Optimum Reserve Estimation in Micro-Grids containing Renewable Distributed Generation Resources

Mohammad Dehghani Sanij*, Mahdi Dehghani-Ashkezari1 and Hamed Hashemi-Dezaki2
1Ashkezar Branch, Islamic Azad University, Ashkezar, Iran; Mohammad.d@aut.ac.ir, ma_dehghani2006@yahoo.com
2Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran; hamed.hashemi@aut.ac.ir

Abstract

Background/Objectives: The increased application of renewable resources in new distribution networks which are known as micro-grids, have caused challenges in utilizing such resources. This is mainly due to the uncertainty and probabilistic behavior of output power of renewable resources. Methods/Statistical Analysis: In this research, a novel stochastic method is proposed for estimating the required Spinning Reserve in a micro-grid containing renewable generation resources. As the amount of intended reserve increases, the cost of load supplying will increase while the outage penalty will decrease. Results: In this study, the amount of Spinning Reserve is determined based on balancing between the reliability cost and the cost of reserve supply. The uncertainty associated with loads, units and renewable resources is considered in this estimation. Another advantage of the proposed method is reduction of calculations due to the fact that different uncertainties will be summed together. The Mixed Integer Linear Programming (MILP) optimization problem is solved by means of Baron/GAMS algorithm. The introduced method is applied to an illustrative test system as well as IEEE 30-bus. Conclusion/Application: The test results illustrate the effectiveness of proposed method from the cost reduction and secure operation perspective.

Keywords: Generation Resources, Micro-Grids, Optimum Reserve Estimation, Renewable Distributed, Renewable Resources, Stochastic Method

1. Introduction

Dominance of Distributed Generators (DGs) in two past decades caused micro-grids to become more attractive within modern distribution networks. Micro-grids can be defined as entities containing low voltage networks, loads, Energy Storing Systems (ESSs) and several small systems of modular production (which present power and heat). Micro-grids can decrease carbon distribution, improve the quality of power, increase local reliability, decrease Energy distribution costs and postpone the development of traditional systems1-3. Utilization of micro-grid requires an Energy management strategy. Energy management strategy must encompass short-term and long-term power balances. Keeping appropriate level of Spinning Reserve (SR) is one of the most important aspects of energy management4.

Spinning Reserve refers to unusable capacity of power system for voluntary response to events in a specified period of time4. Spinning Reserve is an important resource that can fulfill the maintenance of power systems without load loss. Increased requirement to Spinning Reserve can decrease the probability of load loss, while supplying the Spinning Reserve is costly since extra generators shall be inserted into the circuit; therefore, the outputs of units will not be adjusted according to their optimization range.

* Author for correspondence
Many methods are proposed for determining required spinning reserve within big power systems. Traditionally, deterministic methods are utilized for determining the amount of Spinning Reserve\(^6\). The amount of required Spinning Reserve is defined as the capacity of the biggest unit in circuit, or a part of load, or a combination of both. Although this method is easy to implement, it takes into account neither the random nature of system behavior nor economic optimization.

Different probabilistic methods are developed for Spinning Reserve optimization. H.B. Gooi et al.\(^7\) have optimized required Spinning Reserve for the first time in a Unit Commitment (UC) issue by a probabilistic criterion. In this method, risk assessment is utilized in order to calculate optimized Spinning Reserve. F. Bouffard and F. D. Galiana\(^a\) and D. N. Simopoulos et al.\(^9\) have optimized required Spinning Reserve in a UC issue by reliability limitation. Limitations forming reliability criteria such as Loss of Load Probability (LOLP) or average power loss (EENS) shall be placed under a pre-defined threshold. But, design of complicated reliability criteria ceiling is difficult for different power systems. However, limiting such intervals increases the total cost of load supply while cost reduction leads to risk increase. J. X. Wang et al.\(^10\) and M. A. Ortega-Vazquez et al.\(^11\) optimize required Spinning Reserve by balancing between reliability and cost. This approach prevents autonomous selection of risk ceiling. However, it is assumed that reserve market is independent from energy market. Ignoring the relationship between energy market and reserve market may lead to results that are either less than optimum or impractical. Regarding above discussions, estimating the optimum Spinning Reserve required in micro-grids is not easily done.

