Passive Target Tracking using Unscented Kalman Filter based on Monte Carlo Simulation

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Abstract

In surveillance sonar passive tracking for bearing measurements has been done so far by using kalman filter and its modifications. In this paper, target is assumed to be moving mostly at constant velocity in underwater scenario. Unscented Kalman Filter has been utilised to track underwater submarine target/moving ship. Monte Carlo Simulation in PC environment is used to prove the effectiveness of UKF. Cases taken up are Ownship/Target S or L manoeuvring. Methods applied include for target tracking, mathematical modelling, and bearings only techniques. Simulation results have been obtained and comparison has been made based on typical scenarios. Different configurations of underwater scenarios can be taken up for further research.

Keywords: Bearings, Line of Sight, Monte Carlo Simulation, Ownship, S-Manoeuvring, Unscented Kalman filter

1. Introduction

In practical applications the problem of bearings-only tracking arises in Ownship manoeuvring like submarine tracking using passive sonar. The problem is referred as Target Motion Analysis (TMA) and the target using noisy bearing measurements1. In this work Sound Navigation and Ranging (SONAR) plays an important role. The main purpose of sonar is the detection or classification (estimation of position, velocity, and identity) of submerged, floating, or buried objects. The two most common types of sonar systems are passive and active. From last two decades in field of nonlinear estimation the passive measurements of the Line-of-Sight (LOS) suggested bearings collected by a moving observer. Such a problem commonly known as Bearings-only Tracking (BOT)1. Passive sonar includes a receiver but no transmitter. The signal to be detected is then the sound emitted by the target. In an active sonar system, waves propagate from a transmitter to a target and back to a receiver. TMA is done by using the measurements required from the target, and these measurements are Doppler frequency and bearings.

In underwater scenario the noisy measurements are very high, when compare with missile platforms. The linearized Kalman Filter is a special kind of optimal filter observes the noisy measurements2. The covariance of these noises is known. In real world no system is linear. So we are going to use nonlinear system. Such as Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). The state distribution is approximated by GRV (Gaussian Random Variable)3. But it is correct up to 1st order. So it is not used for higher nonlinearities. So UKF is used. In the UKF the state distribution is approximated by the sigma points are transformed through the nonlinear function. Here the UKF is used to estimate the state from the noisy measurements is occurred4.
1.1 Input to Simulator

Target Parameters [R, C, B and S] and Ownship parameters [C and S] are read and taken as input by the simulator. Assumed error in bearing measurement (Sigma_b) and range measurement (Sigma_r) are also fed as input.

1.2 Assumptions

Following are the assumptions made in the simulator

- At start, ownship is at the origin.
- Target is moving at constant velocity.
- We follow True North convention.

From Figure 1 ownship follows either L-manoeuvre or S-manoeuvre recommendation. For every second change in x and y component of ownship position is found, and added to the previous x, y components of ownship position.

For \( t_s = 1 \text{ sec} \)

\[
\begin{align*}
\Delta x_o &= v_o \sin(o_c) t_s \\
\Delta y_o &= v_o \cos(o_c) t_s
\end{align*}
\]

Where

- \( \Delta x_o \) is change in x-component of ownship position in 1 sec.
- \( \Delta y_o \) is change in y-component of ownship position in 1 sec.
- \( v_o \) is ownship velocity.
- \( o_c \) is ownship course (True North).
- \((x_o,y_o)\) is ownship position.

\( x_o = (x_o + \Delta x_o) \) and \( y_o = (y_o + \Delta y_o) \). In typical situation, ownship motion is often restricted to straight line segments and each segment is termed as leg. It is well known that to obtain an observable process, for the purpose of estimating the complete target state, an ownship manoeuvre is required. Generally, in Navy S-manoeuvre for ownship is adapted is show in Figure 2 and 3.

1.3 Simulator

1.3.1 Ownship Manoeuvring

Figure 2. Ownership in S-manoeuvre.

\[
Y \text{ (True North)}
\]

1.3.2 Target Manoeuvring

Figure 3. Range between observer and target.

Where

- R is Range.
- C is Course.
- S is Speed.
- B is Bearing.

