1. Introduction

The analysis of control methods of the distributed systems, problems of routing in large networks and features of multi-parameter routing in real time all show that one of the most important tasks from the point of view of efficiency and reliability of the management process and functioning of a network as a whole, and while quickly adapting to the network condition changes, is the delivery of the information to the receiver in time. However fast refusal of this or that equipment is identified and however precisely administrative decision was accepted but as the controlling information will not achieve a target in time management efficiency aspires to zero. The solution of the traffic management and routing task will allow substantially provide net configuration handling system effectiveness.

2. Markov’s Analytical Model

The main requirement on delivery of the information when network is functioning in transient mode is the optimization of delivery time. However search of an extremum of the aim function of minimization of delivery time in a network has to be implemented for a preset boundary conditions of reliability of delivery, requirements of a quality of service etc. Thus, the subsystem of routing should be based on usage of algorithms of multi-parameter routing providing minimum delivery time of the information to the receiver while fulfilling the accompanying delivery requirements.

Carrying out comparative analysis of existing algorithms of routing and switching would allow defining the most approaching algorithm of routing for a concrete network, traffic and functioning conditions. The probing of algorithms of routing becomes possible due to usage of a specialized system of the analysis of routing processes. The designed analytic model of processes of multi-parameter routing in networks based the theory of finite Markov chains, queuing theory etc., allows defining probability - time characteristics of a networks. Structure and the methods of model construction provide its practical appliability for auto configuration of routing algorithms and traffic control for specific topological structures, channels characteristics and traffic between routers of a network. The analytic model allows to
Investigation of Multiparameter Routing Processes Based on the Markov’s Analytical Model

Research efficiency of routing processes both in highly reliable networks and networks with refusals of separate communication channels and routers. The simulation analysis time while probing into a network using given analytical model is much less, than that of the other simulation models. The effective method of identification of parameters of model of the routing process has been developed. The method is founded on the numerical solution of a nonlinear system of the stream equations describing mass processes of routing in networks.

2.1 Matrix of Transition Probabilities
Assuming that the time $T$ during which the packet will stay in a network does not depend on the time $T(\kappa)$ it stays in each router visited by packet and these values are independent among themselves, according to the theory of finite Markov processes response time of a packet in a network represents a finite Markov chain. The matrix of transition probabilities $P$ together with a priori allocation of routers describes the Markov process determines a procedure of delivery of a packet to a specific target router.

For a specific network it is possible to construct a matrix of transition probabilities $P$, which describes discrete Markov process with two ergodic states, one of which - target router $l$, and another - loss of search. The remaining routers will derive a set of non-recurrent conditions, the probabilities of junctions in which are represented by matrix $R$. Besides, for determination of streams in edges of a network one dummy finite Markov chain state is added, the zero state, which is necessary for the closed definition of streams in $n$ network. That is the state packets come from. An initial states at the definition of initial allocation of streams is the matrix of intensities $\lambda = \|\lambda_{ij}\|_{n,n}$,

$$\lambda_{ij} = \begin{cases} \lambda_{ij} & \text{if } (i,j) \text{ is in the network} \\ 0 & \text{otherwise} \end{cases}$$

where $n$ - number of routers in a network,

$\lambda_{ij}$ - intensity of the requests stream from the router $i$ in a direction of a router $j$.

The probabilities of junctions in routers from zero state are determined on the basis of a matrix of loads

$$P^i = \begin{bmatrix} E & O \\ R & Q \end{bmatrix}$$

Thus matrix $P$ looks like the following:

$$P^i = \begin{bmatrix} E & O \\ R & Q \end{bmatrix}$$

where $E$ - unit matrix, dimension 2x2,

$O$ - zero matrix, dimension 2xn,

$R$ - the matrix, dimension nx2, maps junctions from non-recurrent states to ergodic ones,

$Q$ - the matrix, dimension ran, reflects behavior of the process up to an output of their set of non-recurrent conditions,

$l$ - an index that meaning that the matrix is constructed for a target router $l$.

2.2 Probability-Time Network Characteristics
When analyzing the functioning of a network as a whole, the originating of the requirements on packets transmission becomes massive, thus it is necessary to take up the collection of finite Markov chains of processes. In this case only one nested finite Markov chain corresponds to each target router. The states of a chain are identified with routers, and all processes, as a rule, are defined on the same states. The complete description of routing processes in a network with $n$ by routers assumes presence of $n$ transition matrixes of type (3). Thus the set of equations describing mass processes of routing in a network is nonlinear.