Surender Reddy has Clear joint energy and reserve market considering wind and load forecast error. Unit outage rate (which is an important uncertainty source) has not been considered\(^12\). Reserve resources in the study of Surender Reddy\(^12\) are thermal unit and demand side reserve. In study conducted by J. M. Morales et al.\(^13\), a methodology is presented to identify the appropriate reserve level and cost in a power system with high penetration of wind power. In other work by J. Wang et al.\(^14\), a day-ahead Security-Constrained Unit Commitment (SCUC) model for energy and ancillary services auction is proposed, which can be used by an ISO to optimize reserve requirements in electricity markets. In the study of M. Parvania et al.\(^15\), a short-term stochastic SCUC model is presented which simultaneously schedules generating units’ energy, SRs and reserve provided by demand response resources. A comprehensive formulation to model the demand response in the market clearing process and the benefits of demand response.

The capability of micro-grids to supply Spinning Reserve in depends on their compositions (instance eg., the combination of different resources). Moreover, random output power of renewable units such as Wind Turbines (WT) and Photovoltaic (PV) creates significant uncertainty in micro-grids. In this study, a probabilistic method has been presented for Spinning Reserve estimation for a planning model which has been established recently. Required optimized Spinning Reserve is determined through balancing between reliability and cost.

## 2. Probabilistic Modeling of Load, WT DG Units, and PV DG Units

### 2.1 Load Probabilistic Model

The amount of uncertainty in load prediction of a micro-grid can be assessed using the system history. The normal distribution model is utilized in seven steps similar to big power systems. Regarding error increase of load forecasting at micro-grid level, the standard deviation shall be rather high\(^6\).

### 2.2 WT DG Units Probabilistic Model

The output power of Wind Turbines changes based on wind speed variations\(^16,17\). Since Wind Turbine units are renewable, they don’t provide Spinning Reserve. Previous researches have indicated that the features of wind speed in a specific place have the closest relation with Weibull distribution. Therefore, in this research, we assume that wind speed is predicted using Weibull distribution of average wind speed. The Probability Density Function (PDF) of wind speed described by Weibull distribution is as formulated as follows:

\[
    f(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{(k-1)} e^{-(v/c)^k}, \quad 0 < v < \infty
\]

\(v, k, c\) represent wind speed (m/s or miles/h), figure factor (without dimension) and scale factor (a portion of speed unit), respectively. To simplify the calculations, wind speed distribution is normalized by average speed of Weibull distribution, \(v_{mean}\) Probable points are incremented from \(v_{mean}\) in fixed steps to the time point most are surrounded from distribution. Therefore, distribution curve can be divided to specified intervals whose number depends on the intended accuracy. The probability of each
interval can be calculated by numerical integral by means of related software. In this study, 5-interval distribution of wind speed (Figure 1.) is utilized.

Figure 1. Wind speed distribution probability curve.

Regarding wind speed distribution and speed to power conversion function, we can obtain power distribution of Wind Turbine. In this study, the function used for converting speed to productive power is as follows:

\[ \omega = \begin{cases} 0, & \text{for } \nu < \nu_i \text{ and } \nu > \nu_e \\ \omega_i \frac{(\nu - \nu_i)}{(\nu_e - \nu_i)}, & \text{for } \nu_i \leq \nu \leq \nu_e \\ \omega_e, & \text{for } \nu \leq \nu_i \text{ and } \nu \leq \nu_e \end{cases} \]  

\[ (2) \]

\[ \omega, \nu, \nu_i, \nu_e \text{ and } v_o \text{ represent the output power of Wind Turbine (kw), nominal power, minimum wind speed, nominal wind speed and maximum wind speed, respectively.} \]