From input range and bearing initial position of target is known.

For \( t_s = 1 \text{ sec} \)

\[
\begin{align*}
\Delta x_t &= \text{range} \sin(\text{bearing}) \\
\Delta y_t &= \text{range} \cos(\text{bearing})
\end{align*}
\]

\((x_t, y_t)\) is target position with respect to ownship as the origin.
For every 1 sec, changes in \( xt \) and \( yt \) are calculated and added to previous target position.

\[
dxt = vt\sin(tcr)\times ts \\
dyt = vt\cos(tcr)\times ts
\]

\[
xt = xt + dxt \quad \text{and} \quad yt = yt + dyt
\]

Where
- \( dxt \) is change in \( x \) component of target position in one second.
- \( dyt \) is change in \( y \) component of target position in one second.
- \( vt \) is target velocity.
- \( tcr \) is target course with respect to true north.

Range and Bearing are generated by the following formulae:

True range

\[
\sqrt{(xt - xo)\times (xt - xo) + (yt - yo)\times (yt - yo)}
\]

True bearing

\[
\tan^{-1}\left(\frac{xt - xo}{yt - yo}\right)
\]

### 2. Target Tracking

Tracking is the processing of measurements obtained from target for estimating range, bearing, course and speed of its current state. Modern underwater vehicles use multi sensors to track multi targets. For example, modern ships use Towed Array (TA) along with Hull Mounted Array (HMA). TA is used to obtain measurements from target at long ranges when compared to that of HMA. In underwater environment noisy acoustic energies are generated by autonomous underwater vehicles. Sensors are used to sense the noisy acoustic energies or sounds. And produces a signal, and is fed to signal process which produces measurements. Data associator or tracker is to track the target and estimates the state uncertainties shown in Figure 4.

Tracking consists of
- Estimation of the current state of a target (example position, velocity) based on uncertain measurements (example range, bearing).
- Calculation of the accuracy associated with state estimate (example minimum square error matrix).

Where
- \( B \) : Bearing
- \( LOS \) : Line of Sight
- \( OCR \) : Own Submarine Course
- \( TCR \) : Target Course

### 3. Mathematical Modelling

#### 3.1 State and Measurement Equations

The target is assumed to be moving with constant velocity as shown in the Figure 1 and is defined to have the state vector shown in Figure 5. Let the target state vector be

\[
x_{k} = \begin{bmatrix} \dot{x}(k) \\ \dot{y}(k) \\ R_{x}(k) \\ R_{y}(k) \end{bmatrix}
\]

Where \( \dot{x}(k) \) and \( \dot{y}(k) \) are target velocity components and, \( R_{x}(k) \) and \( R_{y}(k) \) are range components respectively. The target state dynamic equation is given

\[
x_{k+1} = \Psi x_{k} + b(k+1) + \Gamma \omega(k)
\]

Where \( \Psi \) and \( b \) are transition matrix and vector respectively. The transition matrix is given by

\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
t & 0 & 1 & 0 \\
0 & t & 0 & 1
\end{bmatrix}
\]

Where \( t \) is sample time

\[
b(k+1) = \begin{bmatrix} 0 & 0 & -\dot{x}(k+1) - x(k) - \dot{y}(k+1) - y(k) \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Figure 4. Target tracking overview.

Figure 5. Target and observer encounter.
And

\[
\Gamma = \begin{bmatrix}
t & 0 \\
0 & t \\
t^2 / 2 & 0 \\
0 & t^2 / 2
\end{bmatrix}
\]

Where \(x_o\) and \(y_o\) are observer position components. The plant noise, \(\omega(k)\) is assumed to be zero mean white Gaussian with

\[
E \left[ \Gamma(k) \omega^T(k) \Gamma^{-1}(j) \right] = Q \delta_{ij}
\]

True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement, \(B_m\) is modeled as

\[
B_m(k+1) = \tan^{-1} \left( \frac{R_x(k+1)}{R_y(k+1)} \right) + \varsigma(k)
\]

Where, \(\varsigma(k)\) is error in the measurement and this error is assumed to be zero mean Gaussian with variance \(\sigma^2\).

