1. Introduction

In chemical process industries, most of the important processes are multivariable in nature. Distillation is one of these paration process widely implemented in the petroleum and chemical industries for purification of final products. Due to its industrial significance, the distillation column is one of the generally chosen very important multivariable processes by the researchers.\(^1\)\(^-\)\(^7\)

Controller design process for the stable, unstable and non-linear single input single output systems are simple compared with the Two Input Two Output (TITO) systems. Hence, due to its complexity, the numbers of controller design procedures available for TITO systems are very small compared to single input single output systems.

The control literature presents the traditional and modern approach based design and implementation of the PI/PID controllers for a class of systems.\(^8\)\(^-\)\(^18\) In this paper, PI controller design for TITO is addressed. The existing controller design procedure for TITO process can be categorized as 1. Centralized and 2. Decentralized methods. In centralized system, controller design task is quite complex compared with the decentralized system due to the interaction between the loops. In the proposed work, heuristic algorithm assisted centralized PI controller design is discussed for the TITO systems existing in the literature.

In this paper, recent heuristic method, known as Teaching Learning Based Optimization (TLBO) technique proposed\(^19\)\(^,\)\(^20\) is adopted to solve the controller design problem. This algorithm is theoretically similar to the teaching-learning scenario existing in the class room learning process. In order to obtain best optimal value, the controller design process is repeated 10 times and the mean of controller values, such as \(K_{p1}, K_{i1}, K_{p2}, K_{i2}\) are recorded. The simulation study is carried out using the Matlab software and the performance of the proposed method is validated with Particle Swarm Optimization (PSO), Bacterial Foraging Optimization (BFO) and Firefly Algorithm (FA) existing in the literature. In order to execute a fair comparison, all the algorithms are
assigned with similar population size, iteration number, stopping criteria and objective function.

The remaining sections of this paper are arranged as follows. Section 2 describes the distillation process and its benchmark versions. Section 3 presents the overview of the TLBO algorithm considered in this study, the simulated results of this work is presented in section 4 and the conclusion of the present work is discussed in section 5.

2. Process Description

In chemical and petroleum industries, distillation is the separation methodology used to convert the raw mixtures into usable final products. The mixture is applied on the distillation column, with a number of trays and a number of temperature regions. Based on the column temperature, the mixture is split into the Liquid (L) and the Vapour (V) products. The process for the distillation column is based on L-V structure or the energy balance method. In this control configuration, the vapour flow rate and the liquid flow rate are the control inputs.

Figure 1. Schematic of distillation column.

Figure 1 shows the schematic of the industrial scale distillation column. In this paper the benchmark distillation columns, such as WW and VL are chosen.

Figure 2 shows the structure of TITO centralized control system. In this work, the controllers in TP loop ($PI_1$) and BP loop ($PI_2$) are designed using TLBO algorithm.

3. Teaching Learning Based Optimization

TLBO is formulated by imitating the teaching-learning system existing in the classroom scenario and its pseudo code is depicted below. Similar to other heuristic algorithms, the TLBO employs a population based
approach to attain the collective explanation during the exploration. A complete clarification about the TLBO can be found in the recent literature\textsuperscript{19-22}. In this work, conventional TLBO is adopted to tune the PI controllers for the TITO process.

The TLBO has two necessary stages, such as teacher stage and learner stage as shown below:

\textbf{START};

\begin{itemize}
\item Initialize algorithm parameters, such as number of learners (N), parameters to be optimized (D), Maximum number of iteration (Miter) and objective function (J\textsubscript{min});
\item Randomly initialize \(N\) learners for \(x_i\) (\(i = 1, 2, \ldots, n\));
\item Evaluate the performance and select the best solution \(f(x_{i\text{best}})\);
\end{itemize}