2.3 Probabilistic Models of Photovoltaic

PV output mostly depends on radiation, and since radiation varies during a day, this resource is also a renewable one like Wind Turbine. Hourly radiation distribution in a specific place usually follows bimodal distribution\(^{17,18}\) that can be considered as linear combination of two unimodal distribution functions\(^{16}\). Unimodal distribution functions can be modeled through Beta, Weibull and log-normal PDFs. Here, double Weibull distribution is utilized based on Equation 3.

\[ f(g) = \omega \left( k_1/c_1 \right) \left( g/c_1 \right)^{(k_1-1)} e^{-\left( g/c_1 \right)^{k_1}} + (1-\omega) \left( k_2/c_2 \right) \left( g/c_2 \right)^{(k_2-1)} e^{-\left( g/c_2 \right)^{k_2}} \quad 0 < g < \infty \]  

\[ (3) \]

Where g is radiation (kW/m\(^2\)), w is weight factor, \(k_1\) and \(k_2\) are form factors and \(c_1\) and \(c_2\) are scale factors. Similar to curve model of wind speed distribution, the radiation distribution function can become discrete. The 5-interval radiation distribution function utilized in this study is represented in Figure 2.

Figure 2. Radiation distribution probability curve.

Regarding radiation distribution and radiation-to-power conversion function, we can obtain power distribution of PV. The radiation-to-power conversion function utilized in this study is as follows\(^{19}\):

\[ p = \eta_{pv} S_{pv} g \]  

\[ (4) \]

\[ p \text{ is output power of PV (kw), } \eta_{pv} \text{ is efficiency (\%) and } S_{pv} \text{ is total area of PV (m}^2\text{).} \]

3. Problem Formulation

Micro-grids can be utilized in two modes: disconnected from network or connected to network. In disconnected mode, the goal is to minimize total cost while in connected mode, the goal is to maximize profit which is equal to the difference between income and cost\(^{20}\). In this study, only connected mode is considered. The disconnected mode can also be modeled and resolved based on this method.

3.1 Mathematical Model of Optimal Reserve Estimate

Mathematical model of the issue in connected mode includes objective function and constraints are presented in this part. Objective function is as follows:

\[ \max \left\{ \sum_{t=1}^{T} \sum_{i=1}^{I} MP(t_i)P_{ij} - \sum_{t=1}^{T} \sum_{i=1}^{I} \left[ C_{ij}(P_{ij}, U_{ij}) + SC_{ij}K_{ij} \right] \right\} \]

\[ \quad \quad - \sum_{t=1}^{T} \sum_{i=1}^{I} q_{ij} R_{ij} - ENS.VOLL \]  

\[ (5) \]
The first term of objective function represents the total income. The second term represents utilization costs which include production cost as well as units implementation cost. Production cost function can be represented by a linear or piece wise linear function. The third term represents reserve cost and the final term represents the expected outage cost that is equal to Value of Load Loss (VOLL) multiplied by EENS. VOLL is usually estimated through questionnaires collected from consumers. In objective function, utilization costs, reserve cost and expected outage cost are having contradictory goals. When Spinning Reserve increases, utilization and reserve costs will increase while expected outage cost will decrease. Automatic optimization determines the optimum required Spinning Reserve based on internal analysis of cost profit. In continual, we will discuss the limitations of the issue. LOLP limitation which is based upon recommended LOLP formula is indicated in next part as follows:

$$LOLP_t \leq LOLP_t^{max}$$  \hspace{1cm} (6)

Is the LOLP in period of t, and $LOLP_t^{max}$ is the maximum legal LOLP in t period. LOLP limitation is considered as an extra security limitation and it can be deactivated by setting high limits. Balance limitation of power is as follows:

$$\sum_{i=1}^{S} P_{i,t} = P^D_t \hspace{1cm} \forall t = 1,....,T$$  \hspace{1cm} (7)

