The Robust Input/Output Linearization Control for a Time-Varying Nonlinear System with Disturbance

Jong-Yong Lee¹, Young Soo Park¹, Kye-Dong Jung¹ and Chi-Gon Hwang²*

¹Division of General Education, Kwangwoon University, 20 Kwangwoon-ro, Nowon-gu, Seoul - 139-701, Republic of Korea; jyonglee@kw.ac.kr, yspark@kw.ac.kr, gdchung@kw.ac.kr
²Department of Internet Information, Kyungmin College, Uijungbu - 480-702, Republic of Korea; duck1052@kw.ac.kr

Abstract
In this paper, a robust input-output linearization for time-varying nonlinear system is proposed. To this end, feedback linearization for the time-invariant nonlinear systems summarizes. Using the existing linearization, it is proposed the extended input-output linearization technique for time-varying nonlinear system. And the robust linearization for disturbance is proposed. This method is the linearization techniques for a time-invariant nonlinear system. An example of the proposed technique was verified.

Keywords: Extended Input-Output Linearization, Robust Input-Output Linearization, Time-Varying Nonlinear System

1. Introduction
Most of the natural system is expressed as a function of the non-linear and time. In addition, the system of industrial processes is a time-varying nonlinear system. Many studies have been made in order to interpret such a system. First, a time-varying non-linear system analysis based on the equilibrium point was converted to time-invariant linear system. In this case, this expression system has the advantage of simplicity, and to control the system by using a number of control techniques. However, there is a problem that should be corrected the errors occurred in the linearization process. Next there is a way to interpret the invariant nonlinear systems when using differential geometry. But the disadvantage of this technique is to loosen the ordinary differential equation, has the advantage of directly interpreting the system.

Recently, analysis of the time-varying non-linear system using the differential geometry by many researchers has been proposed.

Analysis of non-linear time-varying systems is considered in existing could not ensure the robustness does not take into account the disturbance. In this study, the differential geometry, we consider the case in which the disturbance to the time-varying non-linear system. In this paper, we summarize the analysis of the existing non-linear time-varying systems, and propose a method of linearizing a non-linear time-varying system disturbance is present.

The paper is organized as follows. In Section 2, we summarizes for an existing linearization techniques and propose a robust feedback mechanism for feedback linearization for the disturbance in the system. Section 3 presents the conclusions.

*Author for correspondence
2. Feedback Linearization for the Nonlinear System

In general, the linearization technique using differential geometry is called feedback linearization method. This is because the linearization using the feedback of the system state and the output. Past it been discussed by many researchers, this technique has been applied to a robot systems, power systems, and chemical processes, etc.\(^1,4,13,14\)

In addition, non-linear control using the feedback linearization technique has been studied with respect to the time-invariant systems. In Section 2.1 with respect to the non-linear time-invariant systems, summarize the feedback linearization technique using conventional differential geometry, and in Section 2.2 and summarized the feedback linearization technique for nonlinear time-varying systems. And Section 2.3, we propose a robust feedback linearization technique for nonlinear systems with time-varying disturbance. Feedback linearization technique for nonlinear system in the first invariant as follows:

### 2.1 Classical Feedback Linearization for the Time-Invariant Nonlinear System

The feedback linearization method proposed by Isidori\(^9\) has been extended by many researchers, was a major turning point for the nonlinear control systems. Let’s first single input/output system at a constant expression as follows:

\[
\dot{x} = f(x) + g(x), \quad y = h(x) \quad (1)
\]

Where, \(x \in \mathbb{R}^n\), \(u\) and \(y\) is the state, the control input and the output. The input function \(f\) and output function \(g\) are smooth vector fields on \(\mathbb{R}^n\) and \(h\) is a smooth nonlinear function. Differentiating \(y\) with respect to time \(t\), we obtain as follows:

\[
\dot{y} = \frac{\partial}{\partial t} f(x) + \frac{\partial}{\partial t} g(x) u = L_f h(x) + L_g h(x) u \quad (2)
\]

