Multiuser Detection over Generalized-K Fading Channels with Laplace Noise

Srinivasa R. Vempati1*, Habibulla Khan2 and Anil K. Tipparti3

1Department of ECE, Anurag Engineering College, Kodad - 508206, Telangana, India; s.r.vempati@ieee.org
2Department of ECE, KL University, Vaddeswaram - 522502, Andhra Pradesh, India; habibulla@rediffmail.com
3Department of ECE, SVS Group of Institutions, Hanamkonda - 506001, Telangana, India; tvakumar2000@yahoo.co.in

Abstract

Background: Combined effect of fading and shadowing degrades the performance of multiple access wireless communication systems. The presence of impulsive type non-Gaussian noise along with inter symbol interference and multiple access interference further worsens the system performance. Methods: This paper presents a multiuser detection technique for direct sequence-code division multiple accesses systems over generalized-K fading channels in presence of impulsive noise modeled by Laplace distribution. Maximal ratio combining receive diversity technique is also incorporated to mitigate the effects of simultaneous presence of fading and shadowing. An M-decorrelator is proposed to robustly detect the binary phase shift keyed symbols. Performance of proposed M-decorrelator is evaluated by computing the average probability of error. Findings: The proposed M-decorrelator performs better in the simultaneous presence of fading, shadowing and impulsive noise when compared to least squares, Huber and Hampel M-estimator based detectors.

Keywords: Impulsive Noise, Laplace Noise, M-Estimator, Multiuser Detection, Probability of Error

1. Introduction

The Multiuser Detection (MUD) technique for Direct-Sequence (DS) Code-Division Multiple Access (CDMA) systems in non-Gaussian noise has been developed based on the Huber, Hampel and a new M-estimator based detectors1-3 by modeling the impulsive noise with two-term Gaussian mixture model1. The performance of M-decorrelator in simultaneous presence of fading and shadowing with impulsive noise is presented in4,5 by modeling the impulsive noise with two-term Gaussian mixture model. The Laplace noise can also be commonly used as a model for impulsive noise which prevalent in both indoor and outdoor radio and underwater transmission scenarios6,7. Hence, this paper considers mitigation of Multiple Access Interference (MAI), fading and shadowing effects in DS-CDMA channels with non-Gaussian noise modeled by Laplace noise.

The next part of this paper is arranged as follows. The DS-CDMA system in fading channel with Laplace noise is presented in Section 2. In Section III, the GK fading channel model is presented. Section IV presents the penalty function and influence function of the proposed M-estimator, and the average probability of error of proposed M-decorrelator over GK fading channels. The simulation results and the performance analysis of proposed M-decorrelator is discussed in Section 5. Section 6 presents the concluding remarks and future directions.

2. System Model

This section presents the DS-CDMA system, with L-users, operating with coherent Binary Phase Shift Keying (BPSK) modulation scheme. If all the L-users are active, the received baseband, continuous-time signal is a superposition of all L signals given by

\[ r(t) = X(t) + z(t) \]  

where \( X(t) \) is the useful signal comprised of the data signals of \( L \) active users and the \( z(t) \) is the Additive White Noise.
Gaussian Noise (AWGN). The signal \( X(t) \) can be written as

\[
X(t) = \sum_{i=1}^{M} w_i(t) \sum_{j=0}^{N-1} b_j(i) s_j(t - iT_i - \tau_i)
\]

where \( M \) is the number of data symbols per user in the data frame of interest, \( T_i \) is the symbol interval, \( b_j(i) \) is the \( j \)th bit of the \( i \)th user, \( w_i(t) \) is the received signal amplitude, \( s_j(t) \) is the normalized signaling waveform of the \( i \)th user and \( \tau_i \) is the delay of the \( i \)th user. It is assumed that the signaling interval of each user is \( T \) seconds, and the input alphabet is antipodal binary: \([-1, +1]\). Further, Direct-Sequence Spread Spectrum (DSSS) Multiple Access (MA) signatures can be written as

\[
s_j(t) = \sum_{n=1}^{N} a_n^j u_T(t - (n-1) T_c), \quad t \in [0, T_s]
\]

where \( \{a_n^j\}_{n=1}^{N} \) is a signature sequence of +1s and -1s assigned to the \( i \)th user, and \( u_T(t) \) is a unit-amplitude pulse of duration \( T_c \) with \( T_s = N T_c \). At the receiver, the received signal \( r(i) \) is first filtered by a chip-matched filter and then sampled at the chip rate, \( 1/T_c \). The resulting discrete-time signal corresponding to the \( n \)th chip of the \( i \)th symbol is given by

\[
r_n(i) = \sqrt{\frac{2}{T_c}} i^{n T_c + (n+1) T_c} r(t) e^{-j \alpha t} dt, \quad n = 1 \ldots N.
\]