### 4. Unscented Kalman Filter Algorithm

The unscented kalman filter is back-up to the EKF was first introduced. It was extended further which shares its computational simplicity while avoiding the need to derive and compute Jacobian and achieving greater accuracy in many scenarios the basis of the UKF is the Unscented Transformation (UT), the moments of non-linearly transformed random variable. The approach is illustrated in figure UKF attempts to remove some of imperfection of EKF in the estimation of non-linear systems. The main idea of UKF is to generate several sampling points (sigma points) around the current state estimation based on mean and covariance. The non-linear function is applied to each point in turn to yield the transformed points observing from Figure 6.

The samples are not drawn at random but according to specific deterministic algorithm. Consider a random variable \(x(\text{dimension } \beta)\) propagating through a nonlinear function \(y = o(x)\). Assume \(X\) has mean \(\bar{x}\) and covariance \(P_x\).

To calculate the statistics of \(y\), a matrix \(\eta\) of \(2\beta + 1\) sigma vectors \(\eta_i\) (with corresponding weights \(W_i\)), is formed according to the following equation.

\[
\eta_0 = \bar{x} \\
\eta_i = \bar{x} + \sqrt{\beta/\lambda} P_{x}^{1/2} \xi_i \quad i = 1, \ldots, 2\beta \\
\eta_i = \bar{x} - \sqrt{\beta/\lambda} P_{x}^{1/2} \xi_i \quad i = \beta + 1, \ldots, 2\beta \\
W_0^{(m)} = \lambda/\lambda + \kappa \\
W_0^{(c)} = \lambda/\lambda + (1 - \gamma^2 + \xi) \\
W_i^{(m)} = W_i^{(c)} = 1/2 (\beta + \lambda) \quad i = 1, \ldots, 2\beta
\]

Where \(\gamma = \varpi_2 (\beta_1 + \kappa)\) — \(\beta_1\) is a scaling parameter \(\varpi\) determines the spread of the sigma points around \(\bar{x}\) and is usually set to a small positive value, \(\kappa\) is a secondary scaling parameter which is usually set to \(0\) and \(\xi\) is used to incorporate prior knowledge of the distribution of \(x\) (for Gaussian distributions, \(\xi = 2\) is optimal). \(\sqrt{\beta + \lambda} P_{x}^{1/2}\) is the \(i^{th}\) row of the matrix square root. \(W_0^{(m)}, W_0^{(c)}, W_i^{(m)}\) and \(W_i^{(c)}\) are the weights of initialized target state vector, covariance matrix of initialized target state vector, target state sigma point vector and covariance matrix of target state sigma point vector respectively. These sigma vectors are propagated through the nonlinear function

\[
y_i = o(\eta_i) \quad i = 1, \ldots, 2\beta
\]
The mean and covariance are approximated using a weighted sample mean and covariance of the posterior sigma points. UKF is a straightforward extension of the unscented transformation to the recursive estimation. In UKF, the state random variable is redefined as the concatenation of the original state and noise variables. The unscented transformation sigma point selection scheme is applied to this new augmented state random variable to calculate the corresponding sigma matrix. The standard UKF implementation consists of the following steps:

1. **Calculation of the (2n + 1) state vectors with sigma points starting from the initial conditions**, where n is dimension of target state vector

   \[
   \mu(k) = \begin{bmatrix} \mu_S(k) \\ \sqrt{(n+\lambda)} i(k) \end{bmatrix} \\
   \Sigma_S(k) = \sqrt{(n+\lambda)} i(k) 
   \]  

   Transformation of these sigma points through the process model using equation.