Where, \(L_f h(x) : \mathbb{R}^n \rightarrow \mathbb{R}\) and \(L_g h(x) : \mathbb{R}^n \rightarrow \mathbb{R}\), defined as the Lie derivatives of \(h\) with respect to function \(f\) and \(g\), respectively. Let the open set \(U\) be an open set containing the equilibrium point \((x_0)\), this point is mean that \(f(x)\) becomes null \(f(x_0) = 0\). Thus, in Eq (2), if the Lie derivative of the output function \(h\) with respect to the input function \(g - L_f h(x)\) is bounded away from zero for all \(x \in U\), then the state feedback law as follows:

\[
u = \frac{1}{(L_g h(x))(-L_f h(x) + v)} \quad (3)
\]

Using a control law expressed by Equation (3), when replaced by the external input \(v\), is constant when a nonlinear system expressed by equation (2) is represented by invariant linear system when the external input \(v\) and the output \(y\). At this time, the relative degree of the system is 1. The relative degree of nonlinear time-invariant system (1) is defined as the number of times the output has to be differentiated before the input appears in its expression. This is equivalent to the denominator in (3) being bounded away from zero, for all \(x \in U\). In general, the relative degree of a nonlinear system at \(x_0 \in U\) is defined as an integer \(\gamma\) satisfying as follows:

\[
L_g L_f h(x) \equiv 0, \quad \forall \ x \in U, \ i = 0, \cdots, \gamma - 2, \ L_g L_f ^{\gamma - 1} h(x_0) \neq 0 \quad (4)
\]

Thus, if the nonlinear time-invariant systems (1) has relative degree equal to \(\gamma\), then the differentiation of output \(y\) in (2) is continued until as follows:

\[
y^{(\gamma)} = L_f h(x) + L_g L_f ^{\gamma - 1} h(x) u \quad (5)
\]

Here, the control law is as follows.

\[
u = \frac{1}{(L_g L_f ^{\gamma - 1} h(x))(-L_f h(x) + v)} \quad (6)
\]

The final input-output systems becomes as follows:

\[
y^{(\gamma)} = v \quad (7)
\]

The expression (7) is expressed by the chain integrators system \(\gamma\). This input can be appreciated that with a type of the condition expression Brunocsky. In addition, if equation (1) expressed as the equation (7), then \(\gamma\) is a observable system through the state feedback, \((n - \gamma)\) is the unobservable system. Let’s look at a robust feedback linearization for nonlinear time-invariant system with the disturbance. In general, for a nonlinear system affected by a measurable disturbance \(d\):

\[
\dot{x} = f(x) + g(x) u + p(x) d, \quad y = h(x) \quad (8)
\]

Where \(p(x)\) is a smooth vector field.

Similar to the relative degree of the timeinvariant nonlinear system, the disturbance relative degree is defined as a value \(k\) for which the following equation (9) holds:

\[
L_p L_f ^{k} h(x_0) \equiv 0, \ i = 0, \cdots, k - 1, \ L_p L_f ^{k - 1} h(x_0) \neq 0 \quad (9)
\]
Thus, a comparison between the input relative degree and the disturbance relative degree gives a measure of the effect that each external signal has on the output\(^5\). If disturbance relative degree \((k)< input relative degree \((\gamma)\), the disturbance will have a more direct effect upon the output, as compared to the input signal. Therefore a control law as given in Equation (6) cannot ensure that the disturbance rejection. In this case complex feed forward structures are required and the effective control low must involve anticipatory action for the disturbance.

The control law in Equation (6) is modified to include a dynamic feedforward and state feedback component which differentiates a state and disturbance dependent signal up to the difference between the input relative degrees of the disturbance relative degree \((\gamma – k)\) times, in addition to the pure static state feedback component. In the particular case, the same relative order and disturbance relative degree \((k = \gamma)\), both the disturbance and the manipulated input affect the output in the same way. Therefore, a feed forward and state feedback element which is static in the disturbance is necessary in the control law in addition to the pure state feedback element\(^6\).

\[
u = \frac{1}{\left(L_gL_t^{\gamma - 1}h(x)\right)\left(-L_t^\gamma h(x) + v - L_pL_t^{\gamma - 1}p(x)d\right)}
\]

(10)

So far, it applies a time-varying nonlinear system on the basis of the mentioned information, we discussed with respect to the robust linearization of the time-varying nonlinear system with disturbance.