For synchronous case (i.e., \( \tau_1 = \tau_2 = \ldots = \tau_i = 0 \) ) and with slow fading assumption, (4) can be expressed in matrix notation as

\[
\mathbf{r}(i) = A \mathbf{\theta}(i) + \mathbf{z}(i)
\]

where

\[
\mathbf{z}(i) = [z_1(i), \ldots, z_N(i)]^T
\]

\[
\mathbf{\theta}(i) = \frac{1}{\sqrt{N}} [b_1(i) g_1(i), \ldots, b_L(i) g_L(i)]^T
\]

with \( z_n(i) \) is an independent and identically distributed (i.i.d.) complex random variables sequence having probability density function (PDF) of the form

\[
f(x; \varphi, \lambda) = \frac{1}{2c} \exp \left( -\frac{|x| \lambda}{\varphi} \right)
\]

where \( \lambda \) is the location parameter and \( c \) is the shape parameter and is related to the variance of the variable as \( \varphi^2 = \frac{\lambda^2}{2} \).

### 3. Generalized-K Fading Channel

For the shadowed fading channels, \( \alpha_i(i) \) are i.i.d. random variables with GK distribution having the PDF

\[
P_{\alpha}(\alpha_i) = \frac{2}{\Gamma(m)\Gamma(\mu)} \left( \frac{m\mu}{\Omega_o} \right)^{m+\mu} \alpha_i^{m+\mu} K_{m+\mu} \left( \frac{m\mu}{\Omega_o - \alpha_i} \right)
\]

where, \( m \) is the fading parameter, \( \mu \) indicates the shadowing levels, \( \Omega_o \) is the average SNR, \( K_{m+\mu}(\cdot) \) is the modified Bessel function and \( \Gamma(\cdot) \) is the Gamma function. It is assumed that the effect of fading is mitigated by using a \( D \)-branch Maximal Ratio Combining (MRC), the output of the combiner can be written as

\[
R = \sum_{j=1}^{D} \alpha_j
\]

where \( \alpha \) are random variables each having a PDF of the form (10). The PDF of random variable \( R \) is given by

\[
P_R(r) = \frac{2}{\Gamma(m)\Gamma(\mu)} \left( \frac{m\mu}{\Omega_o} \right)^{m+\mu} \times
\]

\[
\times K_{m+\mu} \left( \frac{m\mu r}{\Omega_o - m\mu} \right)
\]

where

\[
m m = D m + (D - 1) \left[ -0.127 - 0.95 m - 0.0058 \mu \right]
\]

and

\[
\mu_m = D \mu.
\]

### 4. M-Decorrelator

This section of the paper presents the proposed \( M \)-estimator and the average probability of error of the \( M \)-decorrelator.

#### 4.1 M-Estimator

\( M \)-estimator is, a generalization of Maximum Likelihood (ML) estimator, to estimate the unknown parameters by minimizing a sum of function \( \rho(\cdot) \) of the residuals.
\[
\hat{\theta} = \arg \min_{\theta \in \Omega} \sum_{j=1}^{N} \rho \left( r_{j} - \sum_{l=1}^{L} s_{l}^{j} \theta_{l} \right) \tag{15}
\]

where \( \rho(\cdot) \) is the penalty function. The influence functions (IF), \( \psi(\cdot) = \rho'(\cdot) \), of proposed estimator along with Least-Squares (LS), Huber (HU) and Hampel (HA) estimators is shown in Figure 1. The penalty function and influence function of the proposed M-estimator can be expressed as\(^3\)

