Abstract

**Background/Objectives:** Pseudo Linear Estimator (PLE) is developed for active sonar applications. **Methods/Statistical analysis:** The PLE offers features of Extended Kalman Filter (EKF). **Findings:** The results of PLE are compared with that of EKF. The results of MC simulation are presented for typical scenarios. **Application/Improvements:** In PLE, there is no need to initialize target state vector and its covariance matrix with prior (approximate) knowledge and hence its performance is found to be better than that of EKF.

**Keywords:** Estimation, Kalman Filter, Pseudo Linear Estimator, Simulation, Target Tracking

1. Introduction

The algorithm PLE is extended to three dimensional application of Micro Array Vehicle (MAV). Active sensors of MAV generate noisy range and bearing measurements. The MAV processes these measurements and estimates target dynamics. There are many methods available⁴ to obtain target motion parameters in the above situation. Lindgren and Gong, Aidala, Aidala & Nardone, and Nardone, Lindgren & Gong developed Pseudo Linear Estimator (PLE) in batch processing. Rao converted PLE to sequential processing.

In this paper, pseudo linear measurements are derived for both range and bearing measurements. In EKF, the state vector is initialized using the measured range and bearing measurements. The speed components are initialized using the present & previous measurements and measurement time interval. The time to obtain the convergence in the solution decreases if the assumed initial target state vector is nearer to the actual target state vector. Secondly, EKF requires initialization of target state covariance matrix. Researchers assume that initial target state vector follows uniform / Gaussian density function and accordingly the covariance matrix of initial target state is calculated. Sometimes, it is simply assumed to be unit diagonal matrix. In PLE, there is no need to initialize target state vector and its covariance matrix with prior (approximate) knowledge and hence its performance is expected to be better than that of EKF.

2. Mathematical Modelling of Pseudo Linear Estimator

Let the target state vector be

\[ X_{t} (k) = \begin{bmatrix} x_{t}(k) \\ y_{t}(k) \end{bmatrix}, \]

The target state dynamic equation is given by

\[ x_{t}(k + 1) = \Phi(k + 1|x_{t})x_{t}(k) \]

where

\[ B_{m}(k) = B(k) + G(k) \]
Application of Pseudo Linear Estimator for Target Tracking

\[ z'(k) = H'(k)X_i(k) + \gamma'(k) \]

\[ z'(k) = x_0(k) \cos B_m(k) - y_0(k) \sin B_m(k) \]

\[ H'(k) = \begin{bmatrix} 0 & 0 & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \]

\[ z'(k) = H'(k) \varphi(k,0) X_i(0,k) + \gamma'(k) \]

\[ \phi(k,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 + ts_2 + \ldots + ts_k & 0 & 1 & 0 \\ ts_1 + ts_2 + \ldots + ts_k & 0 & 1 & 0 \end{bmatrix} \]

\[ R^2(k) = (x_i(k) - x_0(k))^2 + (y_i(k) - y_0(k))^2 \]

\[ R_m(k) = R(k) + \zeta(k) \]

\[ \begin{align*}
R(k) &= (x_i(k) - x_0(k)) \left( \frac{x_i(k) - x_0(k)}{R(k)} \right) + (y_i(k) - y_0(k)) \left( \frac{y_i(k) - y_0(k)}{R(k)} \right) \\
&= (x_i(k) - x_0(k)) \sin B(k) + (y_i(k) - y_0(k)) \cos B(k)
\end{align*} \]

\[ z''(k) = H''(k)X_i(k) + \zeta'(k) \]

\[ z''(k) = x_0(k) \sin B_m(k) + y_0(k) \cos B_m(k) + R_m(k) \]

\[ H''(k) = \begin{bmatrix} 0 & 0 & \sin B_m(k) & \cos B_m(k) \end{bmatrix} \]

