GPRS, UMTS, GSM, EDGE, HSPA and LTE.

Keywords: Bit Error Rate, LTE, OFDMA, PAPR, SC-FDMA, THP

1. Introduction

Single Carrier Frequency Division Multiple Access (SC-FDMA) and Orthogonal Division Multiple Access (OFDMA) are the most important part of present and future mobile communication. OFDMA is fit for downlink and SC-FDMA is well utilized in uplink1-4. Single carrier structure of SC-FDMA results in a lower Peak-to-Average Power Ratio (PAPR) and low sensitivity to phase noise while compared to OFDMA. Thus it makes more attractive for low power and low cost devices. To study the uplink characteristics (BER, PAPR, Data Rates and SNR) of SC-FDMA in uplink, various modulation (PSK, QPSK, BPSK, and QAM) and coding techniques (DPC, THP, and RS) are developed.

The typical structure of uplink SC-FDMA is shown in Figure 1. The system is same as to OFDMA apart from an FFT block is inserted before sub-carrier mapping and an IFFT block is located after sub-carrier de-mapping.

The transmission part of an SC-FDMA system modulates symbols into blocks and performs an N-point DFT for frequency domain representation of the input symbols. N-DFT outputs are mapped to one of the M > N orthogonal subcarriers5. The receiver converts the received signal to the frequency domain before de-mapping the subcarriers. Then it performs Minimum Mean Square Error (MMSE) equalization or Decision Feedback Equalization (DFE) to get original transmitted data6-8.

2. Modulation and Pre-coding Schemes

To analyze the uplink characteristics of SC-FDMA we have concentrated on the various modulation and pre-coding techniques. We have used two steps to study the SC-FDMA as stated in the Figure 1. In the first step, by applying the Binary Phase Shift Keying, Quadrature
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Phase Shift Keying, Quadrature Amplitude modulation we analyzed the uplink SC-FDMA performance without applying any pre-coding schemes.

The simulation results show that the 16,64QAM has the high performances in reducing the bit error rate. Also, the Peak to Average Power Ratio, Signal to Noise Ratio, Bit Error Rate (BER), and Power Spectral Density for all modes of modulation schemes are calculated for SC-FDMA and plotted in Figure 2.

Figure 1. Typical structure of uplink SC-FDMA system.

Figure 2. Various Modulation Schemes for testing SC-FDMA System.

And in the second step, to solve the multipath effect or Inter symbol Interference and attain high throughput, pre-coding techniques and equalization techniques can be used. However, the linear equalization has some limitations in case of ISI. Thus, this proposed research focus on Tomlinson-Harashima pre-coding technique which is a nonlinear equalization and it overcomes drawbacks of another nonlinear Equalizer called Decision-Feedback Equalizer (DFE). Also we have simulated the Reed Solomon Coding in both transmitter and receiver side.

BER is the main parameter for representing performance of SC-FDMA system of LTE. In the proposed research we have analyzed that the BER in 16-QAM and 64-QAM increases over in both OFDMA and SC-FDMA LTE system. The BPSK and QPSK produce low BER. As shown in Figure 3.

Figure 3. BER measurement of SC-FDMA in various modulation techniques.

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3. Tomlinson-Harashima Precoder Design for SC-FDMA

Tomlinson-Harashima pre-coding is a nonlinear equalization technique used at the transmitter side of SC-FDMA where the equalization is performed. TH pre-coder eliminates error propagation through the transmitter by feeding back the symbols that are already transmitted to reduce interference at the receiver. The proposed THP-SCFDMA block and mathematical model of the TH pre-coder is shown in Figure 4 and 5.
4. Mathematical Concept

THP is nearly indistinguishable to the $1/H(z)$ (inverse channel filter) except an addition of offset free modulo 2M adder instead of the conventional adder. The mathematical equations performed in THP are derived as follows.

$$x[k] = a[k] + d[k] - \sum_{j=1}^{p} h[k], x[k-j]$$

$$x[k] = v[k] - f[k]$$

$$r[k] = \sum_{j=0}^{p} h[k], x[k-j] + n[k]$$

$$r[k] = x[k] + \sum_{j=1}^{p} h[k], x[k-j] + n[k]$$

$$r[k] = x[k] + f[k] + n[k]$$

$$r[k] = v[k] - f[k] + n[k]$$

$$r[k] = v[k] + n[k]$$

We have investigated the SC-FDMA’s BER and PAPR characteristics using THP with MMSE and result information show that THP-MSME offers a greater gain (3-4 dB) in BER over a Decision Feedback Equalizer. The THP outputs in LTE uplink with different modulation techniques are shown in Figure 6 and 7.

Figure 6. THP out with ZF equalizer.

Figure 7. THP with MMSE equalizer.