\[
\rho_{\text{PRO}}(x) = \begin{cases} 
\frac{x^2}{2} & \text{for } |x| \leq a \\
\frac{a^2}{2} - a|x| & \text{for } a < |x| \leq b \\
\frac{ab}{2} \exp \left( \frac{1 - x^2}{b^2} \right) + d & \text{for } |x| > b
\end{cases}
\tag{16}
\]

\[
\psi_{\text{PRO}}(x) = \begin{cases} 
x & \text{for } |x| \leq a \\
\text{sgn}(x) & \text{for } a < |x| \leq b \\
\frac{a}{b} x \exp \left( \frac{1 - x^2}{b^2} \right) & \text{for } |x| > b
\end{cases}
\tag{17}
\]

where the constants \( a \) and \( b \) depends on the robustness measures of the estimator.

\[
P_e = \Pr \left( \hat{\theta} < 0 | \theta > 0 \right) = Q \left( \frac{A}{\sqrt{\mathbf{R}^{-1} \mathbf{j}}} \right) \tag{18}
\]

where \( Q(x) \) is the Gaussian Q-function\(^8\),

\[
\nu^2 = \frac{\int \psi^2(u) f(u) du}{\left[ \int \psi(u) f(u) du \right]^2}
\tag{19}
\]

and \( \mathbf{R} = \mathbf{S}^T \mathbf{S} \) with \( \mathbf{S} \) is an \( N \times L \) matrix of columns \( \mathbf{a} / \).

The asymptotic variance is obtained by substituting (9) and (17) into (19).

For the proposed minmax decorrelating detector, the numerator and denominator integrals of (19) are computed numerically.

### 4.3 Average probability of error

The average probability of error is computed by using the expression derived in\(^3\), which is represented by (20) for completeness

\[
\bar{P}_e = G \cdot \frac{1}{2} \xi^{-0.5} \beta^{-1} \Gamma \left( \frac{1 + d + l}{2} \right) \Gamma \left( \frac{1 - d + l}{2} \right)
\cdot \exp \left[ \frac{\beta^3}{8 \xi} \right] W_{-0.5, d} \left( \frac{\beta^3}{4 \xi} \right)
\tag{20}
\]

where \( d = m - \mu, \quad l = \frac{m + \mu}{2} - 1, \quad \xi = \frac{1}{\nu^2 \left[ \mathbf{R}^{-1} \right]_{11}} \), \( \beta = 2 \frac{12 \xi + \mu}{\Omega_0} \) and \( W_{\lambda, \xi}(\cdot) \) is the Whittaker function\(^9\).

### 5. Simulation Results

This section presents the performance of proposed M-decorrelator by computing the average probability of error, (20), for different values of Laplace noise parameters and diversity order.

In Figure 2, the average probability of error versus the Signal-to-Noise Ratio (SNR) corresponding to the user# 1 under perfect power control of a synchronous DS-CDMA system with six users \( (L = 6) \) and a processing gain \( (N = 31) \) is graphed for Laplace noise parameters \( \sigma = 0.95 \) and \( \lambda = 0 \), fading severity parameter \( m = 1.4 \) and shadowing level \( \mu = 2 \). From Figure 2, it is clear that when the noise variance is high \( (\sigma = 0.95) \), the performance of the proposed M-decorrelator is better when compared to the decorrelating detectors with LS, HU and HA estimators under no diversity \( (D = 1) \) condition. Further, the perfor-
performance of the proposed detector is increased with increase in diversity order to 3.

Similarly, in Figure 3, the average probability of error is plotted for \((\sigma = 0.5, \lambda = 0)\). When the noise variance is reduced to 0.5, the performance of the proposed detector is superior compared to other detectors. Further, the proposed detector outperforms with increased SNR.

It is clear from the simulation results that the proposed M-decorrelator performs better when compared to LS detector, minimax detector with HU and HA estimators in the combined presence of fading and shadowing with Laplace noise.

6. Conclusion

A MUD technique for DS-CDMA systems over GK fading channels in the presence of additive Laplace noise is presented. An M-decorrelator is proposed and its performance is analyzed by computing average probability of error over GK fading channels. Simulation results presented show that the proposed M-decorrelator performs better when compared to LS detector and the minimax decorrelator with HU and HA estimators under the combined effect of fading and shadowing in presence of Laplace noise.

7. References