2. **The prediction of the state estimate at time k+1 with measurements up to time k** is given as

   \[
   X(k+1) = \sum_{i=0}^{2n} W_i^{(m)} \left[ X_S(i,(k+1,k)) - X_S(k+1,k) \right] \\
   \]  

   As the process noise is additive and independent, the predicted covariance is given as

   \[
   P(k+1,k) = \sum_{i=0}^{2n} W_i^{(c)} \left[ X_S(i,(k+1,k)) - X_S(k+1,k) \right] \\
   \left[ X_S(i,(k+1,k)) - X_S(k+1,k) \right]^T + Q(k) 
   \]  

3. **Updation of the sigma points with the predicted mean and covariance**: The updated sigma points are given as

   \[
   X(k+1,k) = \begin{bmatrix} X_S(k+1,k) \\ \sqrt{(n+\lambda)} P(k+1,k) X_S(k+1,k) \end{bmatrix} \\
   \]  

   Transformation of each of the predicted points through measurement model equation.

   **Prediction of measurement given as**

   \[
   \hat{z}(k+1,k) = \sum_{i=0}^{2n} W_i^{(m)} \cdot Y(k+1,k) 
   \]  

   Where

   \[
   Y(k+1,k) = h(X_S(k+1,k)) 
   \]  

4. **Since the measurement noise is also additive and independent, the innovation covariance is given as**

   \[
   P_{yy} = \sum_{i=0}^{2n} W_i^{(c)} \left[ Y(i,(k+1,k)) - \hat{z}(k+1,k) \right] \\
   \left[ Y(i,(k+1,k)) - \hat{z}(k+1,k) \right]^T + \sigma_n^2(k) 
   \]  

5. **The cross covariance is given as**

   \[
   P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} \left[ X_S(i,(k+1,k)) - X_S(k+1,k) \right] \\
   \left[ Y(i,(k+1,k)) - \hat{z}(k+1,k) \right]^T 
   \]  

   **Kalman gain is calculated as**

   \[
   G(k+1) = P_{xy} P_{yy}^{-1} 
   \]  

6. **The estimated state is given as**

   \[
   \hat{X}(k+1,k) = X(k+1,k) + G(k+1) \left( \hat{z}(k+1,k)\right) 
   \]  

7. **Estimated error covariance is given as**

   \[
   P(k+1,k+1) = P(k+1,k) - G(k+1) P_{yy} G(k+1)^T 
   \]  

All raw bearings and elevation measurements are corrupted by additive zero mean Gaussian noise with a r.m.s level of 0.33 degree. Observer is assumed to be stationary or S-manoeuvring shown from Figure 7.

Corresponding to tactical scenarios in which the target is at the initial range of 5000 meters (m) at initial bearing zero and 5 degrees respectively. The target is assumed to be moving at a constant course of 135 degrees (deg) from the Table 1. The results have been ensemble averaged over several Monte Carlo runs shown from Figures 8, 9, 10, 11. The errors in estimates are plotted in Figures. It is observed that this required accuracy is obtained from 1000 seconds onwards and so this algorithm seems to be very much useful for underwater passive target tracking when Observer is stationary or S-manoeuvring, for a moving target.
4.1 Simulation Table

Table 1. Target and observer scenarios

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>PARAMETER</th>
<th>SCENARIO-1</th>
<th>SCENARIO-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial range (m)</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>2</td>
<td>Initial bearing (deg)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Target course (deg)</td>
<td>135</td>
<td>135</td>
</tr>
<tr>
<td>4</td>
<td>Ownship course (deg)</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

4.2 Simulation Results

Figure 7. Target and observer movements.

Figure 8. Error in range.

Figure 9. Error in course.

Figure 10. Error in bearing.

Figure 11. Error in speed.
5. Conclusion

The paper deals with simulation of the motion of the target and determining the initial target parameters namely bearing and frequency. The noise is added to the parameters to get the noisy measurements. BOT is a one of the right method to obtain target motion parameters without using ownship manoeuvre. This method can be easily adopted for underwater passive target tracking application. In this paper, an approach using a UKF (which is useful for nonlinear applications) is proposed to estimate target motion parameters without using ownship manoeuvre in passive target tracking. Monte-Carlo simulation was carried out in the scenarios. Therefore, we may conclude that UKF is robust algorithm.

6. Acknowledgments

The author likes to gratifying thank Sr. Professor Dr. S. Koteswara Rao, scientist of NSTL and Professor Dr. Rudra Pratap Das for their useful support throughout this work. The authors are grateful to their collaborators for their positive comments on the paper.

7. References