5. Reed Solomon Encoder Decoder based SC-FDMA

The Reed Solomon coding generates a novel systematic code that would detect and correct various symbol errors.
A novel work of the RS coder is that it replaces an incorrect byte with correct byte, even a bit or whole 8 bit (byte) in the incoming data is corrupted at the correction time. Hence it proves its performance on clearing burst noise.

Figure 8. BER performance with RS Coder.

The RS coder is shown in Figure 8 demonstrates a proposed simulation model for our SC-FDMA uplink. Inter modulation used here is M-QAM before sending the data set through the channel. Then demodulation and RS Decoding takes place as it in the Figures (9-11) to get the original data in the output data side.

Figure 9. BER performance with RS Coder.

Figure 10. BER performance without RS.

6. Proposed SC-FDMA System Combined with RS and THP

In this chapter we implement a novelty system for SC-FDMA uplink for reducing the Inter Symbol Interference (ISI) and lowering the Bit Error Rate (BER). Hence our proposed system's block is formed by data source, RS Encoder, Modulator, Tomlinson-Harashima Pre-coder and SC-FDMA block with Channel to transmit as shown in Figure 12.

Figure 11. BER vs. SNR of SC-FDMA with RS and 16 QAM.

In the proposed SC-FDMA system with Reed Solomon coder and THP, first data symbols are encoded with reed Solomon encoder as designed above. In our simulation, we took RS (255,223) and m=8. The reason for choosing here this RS is that error performance is enhanced when the code block size increases. The BER before RS and after RS are analyzed. Then the RS encoded output fed to the modulator to create modulated complex symbols. Depending upon the channel the LTE has tested various adaptive base band modulation schemes.
The common modulation schemes used by LTE are QPSK, BPSK, and M-QAM. Our system is used the same and get modulated. Then the serial data which is modulated is converted into N parallel data streams and fed to the Tomlinson pre-coder for estimating the channel coefficients in order to know the current feedback of the inter transmitter link. The next step of this system is adding the THP part as created before. The Tomlinson-Harashima pre-coder is used to identify the channel performance in the intermediate point of transmission in order to focus on eliminating ISI which affect the uplink performances. Then it does an N-point Discrete Fourier Transform (DFT), this step will convert time domain symbols to frequency domain symbols. Interleaved sub-carrier mapping which maps N point DFT output symbols to one of M orthogonal sub-carriers.

After that Inverse DFT (IDFT) is performed and Cyclic Prefix (CP) is added for the purpose of eliminating Inter Symbol Interference (ISI). Then the final data is transferred through the channel. And at the receiver side the inverse operation of transmitter side is performed with RS Decoder and most common equalizer called Minimum Mean Square Error (MMSE) to get the final output.

The Figure 13 shows the BER performance of combining RS and THP together with the modulator at the SC-FDMA uplink system. Thus it is possible to observe that BER after the use of THP and RS has a free error transmission. Also the BER achieved by applying only THP does not have that much improvement without using RS.

### 7. Peak to Average Power Ratio (PAPR)

As the power management in the transmission side is the huge requirement of the LTE, there is a strong performance evaluation for the proposed THP+RS based system on PAPR values. The PAPR is measured by Complementary Cumulative Distribution Function. And it is noted to be \( \Pr \{ \text{PAPR} > \text{PAPR}_0 \} \).

### 8. Bit Error Rate (BER), Signal to Noise Ratio (SNR) and Error Probability (Pe)

- **Bit Error Rate (BER)**
  
  \[
  \text{BER} = \frac{\text{Total Error Bits}}{\text{Total number of Bits Transmitted}}
  \]
  
  - **Signal to Noise Ratio (SNR)**
  
  \[
  \text{SNR} = \frac{E_b}{N_0}
  \]
  
  - **Error Probability (Pe)**
  
  \[
  \text{Pe} = \Pr \{ \text{BER} > \text{BER}_0 \} \]
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For all modulation scheme used in the paper, the BER is described in terms of SNR.

The probability of error (Pe) is the rate of errors occurs in the received signal.

\[ Pe = 2Q \left( \sqrt{\frac{2E_b \log_2 M}{N_0} \sin \frac{\pi}{M}} \right) \]

9. Conclusion

In this proposed paper, an investigation on the attainable BER is measured using two methods. In the first method, THP algorithm with both ZF and MMSE and Reed Solomon coder are separately tested the SC-FDMA performance for its BER and PAPR. In the second method, integration of both THP and RS coding are tested with various modulation schemes. It is noted that the BER is comparatively low in the second method than first method. BPSK, QPSK and M-QAM are used in our simulation to test the entire SC-FDMA system and the results are plotted. And finally we conclude that it is possible to combine RS and THP for SC-FDMA to achieve great improvement in the Bit Error Rate and Inter Symbol Interference of the system.

10. References