By Wavelet based Fusion Directional Adaptive Interpolation Super Resolution Image Construction

S. Caroline Kirupa Shiny\textsuperscript{1*} and R. Renita Rexy\textsuperscript{2}

\textsuperscript{1}Department of Electronics and Communication Engineering, Jerusalem College of Engineering, Chennai - 600100, Tamil Nadu, India; carolinesam88@gmail.com
\textsuperscript{2}Department of Electronics and Communication Engineering, Bharath University, Chennai - 600073, Tamil Nadu, India; renitarexy.ece@bharathuniv.ac.in

Abstract

Super resolution is the process of recovering a high resolution image from multiple low resolution images of the same scene. This process consists of three steps: Image registration, Image fusion and Interpolation. Image registration is the first step in super resolution process in which the reference and sensed images are geometrically aligned. This alignment is done based on maximization of an appropriate similarity measure. The similarity measure used in this paper is Pearson correlation coefficient. Methods used for fusion are Discrete Wavelet Transform (DWT) based decomposition at one level and DWT based decomposition at 2 level. Interpolation is the final step in super resolution process. Various interpolation algorithms namely, nearest neighbour, bi-cubic, edge oriented interpolation, angular variation based interpolation and directional adaptive interpolation were applied to the images and are compared quantitatively. The overall performance of super resolution algorithms are compared using sharpness index metric.

Keywords: Image Registration, Pearson Correlation Coefficient, Sharpness Index, Sobel Operator

1. Introduction

In most electronic imaging applications, images with High Resolution (HR) are desired and often required. High Resolution means pixel density within an image is high, and therefore an HR image can offer more details that may be critical in various applications. Super resolution reconstruction is the process of recovering a high resolution image from multiple low resolution images of the same scene. This process consists of three steps: image registration, fusion and interpolation.

Accurate image registration is a crucial step in the super resolution process. Images captured by various sensors of the same scene may be oriented along different directions. Registration is the process of geometrically aligning the input images as in\textsuperscript{13}. Image registration methods are classified into two main approaches: Intensity based methods and feature based methods as in\textsuperscript{1}.

Existing super-resolution fluorescence microscopes compromise acquisition speed to provide subdiffractional sample information. We report an analog implementation of structured illumination microscopy that enables Three-Dimensional (3D) super-resolution imaging with a lateral resolution of 145 nm and an axial resolution of 350 nm at acquisition speeds up to 100 Hz.

Image fusion is the process of combining two or more images into a single image retaining important features from each. Image fusion is performed to integrate the information from the low resolution images, prior to the interpolation step as in\textsuperscript{1}. The objective is to maintain the
important information from the low-resolution images, which would not occur if the interpolation was carried out on the registered image directly.

The final step in the super-resolution process is the interpolation step. It is a technique which estimates a new pixel from the surrounding pixels available in the low resolution image. Nearest neighbor and bi-cubic interpolation are the two conventional methods used for interpolation.

Section 2 describes the image registration algorithm, the image fusion algorithms are discussed in section 3 and the interpolation algorithms are presented in section 4. Super resolution algorithms are compared in section 5 and the conclusions are presented in section 6.

2. Image Registration

Image registration is done as in1. It is done by considering two low resolution images namely, A and B. These two images must be geometrically aligned prior to fusion step. Thus our aim is to calculate the coordinates of image B that corresponds to the origin of reference image A. Let the dimension of the low resolution images be a"b. Let w (i, j) represent a window with size n×n on image A. Let g (i, j) represent the search area in image B. For both w (i, j) and g (i, j), the upper left corner is located at the origin of a global coordinate system4,5.

Then it is clear that all the possible positions of the window w (i, j) in the search area g (i, j), takes values in the following set,

\[ S = [0, N - 1] \times [0, M - 1] \] (1)

where \( N = a-n+1 \) and \( M = b-n+1 \)

Let w1 (i, j) be a window in the search area g(i, j) that has the same size as that of w. The window w1(i, j) is moved over the search area of image B along the x and y directions as an overlapping window. A condition is searched such that w1 (i, j) matches with w(i, j). This matching is done based on maximization of an appropriate similarity measure. Images are registered by finding the coordinate position at which the value of the similarity measure is maximum. The size of the window used is 64×64. The similarity measure used is Pearson correlation coefficient.

Pearson coefficient is calculated using (2).

\[ r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \] (2)

where \( x_i \) and \( y_i \) represents the gray scale values in matrix A and B respectively, \( \bar{x} \) and \( \bar{y} \) represents mean values of matrices A and B respectively and n represents the total number of values in matrix A or matrix B.