The limitation of Spinning Reserve is also explained by Equation 8:

$$\begin{align*}
R_{s,t} &\leq P_{s,t}^{max} U_{s,t} - P_{s,t} \\
R_{s,t} &\leq U_{s,t} (R_{s,t}^{max} \tau)
\end{align*}$$  \hspace{1cm} (8)

In addition, each productive unit has its own utilization limits including the limitation of maximum or minimum production, the limitation of minimum outage or working time, initial conditions limitation and the limitation of production increase and decrease. Storage resources (ESS) such as battery banks are considered as specific dispatching units to store and produce energy at the same time. ESS shall meet the following limitations. Output power limits:

$$P_{t}^E \leq P_{t}^{max}$$  \hspace{1cm} (9)

Charge equation:

$$C(t+1) = C(t) - d_t \cdot P_t^E$$  \hspace{1cm} (10)

Discharge equation:

$$C(t+1) = C(t) - d_t \cdot P_t^E / \eta_t^E$$  \hspace{1cm} (11)

Starting/Ending limits:

$$C(0) = C_T, C(T) = C_E$$  \hspace{1cm} (12)

Stored energy limits:

$$C_{min} \leq C(t) \leq C_{max}$$  \hspace{1cm} (13)

$P_t^E$ is output power during t period; $P_{t}^{max}$ is limitation of maximum allowed charge and discharge; $\eta_t^E$ is discharge efficiency; $d_t$ is time duration of each period; $C(t)$ is reserved energy in ESS until t period; $C_T$ and $C_E$ are the energy during start and finishing of programming period, respectively; and are the maximum and minimum allowed reserved energy in ESS. Upstream network is also considered as a special dispatching unit that can absorb energy. The capacity of this network is limited according to its capacity of communication line with micro-grid. The price of supplied energy from this resource is equal to market price. Its reliability is a combination of the reliability of upstream network and lines.

3.2 EENS and LOLP Formulation

As we know, the variables of units participation planning issue (UC) include the status of units and their output powers, therefore, in order to calculate EENS and LOLP we can easily act based on probability table of capacity exit convolution (COPT) and load curve. However, due to the presence of EENS and LOLP in formulation of units participation issue, these variables shall be determined at the same time with other variables of the issue. The equations of these two variables include Equation (14) and Equation (15).

$$EENS_s = \sum_{s=1}^{S} p_s b_s (\Delta P_s + \Delta R_s - R_s) d_t$$  \hspace{1cm} (14)

$$LOLP_t = \sum_{s=1}^{S} p_s b_s$$  \hspace{1cm} (15)

$S$ is sum of load loss events probabilities that are determined regarding uncertainties or units’ exit; $P_{s,t}^E$ and $\Delta P_s$ represent probability, power limitation and reserve limitation under s event during t period, respectively. Binary variable of $b_s$ is defined as follows:

$$b_s = \begin{cases} 
1, & \text{if } \Delta P_s + \Delta R_s - R_s > 0 \\
0, & \text{otherwise}
\end{cases}$$  \hspace{1cm} (16)

In this study, only PV, WT and load uncertainties and units’ uncertainties are considered. The impacts of network and market price uncertainties are not considered. $d_t$ is equal to 1 hour. Regarding Equation (14), EENS can be rewritten as below. To simplify the formulation regarding low level of probability, EENS caused by third order and higher outage events is not considered very low,
EENS = EENS₁ + EENS₂ + EENS₃