The numerical decision (5) systems of the nonlinear equations (4) for a preset network, traffic and conditions of functioning allows to implement determination of a probability-time characteristics of a network, carry out a rating of used algorithms of routing, methods of streams control etc.

$$\pi_{jk} = (\lambda_{ij} - C_{jk}) \cdot \mu / \lambda_{ij} \text{ with } p_{jk} \geq 1,$$

$$\pi_{jk} = (1 - p_{jk}) \cdot p_{jk}^{m_{jk}} / (1 - p_{jk}^{m_{jk}+1}) \text{ with } p_{jk} < 1,$$

$$p^{(i)} = \|p^{(i)}_{ik}\|_{n+1,n-1} = \sum_{s=1}^{2^s} \Omega^{(s)}_{ik}$$

where $\lambda_{ij}$ - intensity of stream in an edge $j$,

$1 / \mu$ - average length of packets,

$C_{jk}$ - transmission capacity of an edge $jk$,

$p_{jk}$ - probability of originating of a situation $(X_m)$,
\( \pi_{ik} \) - probability of blocking of the channel \( ik \),
\( P_{jk} \) - channel utilization coefficient.

As you can see from (5) the numeric solution of a system has iterative nature.

\[
\pi_{jk}^{(b)} = (\lambda_{jk}^{(b-1)} - C_{jk}) \cdot \mu / \lambda_{jk}^{(b-1)} \quad \text{with} \quad P_{jk}^{(b-1)} \geq 1,
\]

\[
\pi_{jk}^{(b)} = (1 - p_{jk}) \cdot p_{jk}^{m_{jk}} / (1 - p_{jk}^{m_{jk}+1}) \quad \text{with} \quad P_{jk}^{(b-1)} < 1,
\]

\[
P_{l}^{(b)} = \|p_{l}^{(b)}\|_{\infty}^{n-1,n-1},
\]

\[
\lambda_{jk}^{(b)} = \left( \sum_{i=1}^{n} \lambda_{il}^{(b)} \cdot f_{ij}^{(b)} \cdot q_{ij}^{(b)} / (1 - \sigma_{jk}^{(b)}) + \lambda_{s}^{(b)} \right),
\]

where \( b \)- step number,
\( f_{ij}^{(b)} \) - appropriate row of a fundamental matrix \( F \) on step \( b \),
\( q_{jk}^{(b)} \) - appropriate row of a fundamental matrix \( Q \) on step \( b \),
\( \sigma_{jk}^{(b)} \) - standard deviation of intensity of stream on step \( b \),
\( \lambda_{s}^{(b)} \) - service stream on step \( b \).

3. **Identification Algorithm**

The identification of parameters of model of the routing process close to best values is possible during a repetitive process of search of the solution of a system of nonlinear equations. It becomes possible to find optimal values of routing algorithm configuration parameters for a preset network and traffic after introducing step procedure of corrective action into a repetitive process of search of the solution of a system of the stream equations. The model parameters identification algorithm is represented in Figure 1.

The system of the analysis is developed in the Inprise CBuilder development environment using the object-oriented language C++ and consists of several program modules:

- The module net.cpp - allows to establish and to make modifications to a network configuration (description of a network).
- The module model.cpp - implements simulation analysis of processes of multi-parameter routing and simulation analysis of refusals of communication channels and routers.
- The module process.cpp - represents program implementation of analytic model of processes of routing in a network described in the chapter 2, and also program implementation of researched algorithms of routing (basing on the shortest paths first, and in view of alternative paths, designed in the chapter 3).
- The module appl.cpp - is the base module of a system of the analysis, provides process control of simulation analysis and calls all remaining modules of the program.

\[
Q = (K_{1}T + K_{2}D + K_{3}EL) / R,
\]

where \( T \) - time of delivery of a packet for a current load of a network (seconds),

4. **Integrated Criterion of Optimization**

According to results of probing algorithms of routing the integrated criterion of optimization of the path is created which looks like this:
D - time of delivery of a packet for the not loaded (free) network (seconds),
E - average time packet stays in a router (seconds),
L - number of transit routers,
R - percent of the successfully transferred packets (percent).

And \[ T = \frac{1}{S(1 - B)} \]
for each communication channel,

where S - transmission capacity of the channel (kbps),
B - load of the channel (percent).

Thus, the given integrated criterion allows to take into account not only topology of a network and transmission capacity of communication channels (parameters L and D), but also current real condition of a network (parameter T), and also reliability of delivery of the information (parameter R). A weight coefficient Ki allows to organize a priority of optimization parameters of the path, that allows flexible control of the process of routing. Besides, one-parameter routing execution is also possible using selected criterion.

Similar tasks at different times was considered in the context of the study of distributing function for load balancing in multi-server systems\(^2\), modeling the TCP Protocol\(^3\), lumping of Markov chains\(^4\,5\,6\), in the study of adaptive performance management\(^7\) and for mobile routing modeling\(^8\,9\).

5. References