3. Image Fusion

Two techniques for performing image fusion namely, fusion using wavelet transforms at one level and two levels6,7 are compared in the following sections.

3.1 Fusion using One Level Wavelet Transform

Step 1. Apply DWT decomposition (1 level) to input images, \( f_{11}^{(m, n)} \) and \( f_{12}^{(m, n)} \)

Step 2. For each pixel of \( LL_{11}^{(m, n)} \) (i.e., approximation or low frequency sub band), identify 3×3 windows \( CL_{11}^{(m, n)}, CH_{11}^{(m, n)}, \) and \( HH_{11}^{(m, n)} \), centered at the corresponding pixels in \( LH_{11}^{(m, n)}, HL_{11}^{(m, n)} \) and \( HH_{11}^{(m, n)} \), sub-bands (i.e., detail or high-frequency sub bands), where \( k = 1, 2 \) and represents the images involved in fusion.

1. Find:

\( CA_{11}^{(m, n)} = abs(CL_{11}^{(m, n)}) + abs(CH_{11}^{(m, n)}) + abs(CH_{11}^{(m, n)}) \).

2. Find the mean \( MA_{11}^{(m, n)} \) and standard deviation \( SA_{11}^{(m, n)} \) of \( CA_{11}^{(m, n)} \) of as activity levels.

3. Fuse wavelet coefficients of \( LL_{11}^{(m, n)} \) using the Fusion rule as follows.

\[ F^{(m, n)} = \begin{cases} L_{11}^{(m, n)}; & \text{if } MA_{11}^{(m, n)} > MA_{11}^{(m, n)} \\ \left( L_{11}^{(m, n)} + L_{11}^{(m, n)} \right)/2; & \text{otherwise} \end{cases} \] (3)

Step 3. For each pixel of \( LH_{11}^{(m, n)} \), (i.e., high frequency sub band), find the standard deviation \( SD_{11}^{(m, n)} \) over 3×3 windows, where \( k = 1, 2 \) represent the images involved in fusion. Fuse the wavelet coefficients of \( LH_{11}^{(m, n)} \) sub-band using the fusion rule as follows.
3.2 Fusion using Two Level Wavelet Transform

The following steps are involved in the fusion of images using wavelet transform at two level.

Step 1. Apply DWT decomposition (1 level) to input images, \( f_1(m, n) \) and \( f_2(m, n) \).

Step 2. For each pixel of \( LL_2(m, n) \), identify 3×3 windows centered at the corresponding pixels in \( LH_2(m, n), \ HL_2(m, n) \) and \( HH_2(m, n) \), sub-bands, where \( k = 1, 2 \) and represents the images involved in fusion.

1. Find \( CA_k(m, n) = abs(CLH_k(m, n)) + abs(CHL_k(m, n)) + abs(CHH_k(m, n)) \).
2. Find the mean \( MA_k(m, n) \) and standard deviation \( SA_k(m, n) \) of \( CA_k(m, n) \) as activity levels.
3. Fuse wavelet coefficients of \( LL_2 \) using the Fusion rule as follows

\[
F(m, n) = \begin{cases} 
LL_2^1(m, n); & \text{if } \frac{MA_1^1(m, n)}{SA_1^1(m, n)} > \frac{MA_2^1(m, n)}{SA_2^1(m, n)} \\
LL_2^2(m, n); & \text{if } \frac{MA_1^2(m, n)}{SA_1^2(m, n)} < \frac{MA_2^2(m, n)}{SA_2^2(m, n)} \\
\frac{LL_2^1(m, n) + LL_2^2(m, n)}{2}; & \text{otherwise}
\end{cases}
\]

Step 3. For each pixel of \( LH_k(m, n) \), (i.e., high frequency sub band), find the standard deviation \( SD_k \), over 3×3 windows, where \( k = 1, 2 \) represent the images involved in fusion and \( m = 1, 2 \) represents the levels of DWT decomposition. Fuse the wavelet coefficients of \( LH_k(m, n) \) sub-band using the fusion rule as follows

\[
F(m, n) = \begin{cases} 
LH_k^1(m, n); & \text{if } \frac{SD_1^1(m, n)}{SD_1^2(m, n)} > \frac{SD_1^2(m, n)}{SD_2^2(m, n)} \\
LH_k^2(m, n); & \text{if } \frac{SD_1^1(m, n)}{SD_1^2(m, n)} < \frac{SD_1^2(m, n)}{SD_2^2(m, n)} \\
\frac{LH_k^1(m, n) + LH_k^2(m, n)}{2}; & \text{otherwise}
\end{cases}
\]

Step 4. Repeat step 3 for other high frequency sub bands.