\[
EENS = \sum_{i=1}^{n_{WT}} \sum_{t=1}^{T} P_{WT}^{i} P_{PV}^{i} b_{t,i,m}^{o} x(\Delta P_{WT}^{i} + \Delta P_{PV}^{i} - R_i) 
+ \sum_{i=1}^{n_{WT}} + \sum_{t=1}^{T} + \sum_{k=1}^{K} + \sum_{l=1}^{L} + \sum_{m=1}^{M} P_{WT}^{i} P_{PV}^{i} P_{PV}^{t} b_{l,m,t}^{o} x(P_{i} + R_j + \Delta P_{WT}^{i} + \Delta P_{PV}^{i} - R_i) 
+ \sum_{i=1}^{n_{WT}} + \sum_{t=1}^{T} + \sum_{k=1}^{K} + \sum_{l=1}^{L} + \sum_{m=1}^{M} P_{WT}^{i} P_{PV}^{i} P_{PV}^{t} b_{l,m,t}^{o} x(P_{i} + R_j + \Delta P_{WT}^{i} + \Delta P_{PV}^{i} + \Delta P_{PV}^{i} + \Delta P_{PV}^{i} - R_i) 
\]

\[
EENS₁, EENS₂, \text{ and } EENS₃ \text{ represent mean energy loss}
\]

4. Case Study

The proposed model is tested on an illustrative example and modified IEEE 30-bus test system.

4.1 Illustrative Example

To conduct a case study, a hypothetical micro-grid is utilized in this study. The features of load and energy price are indicated in Tables 1 and 2, respectively.

**Table 1.** Load level of system during 24 hours of study

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load level</td>
<td>70</td>
<td>65</td>
<td>60</td>
<td>65</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>Time</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Load level</td>
<td>140</td>
<td>155</td>
<td>170</td>
<td>175</td>
<td>172</td>
<td>172</td>
<td>172</td>
<td>167</td>
</tr>
<tr>
<td>Time</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>Load level</td>
<td>160</td>
<td>180</td>
<td>200</td>
<td>190</td>
<td>160</td>
<td>140</td>
<td>120</td>
<td>100</td>
</tr>
</tbody>
</table>

**Table 2.** Electricity price in upstream network during 24 hours of study

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity price</td>
<td>3</td>
<td>2.9</td>
<td>2.8</td>
<td>2.7</td>
<td>2.6</td>
<td>2.7</td>
<td>2.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Time</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Electricity price</td>
<td>15</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>15</td>
<td>40</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Time</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>Electricity price</td>
<td>6</td>
<td>4</td>
<td>3.5</td>
<td>4</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 3.** DG unit characteristics

<table>
<thead>
<tr>
<th>Unit type</th>
<th>MT</th>
<th>FC</th>
<th>WT</th>
<th>PV</th>
<th>ESS</th>
<th>Upstream Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Power (kW)</td>
<td>30</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>-20</td>
<td>-100</td>
</tr>
<tr>
<td>Max Power (kW)</td>
<td>180</td>
<td>170</td>
<td>80</td>
<td>70</td>
<td>30</td>
<td>190</td>
</tr>
<tr>
<td>Min Up/Down</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Time (h)</td>
<td>900</td>
<td>900</td>
<td>600</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Ramp Up/Down</td>
<td>10.63</td>
<td>10.63</td>
<td>54.84</td>
<td>54.84</td>
<td>0</td>
<td>MP(1)</td>
</tr>
<tr>
<td>Down Rate (kW/h)</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>Failure Rate (f/</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>year)</td>
<td>b(Cent/kWh)</td>
<td>43.7</td>
<td>28.4</td>
<td>10.63</td>
<td>54.84</td>
<td>0</td>
</tr>
<tr>
<td>c(Cent/h)</td>
<td>425</td>
<td>850</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>SC(Cent)</td>
<td>45</td>
<td>53</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Four DG units are installed on micro-grid: a Wind Turbine, a Photovoltaic, a fuel cell and a micro turbine. The micro-grid includes one ESS which is connected to upstream network. We can consider it as 6-unit system.
Table 3 indicates the summary of their parameters. Here, a production cost function tender is utilized.