\[ \zeta'(k) = \zeta(k) - \gamma(k) [\cos B_m(k)(x_i(k) - x_0(k)) - \sin B_m(k)(y_i(k) - y_0(k))] \]

\[ z''(k) = H''(k) \varphi(k,0) X_i(0,k) + \zeta'(k) \]

\[ Z(k) = [z'(1)z'(2)z'(2)z'(3)z'(3)\ldots z'(k)z''(k)] \]

\[ \hat{X}_i(0,k) = \left[ A^T(k,0)A(k,0) \right]^{-1} A^T(k,0)Z(k) \]

\[ A(k,0) = \begin{bmatrix} H'(1) \varphi(1,0) & H'(1) \varphi(1,0) & H'(2) \varphi(2,0) & H'(2) \varphi(2,0) \ldots \end{bmatrix} \]

\[ A'(k,0)A(k,0) = \begin{bmatrix} \ldots & \sum_{i=1}^{ts_k} \cos B_m(i) & \sum_{i=1}^{ts_k} \sin B_m(i) & \ldots \\ \sum_{i=1}^{ts_k} \sin B_m(i) & \sum_{i=1}^{ts_k} \cos B_m(i) & -\sin B_m(i) & \ldots \\ \sum_{i=1}^{ts_k} -\sin B_m(i) & \sum_{i=1}^{ts_k} \cos B_m(i) & \sin B_m(i) & \ldots \\ \sum_{i=1}^{ts_k} \cos B_m(i) & \sum_{i=1}^{ts_k} \cos B_m(i) & -\sin B_m(i) & \ldots \\ \sum_{i=1}^{ts_k} -\sin B_m(i) & \sum_{i=1}^{ts_k} \cos B_m(i) & \sin B_m(i) & \ldots \\ \sum_{i=1}^{ts_k} \cos B_m(i) & \sum_{i=1}^{ts_k} \sin B_m(i) & \ldots & \sum_{i=1}^{ts_k} \cos B_m(i) & \ldots \\ \sum_{i=1}^{ts_k} -\sin B_m(i) & \sum_{i=1}^{ts_k} \sin B_m(i) & \ldots & \sum_{i=1}^{ts_k} \cos B_m(i) & \ldots \end{bmatrix} \]

\[ p = (ts_1)^2 + (ts_1 + ts_2)^2 + (ts_1 + ts_2 + ts_3)^2 + \ldots = \sum_{i=1}^{k} (ts_i)^2 \]

\[ PSI = A^T(k,0)A(k,0) \]

\[ q = (ts_1) + (ts_1 + ts_2) + (ts_1 + ts_2 + ts_3) + \ldots = \sum_{i=1}^{k} (ts_i) \]

\[ PSI = \begin{bmatrix} p & 0 & q & 0 \\ 0 & p & 0 & q \\ q & 0 & k & 0 \\ 0 & q & 0 & k \end{bmatrix} \]

\[ A'(k,0)Z(k) = \begin{bmatrix} \sum_{i=1}^{k} \cos B_m(i)z'(i) + \sum_{i=1}^{k} \sum_{i=1}^{ts_i} \sin B_m(i)z'(i) \\ \sum_{i=1}^{k} -\sin B_m(i)z'(i) + \sum_{i=1}^{k} \sum_{i=1}^{ts_i} \cos B_m(i)z'(i) \\ \sum_{i=1}^{k} \cos B_m(i)z'(i) + \sum_{i=1}^{k} \sum_{i=1}^{ts_i} \sin B_m(i)z'(i) \\ \sum_{i=1}^{k} -\sin B_m(i)z'(i) + \sum_{i=1}^{k} \sum_{i=1}^{ts_i} \cos B_m(i)z'(i) \end{bmatrix} \]

\[ G = A^T(k,0)Z(k) \]
\[
\hat{X}_t(0,k) = \left[A^T(k,0) A(k,0)\right]^{-1} A^T(k,0) Z(k)
\]
\[
\hat{X}_t(0,k) = \left[PSI\right]^t[G]
\]