Step 5. Apply Inverse DWT to get the fused image.

3.3 Comparison of Image Fusion Algorithms

Mutual Information\(^{11}\) (MI) is used for measuring and comparing the fusion performance. Several test images were fused using the two fusion algorithms and they were compared quantitatively.\(^{15,16}\)

The higher the value of this parameter, the greater is the efficiency of the fusion algorithm being used.

Mutual information calculated for all the two fusion algorithms are shown in Table 1. From this table it is clear that for several test images the mutual information using one level DWT is the highest.

<table>
<thead>
<tr>
<th>Test Images</th>
<th>1 level DWT</th>
<th>2 level DWT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>5.889</td>
<td>5.775</td>
</tr>
<tr>
<td>Lena</td>
<td>4.016</td>
<td>3.946</td>
</tr>
<tr>
<td>Caps</td>
<td>6.283</td>
<td>6.201</td>
</tr>
<tr>
<td>Bike</td>
<td>5.592</td>
<td>4.387</td>
</tr>
</tbody>
</table>

4. Directional Adaptive Interpolation

Image interpolation based on cubic convolution was given by D. Zhou et al. in\(^{13}\) in which the edge detection is done based on a preset threshold value. In the proposed method edge detection is done using Sobel operator. The Sobel operator performs a 2-D spatial gradient measurement on an image and so emphasizes regions of high spatial frequency that correspond to edges. Initially zeros are inserted after every row and column and Sobel operator is applied. To the output of Sobel operator a window of size 5×5 is moved and mean value of the window is calculated. If the mean value is greater than zero, the pixel corresponding to the centre position of the window is classified as edge pixel and if mean is equal to zero, it is classified as a smooth pixel\(^{17}\). Let \( P \) be the image to be interpolated. Compute matrix \( Q \) using matrix \( P \), such that,

\[
Q(2i, 2j) = P(i, j)
\]

In this method, interpolation is performed in two steps. In the first step we compute the ‘b’ pixels, and then the ‘a’ pixels in the second step\(^{10}\). The ‘b’ pixels correspond to the pixels at \( Q(i, j) \) where \((i+j)\) is even and ‘a’ pixels correspond to the pixels at \( Q(i, j) \) where \((i+j)\) is odd.
is odd. The ‘a’ and ‘b’ pixels are shown in Figure 1. The ‘b’ pixels have their 4 interpolation neighbors already known. The ‘a’ pixels will have 4 neighbors only after ‘b’ pixels have been determined.

### 4.1 Interpolation of Diagonal and Axial Pixels

For each ‘b’ pixel to be interpolated, its neighboring pixels covered by a 7×7 window are projected along two different angular orientations i.e., along 45° and 135°. For every orientation, gradients are calculated using (8) and (9).

\[
G_i = \sum_{m=-3}^{3} \sum_{n=-3}^{3} [I(i+m, j-n) - I(i+m-2, j-n+2)]
\]  
\(45^\circ \text{ diagonal} \)  
(8)

\[
G_j = \sum_{m=-3}^{3} \sum_{n=-3}^{3} [I(i+m, j+n) - I(i+m-2, j-n-2)]
\]  
\(135^\circ \text{ diagonal} \)  
(9)

\[
b = \begin{cases} 
\frac{-x_5 + 9x_{19} + 9x_{33} - x_{43}}{16} & ; \text{45° edge pixel} \\
\frac{-x_5 + 9x_{17} + 9x_{33} - x_{49}}{16} & ; \text{135° edge pixel}
\end{cases}
\]  
(11)

Figure 1. 7X7 matrix representing diagonal pixels ‘b’ and axial pixels ‘a’ to be interpolated.