For ESS: $P_{\text{max}} E = 20\text{kW}$, $\eta_E = 0.9$, $C_{\text{min}} = 180\text{kWh}$, $C_{\text{max}} = 260\text{kWh}$ and load standard deviation is $\sigma = 0.8$.

$$ VOLL = \frac{5}{E} \text{LOLP}^{\text{max}} = 0.005, q_i = 0.4, \text{cent/\text{kW}} \tau = 10, \text{min}, E = 0.005. $$

Hourly wind speed mean of WT and PV radiation are presented in Tables 4 and 5, respectively.

**Table 4. Wind speed during 24 hours of study**

<table>
<thead>
<tr>
<th>Time(h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind speed(km/h)</td>
<td>7.0</td>
<td>6.8</td>
<td>6.7</td>
<td>6.7</td>
<td>6.7</td>
<td>6.9</td>
<td>7.1</td>
<td>7.2</td>
</tr>
</tbody>
</table>

**Table 5. The amount of solar radiation during 24 hours of study**

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

For WT, $v_r$, $v_i$ and $v_0$ are respectively 5, 15 and 45 miles/h, it is assumed that for all the hours $J = 10$ and $k = 2$ and $c = v_{\text{max}}/0.9\text{miles/h}$. For PV, $\eta_{PV}^{\text{max}} = 12\%$ and $S_{PV} = 270 \text{m}^2$. It is assumed that for all the hours $K = 10$, $\omega = 0.3$, $k_i = 2$, $k_j = 10$, $c_z = g_{\text{mean}}/0.95$, $c_i = 0.4c_z$.

Simulations are done in several modes in this study. Since the model becomes non-linear by calculation of EENS, initially, the model is performed without considering EENS, and only by calculation of LOLP and putting maximum condition; the results have been presented in 5-1. Afterwards, the results are calculated by considering energy loss and they are presented in 5-2.

### 4.1.1 Results of Simulation without Considering Outage Penalty

The results of this part include the cost of load supply, maximum available reserve, optimum reserve and LOLP amount during programming period. The estimated cost in this mode is 44021.456 dollars. Figure 4 represents optimum reserve and the maximum available reserve, respectively. In addition, Figure 5 represents LOLP amount per hour.

![Figure 3. Optimal hourly reserve (MW).](image)

![Figure 4. The maximum available reserve (MW).](image)

![Figure 5. Hourly LOLP.](image)

### 4.1.2 Simulation Results by Considering Outage Penalty

The results of this part include cost of load supply, maximum available reserve, optimum reserve and LOLP amount during programming period in addition to the amount of EENS. In this part, since outage penalty is added to the objective function, optimum reserve is expected to increase; therefore, LOLP amount will decrease. Overall cost of load supply in this mode is estimated 49281,767...
US$. Figure 6 and Figure 7 represent optimum reserve and maximum available reserve, respectively. Moreover, the amount of mean unfulfilled energy in this mode is calculated 158.42 KWh.

4.1.3 Comparative Studies

By comparing the results of two studies carried out in 5-1 and 5-2, it is observed that cost of load in second mood, in which EENS is added in form of a component of costs in the objective function, is more. This incensement is due to the new cost component and using more units. More turned units can be found through amount of available reservation. In other words, by comparing Figure 7 and Figure 4, it is observed that the amount of available reservation in second study is more than the first one. This will distance us from economic load distribution and therefore, it will rise the costs. It is worth noting that the second study is closer to reality, since practically outage penalties must be paid to the subscriber by the network operator.

4.2 IEEE 30-Bus Test System

The IEEE 30 bus system is used to establish the effectiveness of the proposed approach. The test system consists of six generators, of which, four are CGs located at buses 1, 2, 5 and 8; and two are assumed to be wind farms, located at buses 11 and 13. The reserve cost coefficients and penalty cost coefficients for wind power and load are selected based on the information given in the investigation of Hetzer et al. and PV generation data is as illustrative example.