3. Sequential Mode

Let \( T \) represent the total time elapsed from obtaining first measurement from MAV up to the availability of \( k \)th measurement and is given by
\[
T = ts_1 + ts_2 + ts_3 + \ldots + ts_k
\]

\[
SUMS[1]_k = \cos B_m(1) z'(1)
\]
\[
SUMS[2]_k = -\sin B_m(1) z'(1)
\]
\[
SUMS[3]_k = T \cos B_m(1) z'(1)
\]
\[
SUMS[4]_k = -T \sin B_m(1) z'(1)
\]
\[
SUMS[5]_k = \cos B_m(1) z''(1)
\]
\[
SUMS[6]_k = \sin B_m(1) z''(1)
\]
\[
SUMS[7]_k = T \cos B_m(1) z''(1)
\]
\[
SUMS[8]_k = T \sin B_m(1) z''(1)
\]

\[
SUMS[1]_k = \cos B_m(k) z'(k) + SUMS[1]_{k-1}
\]
\[
SUMS[2]_k = -\sin B_m(k) z'(k) + SUMS[2]_{k-1}
\]
\[
SUMS[3]_k = T \cos B_m(k) z'(k) + SUMS[3]_{k-1}
\]
\[
SUMS[4]_k = -T \sin B_m(k) z'(k) + SUMS[4]_{k-1}
\]
\[
SUMS[5]_k = \cos B_m(k) z''(k) + SUMS[5]_{k-1}
\]
\[
SUMS[6]_k = \sin B_m(k) z''(k) + SUMS[6]_{k-1}
\]
\[
SUMS[7]_k = T \cos B_m(k) z''(k) + SUMS[7]_{k-1}
\]
\[
SUMS[8]_k = T \sin B_m(k) z''(k) + SUMS[8]_{k-1}
\]

\[
G = \left[SUMS[3]_k + SUMS[8]_k\right]^t
\]
\[
SUMS[4]_k + SUMS[7]_k
\]
\[
SUMS[1]_k + SUMS[6]_k
\]
\[
SUMS[2]_k + SUMS[5]_k
\]

Once \( X_t(0,k) \) is calculated, the state vector corresponding to the current measurement can be found using transient matrix. The range, course, bearing and speed of the target are calculated using the current state vector.

4. Tracking of a Manoeuvring Target

So far, it is assumed that target is not manoeuvring. In this section, this assumption is relaxed. Let us assume that the target moves at constant velocity with occasional manoeuvre as shown in Table 1. This problem can be easily solved by using fifteen to twenty measurements with sliding window technique.

5. Simulation and Results

The algorithm is realized using Matlab on a PC platform. The positions of target and MAV are updated every second. The scenarios considered are shown in Table 1. In scenarios 1 and 2, target moves at constant velocity for a period of 240 seconds and then it manoeuvres in course at the rate of 3°/s. Let the noise in the bearing and range measurements are white Gaussian and their values are shown in Table 1. The simulated and estimated target paths for scenario 1 & 2 are shown in Figure 1(a) & Figure 1(b) respectively. The results of scenarios in Monte-Carlo simulation with 100 runs are shown in Figures 2 & 3.