After the estimation of gradients the direction of the edge is calculated for edge pixels using (10) and the value of ‘b’ pixel is calculated using (11).

\[
\text{if } G_i / G_j > G_j / G_i; \text{pixel is on 135° edge} \\
\text{if } G_i / G_j > G_i / G_j; \text{pixel is on 45° edge}
\]  
(10)

\[
b = \begin{cases} 
\frac{-x_5 + 9x_{19} + 9x_{33} - x_{43}}{16} & ; \text{45° edge pixel} \\
\frac{-x_5 + 9x_{17} + 9x_{33} - x_{49}}{16} & ; \text{135° edge pixel}
\end{cases}
\]  
(11)

For the interpolation of smooth pixels, the following equation is used.

\[
b = \frac{(w_1p_1 + w_2p_2)}{(w_1 + w_2)}
\]

Where,

\[
p_1 = \frac{-x_5 + 9x_{19} + 9x_{33} - x_{43}}{16}
\]

\[
p_2 = \frac{-x_5 + 9x_{17} + 9x_{33} - x_{49}}{16}
\]

\[
w_1 = \frac{1}{1 + G_i^4}
\]

\[
w_2 = \frac{1}{1 + G_j^4}
\]

After the interpolation of ‘b’ pixels the ‘a’ pixels are interpolated using the same procedure with its axial neighbors.

### 4.2 Comparison of Interpolation Algorithms

Standard test images of size 256×256 were interpolated using various interpolation algorithms and they were compared visually and quantitatively. For quantitative analysis, correlation coefficient was calculated for the interpolated image and standard test images of size 512×512 using (2). Correlation coefficient value depicts the similarity between two input images. It has a maximum value of 1, when both the input images are exactly similar. The values of correlation coefficient for various interpolation algorithms are shown in Table 2. This table depicts that correlation coefficient value computed for various images using directional adaptive interpolation algorithm is highest. Hence directional adaptive interpolation algorithm out performs other algorithms.

\[
\text{Correlation coefficient } = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}
\]

Table 2. Comparison of correlation coefficient for various interpolation algorithms

<table>
<thead>
<tr>
<th>Test image</th>
<th>Bi-cubic</th>
<th>Edge Oriented</th>
<th>Adaptive directional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>0.9714</td>
<td>0.9838</td>
<td>0.9910</td>
</tr>
<tr>
<td>Lena</td>
<td>0.9709</td>
<td>0.9813</td>
<td>0.98946</td>
</tr>
<tr>
<td>Caps</td>
<td>0.9736</td>
<td>0.9897</td>
<td>0.9969</td>
</tr>
<tr>
<td>Bike</td>
<td>0.9506</td>
<td>0.9626</td>
<td>0.9758</td>
</tr>
</tbody>
</table>

### 5. Comparison of Super Resolution Algorithms

To assess the quality of the final super resolution image
Figure 2. Interpolation results for house image obtained with. (a) Bicubic interpolation. (b) Edge directed. (c) Adaptive directional interpolation.

Figure 3. Interpolation results for light house image obtained with. (a) Bicubic interpolation. (b) Edge directed. (c) Adaptive directional interpolation.

Figure 4. Comparison with conventional interpolation algorithms (Original image: 100x150 pixels, output image: 400x600 pixels). (a) Bilinear interpolation. (b) Bicubic interpolation. (c) Dir8 algorithm.
Table 3. Quantitative comparison of super resolution process

<table>
<thead>
<tr>
<th>Methods</th>
<th>Flower image</th>
<th>Leaf image</th>
<th>Paper image</th>
<th>Plane image</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Level DWT fusion with Edge directed interpolation</td>
<td>1104884</td>
<td>831290</td>
<td>2429894</td>
<td>214646</td>
</tr>
<tr>
<td>2 Level DWT fusion with adaptive directional interpolation</td>
<td>1123564</td>
<td>840889</td>
<td>2447438</td>
<td>216992</td>
</tr>
<tr>
<td>1 Level DWT fusion with Edge directed interpolation</td>
<td>1137896</td>
<td>874495</td>
<td>2650277</td>
<td>228361</td>
</tr>
<tr>
<td>1 Level DWT fusion with adaptive directional interpolation</td>
<td>1335084</td>
<td>979007</td>
<td>2909629</td>
<td>237054</td>
</tr>
</tbody>
</table>

Sharpness index, M is used. Sharpness index is calculated using (13)

\[
M = \sum_{x=0}^{n-1} \sum_{y=0}^{m-1} ||L(x, y)||
\]  

13

where n×m is the size of the image and L(x, y) is the output of the Laplacian filter, whose input is the pixel (x, y). The sharper the image, the larger will be the value of sharpness index. Super resolution images obtained using different interpolation algorithms are shown in Figures 2 and 3.

6. Conclusion

It has been found from Table 3 that super resolution images constructed using Pearson correlation coefficient based registration, 1 Level DWT based fusion and adaptive directional interpolation produces best results with reduced computational complexity.

7. References