<table>
<thead>
<tr>
<th>Table 6. Summary of results for these four case studies</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheduled generation, SR</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>P_1(MW)</td>
<td>105.2</td>
<td>152.1</td>
<td>112.07</td>
<td>171.57</td>
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<tr>
<td>P_2(MW)</td>
<td>47.7</td>
<td>36.98</td>
<td>42.18</td>
<td>26.00</td>
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<tr>
<td>P_3(MW)</td>
<td>40.87</td>
<td>28.45</td>
<td>43</td>
<td>26.65</td>
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<tr>
<td>P_4(MW)</td>
<td>43.05</td>
<td>37.00</td>
<td>39.44</td>
<td>25.22</td>
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<tr>
<td>P_w1(MW)</td>
<td>29.04</td>
<td>22.30</td>
<td>25.65</td>
<td>16.88</td>
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<tr>
<td>P_w3(MW)</td>
<td>22.77</td>
<td>15.35</td>
<td>28.25</td>
<td>27.10</td>
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<tr>
<td>SR_1(MW)</td>
<td>21.52</td>
<td>16.94</td>
<td>18.47</td>
<td>18.40</td>
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<tr>
<td>SR_2(MW)</td>
<td>11.53</td>
<td>16.08</td>
<td>20.29</td>
<td>10.04</td>
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<tr>
<td>SR_5(MW)</td>
<td>15.80</td>
<td>6.62</td>
<td>19.01</td>
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<td>SR_8(MW)</td>
<td>22.57</td>
<td>12.50</td>
<td>9.44</td>
<td>15.45</td>
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<td>Total generation cost($/h)</td>
<td>947.12</td>
<td>966.00</td>
<td>949.74</td>
<td>982.61</td>
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<td>Thermal SR cost($/h)</td>
<td>326.17</td>
<td>261.34</td>
<td>304.08</td>
<td>251.84</td>
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<tr>
<td>Total cost($/h)</td>
<td>1287.09</td>
<td>1289.20</td>
<td>1296.13</td>
<td>1299.50</td>
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<tr>
<td>Total SR required(MW)</td>
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<td>52.20</td>
<td>67.21</td>
<td>46.91</td>
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<td>SR for renewable</td>
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<tr>
<td>Wind</td>
<td>14.95</td>
<td>10.60</td>
<td>12.73</td>
<td>7.43</td>
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<tr>
<td>PV</td>
<td>4.43</td>
<td>2.18</td>
<td>6.67</td>
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<tr>
<td>Excess power available</td>
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<tr>
<td>Wind</td>
<td>0.30</td>
<td>2.69</td>
<td>1.44</td>
<td>4.97</td>
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<tr>
<td>PV</td>
<td>2.16</td>
<td>5.97</td>
<td>0.44</td>
<td>0.71</td>
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<tr>
<td>SR required for loads</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind</td>
<td>2.39</td>
<td>2.39</td>
<td>4.68</td>
<td>4.68</td>
</tr>
<tr>
<td>PV</td>
<td>2.39</td>
<td>2.39</td>
<td>4.68</td>
<td>4.68</td>
</tr>
</tbody>
</table>
In reality, wind farms consist of a number of wind-driven DFIG generators. Here, for simplicity, all wind turbines are considered to be identical.

Table 6 summarizes the results of these four case studies: Four cases have been examined in this case study as follows:

- Case 1: The total cost minimization objective with uncertainty in wind power and ±5% uncertainty in load forecasts;
- Case 2: The total cost minimization objective considering uncertainty in wind power and ±5% uncertainty in load forecasts;
- Case 3: The total cost minimization objective with uncertainty in wind power and ±10% uncertainty in load forecasts;
- Case 4: The total cost minimization objective considering uncertainty in wind power and ±10% uncertainty in load forecasts.