<p>| Table 1. Scenarios chosen for evaluation of algorithm |</p>
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Range (m)</td>
<td>5000</td>
<td>4500</td>
</tr>
<tr>
<td>Initial Bearing (deg)</td>
<td>55</td>
<td>240</td>
</tr>
<tr>
<td>Target Speed (m/s)</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>Target Course (deg)</td>
<td>250 to 180 at 240 seconds</td>
<td>150 to 90 at 240 seconds</td>
</tr>
<tr>
<td>MAV Speed (m/s)</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>MAV Course (deg)</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>Error in Range (rms)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Error in Bearing (rms)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In case of PLE, for scenario 1, it was observed that the required accuracies are obtained in estimated target course and speed after 5th and 3rd measurements respectively. Once the target manoeuvres, the estimated solution is disturbed. The target manoeuvre is completed around 270 seconds. Upon the target manoeuvre, the error in estimated course is increased and reduced to within limits from 35th measurement onwards. There is
very small disturbance in estimated target speed during
target manoeuvre. The error in estimated range is accept-
able from the beginning of the trial. In case of scenario
2, the required accuracies in estimated target course and speed
are obtained from 3rd measurement onwards. Upon
the target manoeuvre, the error in estimated course is
increased and reduced to within limits from 27th measure-
ment onwards.

In case of EKF, for scenario 1, it was observed that
the required accuracies are obtained in estimated tar-
get course and speed after 6th and 3rd measurements
respectively. Once the target manoeuvres, the estimated
solution is disturbed. The target manoeuvre is completed
around 270 seconds. Upon the target manoeuvre, the
error in estimated course is increased and reduced to
within limits from 37th measurement onwards. There is
very small disturbance in estimated target speed during
target manoeuvre. The error in estimated range is accept-
able from the beginning of the trial. In case of scenario
2, the required accuracies in estimated course and speed
are obtained from 7th measurement onwards. Upon
the target manoeuvre, the error in estimated course is
increased and reduced to within limits from 33rd measure-
ment onwards. The required accuracies are obtained
faster in PLE than in EKF.

It is assumed that all measurements are correct. Due
to the clutter, MAV measurements may contain a num-
ber of outliers which can completely distort the solution
when using the proposed method. This problem can be
resolved if the measurements are available along with its
variance to PLE. After obtaining first measurement, the
Recursive Sums, SUMS [1] to SUMS [8] are given by

\[
\begin{align*}
\text{SUMS}[1]_k &= \cos B_m(k)z'(k)/\sigma^2_B(k) + \text{SUMS}[1]_{k-1} \\
\text{SUMS}[2]_k &= -\sin B_m(k)z'(k)/\sigma^2_B(k) + \text{SUMS}[2]_{k-1} \\
\text{SUMS}[3]_k &= T \cos B_m(k)z'(k)/\sigma^2_B(k) + \text{SUMS}[3]_{k-1} \\
\text{SUMS}[4]_k &= -T \sin B_m(k)z'(k)/\sigma^2_B(k) + \text{SUMS}[4]_{k-1} \\
\text{SUMS}[5]_k &= \cos B_m(k)z''(k)/\sigma^2_B(k) + \text{SUMS}[5]_{k-1} \\
\text{SUMS}[6]_k &=\sin B_m(k)z''(k)/\sigma^2_B(k) + \text{SUMS}[6]_{k-1} \\
\text{SUMS}[7]_k &= T \cos B_m(k)z''(k)/\sigma^2_B(k) + \text{SUMS}[7]_{k-1} \\
\text{SUMS}[8]_k &= T \sin B_m(k)z''(k)/\sigma^2_B(k) + \text{SUMS}[8]_{k-1}
\end{align*}
\]

where,

\[
\begin{align*}
z'(k) &= x_y(k)\cos B_m(k)/\sigma^2_B(k) - y_y(k)\sin B_m(k)/\sigma^2_B(k) \\
z''(k) &= x_y(k)\cos B_m(k)/\sigma^2_B(k) + y_y(k)\sin B_m(k)/\sigma^2_B(k) + R_m(k)/\sigma^2_B(k)
\end{align*}
\]

**Figure 1** (a). Target paths for scenario 1. (b). Target paths for scenario 2.
6. Conclusion

In this paper, PLE algorithm is explained in this paper with sequential processing. The algorithm is extended for tracking manoeuvring targets also. The effect of the bias is negligible, as weapons are highly sophisticated with homing capabilities. It is inferred that the algorithm is apt for undersea applications.

7. References