### 2.2 Feedback Linearization for Time-Varying Nonlinear Systems\(^6\)

For a single input single output system time-varying nonlinear system, it given by as follows.

\[
\dot{x} = f(x, t) + g(x, t), y = h(x, t)
\]

(11)

Where, \(x \in \mathbb{R}^n\) is the state, \(u\) is the control input, \(y\) is the output, state function \(f\) and input function \(g\) are smooth vector fields on \(\mathbb{R}^n\), output function \(h\) is a smooth nonlinear function and \(t\) is a time. For simplicity of expression, time \(t\) is omitted. Differentiating \(y\) with respect to time \(t\), we obtain as follows.

\[
\dot{y}(t) = \dot{h}(x, t) = \frac{\partial h(x, t)}{\partial x} \frac{dx}{dt} + \frac{\partial h(x, t)}{\partial t}\

f(x, t) + \frac{\partial h(x, t)}{\partial t} + \frac{\partial h(x, t)}{\partial x} g(x, t)u(t)
\]

(12)

Here, the expanded vector of defined as \(X = [x^T, \dot{x}^T]^{\gamma}, F = [f^T, 1]^T, G = [g^T, 0]^T\), and the equation (12) is expressed as follows.

\[
y(t) = L_{f} (h(x, t)) + L_{g} (h(x, t))u(t)
\]

(13)

Here, \(L_{f}h(x, t): \mathbb{R}^{n+1} \rightarrow \mathbb{R}\) and \(L_{g}h(x, t): \mathbb{R}^{n+1} \rightarrow \mathbb{R}\), defined as the Lie derivatives of \(h\) with respect to \(f\) and \(g\), respectively. Let \(U\) be an open set containing the equilibrium point \(x_0\), that is a point where \(f(x_0, t) = 0\). Thus, in equation Equation (13), if the Lie derivative of \(h\) with respect to \(g - L_{g}h(x, t)\) is bounded away from zero for all \(x \in U\), then the state feedback law as Equation (14).

\[
u = \frac{1}{\left(L_g h(x, t)\right)\left(-L_t h(x, t) + v\right)}
\]

(14)

It yields a linear first order system from the external input \(v\) to the initial output of the system, \(y\). Thus, there exists a state feedback law Equation (14), that the time-varying nonlinear system in Equation (11) is converted to the linear system.

The relative degree of system Equation (11) is defined as the number of times the output has to be differentiated before the input appears in its expression. This is equivalent to the denominator in Equation (12) being bounded away from zero, for all \(x \in U\). In general, the relative degree of a nonlinear system at \(x \in U\) is defined as an integer \(\gamma\) satisfying the equation (15).

\[
L_{g}L_{t}^{\gamma - 1}h(x, t) = 0, \quad \forall x \in U, \quad i = 0, \cdots, \gamma - 2, \quad \gamma \geq 2
\]

(15)

Thus, if the time-varying nonlinear system in Equation (11) has relative degree equal to \(\gamma\), then the differentiation of \(y\) in Equation (12) is continued until as follows.

\[
y^{(\gamma)} = L_{f}^{\gamma}h(x) + L_{g}^{\gamma}L_{t}^{\gamma}h(x, t)u
\]

(16)

Here, the control input equal to as follows Equation (17).

\[
u = \frac{1}{\left(L_g L_t^{\gamma - 1}h(x, t)\right)\left(-L_t h(x, t) + v\right)}
\]

(17)

The final new external input–output relation becomes Equation (18).

\[
y^{(\gamma)} = v
\]

(18)

Equation (18) is linear and can be written as a chain of integrators \(\gamma\) and Brunovsky form. The control law in Equation (17) yields \((n - \gamma)\) states of the nonlinear system in Equation (11) unobservable through state feedback.
Next, we consider the feedback linearization problem for the time-varying nonlinear system with a measurable disturbance. The time-varying nonlinear system considering the disturbance is expressed as equation (19).

$$\dot{x} = f(x, t) + g(x, t)u + p(x, t)d, \quad y = h(x, t)$$  \hspace{1cm} (19)

Here, function $p(x, t)$ a smooth vector field and using the vector $P = [p^T, 0]^T$.