## 5. Conclusion

In this study, a method is introduced for determining required Spinning Reserve in micro-grids. Required optimum Spinning Reserve is determined by maximizing profit regarding units’ uncertainty and uncertainty due to unit forced outage, load and renewable units. In fact, by increasing reserve level, the cost of load supply will increase while outage penalties will decrease. On the other hand, by decreasing reserve level, overall cost of load supply will decrease while system will face higher outage penalties. Therefore, the optimum reserve is a trade-off between outage penalties and cost of load supply; in this research, this reconciliation is being studied.

## 6. References

Nomenclature

A) Indices

i, j Unit Index.

k Number of wind speed distribution index.

l Intervals of load distribution.

m Radiation distribution intervals index.

t Index of optimization period.

B) Variables

B-1) Binary Variables

\( b_{i,k,l,m,t}^0 \) Binary variable, whose “1” value means that the uncertainty of load, photovoltaic modules (PV), and Wind Turbine (WT) led to loss of load.

\( b_{i,k,l,m,t}^1 \) Binary variable, whose “1” value means that the uncertainty of load, photovoltaic modules (PV), Wind Turbine (WT), and unavailability of the i-th unit led to loss of load.

\( b_{i,j,k,l,m,t}^2 \) Binary variable, whose “1” value means that the uncertainty of load, photovoltaic modules (PV), Wind Turbine (WT), and simultaneous unavailability of the i-th and j-th units led to loss of load.

\( b_{i,t}^3 \) Binary variable, whose “1” value means that the aggregate uncertainty during t-th period led to loss of load.

\( b_{i,t}^4 \) Binary variable, whose “1” value means that the aggregate uncertainty and outage of i-th unit led to loss of load.

\( b_{i,j,t}^{5,0} \) Binary variable, whose “1” value means that the aggregate uncertainty and simultaneous outage of i-th and j-th units led to loss of load.

\( b_{i,j,t}^{5,1} \) Binary variable, whose “1” value means that the final combined distribution during t period led to loss of load.

\( K_{i,t} \) Binary variable, whose “1” value means that the i-th unit was set during t period.

\( U_{i,t} \) The status of i-th unit (i.e., being 1 or 0) during t period.

B-2) Continuous Variables

\( p_i^0 \) Probability of all units being operative at hour t.

\( p_i^1 \) Probability of all except i-th unit being operative at hour t.

\( p_{i,j}^1 \) Probability of all except i-th and j-th units being operative at hour t.

\( P_{i,t} \) Output power of i-th unit at hour t.

\( R_{i,t} \) The amount of proposed reserve in i-th unit and during t period.

\( R_t \) The total amount of reserve during t period.

C) Constants

\( d_t \) Duration of each period.

\( l \) Number of production units.

\( K,M,L \) Number of wind speed, radiation, and load, scenarios, respectively.

\( MP(t) \) Active power prices on the open market during t period.

\( p_i^0, p_i^1 \) Possibility and deviation occurs when load fall happens in I.

\( p_{i,w}^0, p_{i,w}^1 \) Possibility and wt power deviation when the wind speed is in the range of k.

\( p_{i,v}^0, p_{i,v}^1 \) Possibility and PV power deviation when the radiation is in the range of m.

\( p_{i,n}^0, p_{i,n}^1 \) Possibility and Deviation of power when nth aggregated distribution of uncertainty has been selected during t period.

\( p_{i,n}^0, p_{i,n}^1 \) Possibility and Deviation of power when nth final combined distribution caused by uncertainty second cut off events and higher have been selected.

\( P_{i}^{\text{max}} \) Maximum power output of i.

\( P_i^0 \) Load forecasting system over the period t.

\( q_i^0 \) Saved price offer of I during t.

\( R_{i}^{\text{up}} \) Rate of I rise and fall.

\( SC_{i} \) cost of setting up i unit.

\( T \) Periodic optimization.

\( u_i \) Uncertainty or ORR of unit I.

\( T \) The amount of time required to deliver the output of available generators to produce store.