Similar to the relative degree of the time-invariant nonlinear system, a disturbance relative degree is defined as a value $k$ for which the following relation holds as follows.

$$L_p^1 \mathcal{h}(x, t) = 0, i = 0, \ldots, k - 1, \quad L_p^k \mathcal{h}(x_0, 0) \neq 0$$  \hspace{1cm} (20)

Here, a comparison between the input relative degree and the disturbance relative degree gives a measure of the effect that each external signal has on the output. If disturbance relative degree ($k$) < input relative degree ($\gamma$), the disturbance will have a more direct effect upon the output, as compared to the input signal. Therefore a control law as given in Equation (14) cannot ensure that the disturbance rejection. In this case complex feed forward structures are required and the effective control low must involve anticipatory action for the disturbance. The control law in Equation (14) is modified to include a dynamic feed forward and state feedback component which differentiates a state and disturbance dependent signal up to the difference between the input relative degree of the disturbance relative degree ($\gamma - k$) times, in addition to the pure static state feedback component. In the particular case, the same relative order and disturbance relative degree ($k = \gamma$), both the disturbance and the manipulated input affect the output in the same way. Therefore, a feed forward and state feedback element which is static in the disturbance is necessary in the control law in addition to the pure state feedback element.

$$u = \frac{1}{L_p^1 \mathcal{h}(x, t)} \left( -L_p^k \mathcal{h}(x, t) + v - L_p^k \mathcal{p}(x, t)d \right)$$  \hspace{1cm} (21)

2.3 Robust Feedback Linearization

To overcome the disadvantages of classical feedback linearization, the robust feedback linearization is performed in a neighborhood of an operating point, $x_o$. The linearized system would be equal to the tangent linearized one around the chosen operating point.

Such system would bear similar physical interpretation as compared to the initial nonlinear system, thus making it more efficient and simple to design a controller.

The time-varying nonlinear system with disturbance vector $d$, is given in the following equation.

$$\dot{x} = f(x, t) + g(x, t)u + p(x, t)d, \quad y = h(x, t)$$  \hspace{1cm} (22)

In robust feedback linearization, the purpose is to find a state feedback control law that transforms the nonlinear system (22) in a tangent linearized one around an equilibrium point, $x_o$.

$$\dot{z} = Az + Bw$$  \hspace{1cm} (23)

In what follows, we assume the feedback linearization conditions are satisfied and that the output of the nonlinear system given in (22) can be chosen as: $y(x) = \lambda(x)$, where $\lambda(x) = [\lambda_1(x) \ldots \lambda_m(x)]$ is a vector formed by functions $\lambda_i(x)$, such that the sum of the relative degrees of each function $\lambda_i(x)$ to the input vector is equal to the number of states of (22).

With the $(A, B)$ pair in (23) controllable, we define the matrices $L(m \times n), T(n \times n)$ and $R(m \times m)$ such that as follows.

$$T(A - BRL)T^{-1} = A_C, \quad TBR = B_C$$  \hspace{1cm} (24)

Here, $T$ and $R$ nonsingular.

By taking the external input as follows.

$$v = LT^{-1}x_c + R^{-1}w$$  \hspace{1cm} (25)

And the state transformation is defined as follows.

$$z = T^{-1}x_c$$  \hspace{1cm} (26)

Using the transformation (26), and the system in (23) is rewritten as follows.

$$\dot{x_c} = A_c x_c + B_c LT^{-1}x_c + B_c R^{-1}w = w(A_c + B_c LT^{-1})x_c + B_c R^{-1}w$$  \hspace{1cm} (27)

Equation (26) yields as follows.

$$z = T^{-1}x_c \to \dot{z} = Tz$$  \hspace{1cm} (28)

Replacing (28) into (27) and using (24), gives as follows.

$$\dot{z} = (A_c + B_c LT^{-1})Tz + B_c R^{-1}v$$
linear system was.

varying linear system to convert the time-invariant input-output linearizing technique, through a time-

for a time-invariant nonlinear system. Also, extended linearization technique for time-varying nonlinear systems summarizes. Using the existing lin

disturbance is linearization, it is proposed the extended input-output linearization technique for time-varying nonlinear system. And the robust linearization for disturbance is proposed. This method is the linearization techniques for a time-invariant nonlinear system. Also, extended input-output linearizing technique, through a time-varying linear system to convert the time-invariant linear system was.

4. References


14. Pop CI, Dulf E, Festila Cl. Nonlinear robust control of the 13c cryogenic isotope separation column. Proceedings of the 17th International Conference on Control Systems and