\begin{itemize}
\item WHILE iter = 1:Miter;
\item \%TEACHER STAGE\%
\item Use \(f(x_{i\text{best}})\) as teacher;
\item Sort based on \(f(x_{i\text{best}})\), select other teachers based on:
\item \(f(x_{i\text{best}}) = f(x_{i\text{best}}) - \text{rand for } f(x_{i\text{best}}) = 2, 3, \ldots, T\);
\item FOR \(i = 1:n\)
\item Calculate \(F_T = \text{rand} \cdot (0,1)\) \(x_i\) \(x_{i\text{best}}\) \(x_i\) \(x_i\) \(0,1)\) \(x_i\) \(x_{i\text{mean}}\);
\item End If \% End of TEACHER STAGE\%
\item \%STUDENT STAGE\%
\item Arbitrarily Select the learner \(x_i\), such that \(j \neq i\);
\item If \((f(x_i) < f(x_j))\), then \(x_i = x_{i\text{new}}\);\item Else \(x_{i\text{new}} = x_i + \text{rand}(0,1)(x_j - x_i)\);
\item End If
\item If \(x_{i\text{new}}\) is better than \(x_i\), then \(x_i = x_{i\text{new}}\);
\item End If \% End of STUDENT STAGE\%
\item End FOR
\item Set \(k = k+1\);
\item End WHILE
\item Record the controller valus, \(J_{\text{min}}\), and performance measures;
\item STOP;
\end{itemize}

In this work, heuristic algorithms such as Particle Swarm Optimization (PSO) and Firefly Algorithm (FA) are considered to validate the performance of TLBO.

- **Particle Swarm Optimization**
PSO is developed by modeling the group activities in flock of birds or school of fish. Due to its high computational capability, it is widely considered by the researchers to solve constrained and un constrained optimization problems. In this work, PSO with the following mathematical expression is considered\textsuperscript{23}:

\[ V_i(t + 1) = W^t V_i^t + C_1 R_1 (P_i^t - x_i^t) + C_2 R_2 (G_i^t - x_i^t) \]

\[ x_i(t + 1) = x_i^t + V_i(t + 1) \]

where \(w\) is inertia weight (chosen as 0.6), \(R_1\) and \(R_2\) are random values \([0,1]\), \(C_1\) and \(C_2\) is allotted as 2.0 and 1.5 correspondingly.

- **Bacterial Foraging Optimization**
Bacterial Foraging Optimization (BFO) algorithm is one of the successful nature inspired heuristic method, developed based on the mathematical model of the foraging activities in Escherichia coli (E.coli) bacteria. In this work, the enhanced BFO algorithm discussed in\textsuperscript{10-13} is adopted.

\[ N = 30; N_x = \frac{N}{2}; N_x = \frac{N}{3}; N_{ed} \approx \frac{N}{4}; N_y = \frac{N}{2} \frac{P_{ed}}{N + N_{ed}}; \]

\[ d_{a}\text{m\tiny{a\textsubscript{e}}} = W_{a}\text{m\tiny{a\textsubscript{e}}} = \frac{N}{N}; \text{ and } h_{\text{r\tiny{e\textsubscript{p\textsubscript{d}}}d\textsubscript{e}l\textsubscript{l}}} = W_{\text{r\tiny{e\textsubscript{p\textsubscript{d}}}d\textsubscript{e}l\textsubscript{l}}} = \frac{N}{N} \]

\[ (7) \]

- **Firefly Algorithm**
FA based technique utilizes the mathematical representation of a firefly, searching for a mate in the assigned search space. The detail of FA can be found in\textsuperscript{24}.

The association of an attracted firefly towards a mate can be expressed as:

\[ X_{i\text{new}} = X_i + \beta \gamma^\gamma (X_{i\text{new}} - X_i^t) + \alpha \text{rand} (1, 2) \]

where \(X_i\) is early location; \(X_{i\text{new}}^t\) is updated location; \(\beta\) is preliminary attractiveness; \(\gamma\) is absorption coefficient; \(\alpha\) is randomization operator and rand is random number \([0,1]\). In this paper, the following values are chosen for FA parameters: \(\alpha = 0.15; \beta = 0.1\) and \(\gamma = 1\).

\[ (8) \]

4. **Result and Discussions**
The controller design problem deals with finding optimal values of \(K_p\) and \(K_i\) for the top and bottom process loops. In this work, the TLBO explores the four dimensional search space in order to find the optimal controller values. For the controller design problem, a weighted sum of cost function is assigned to guide the heuristic search as given below.
\[ J_{\text{min}} = W_1 \cdot M_p + W_2 \cdot t_s + W_3 \cdot \text{ISE} + W_4 \cdot \text{IAE} \quad (9) \]

where \( M_p \) is peak overshoot, \( t_s \) is settling time, ISE is integral squared error and IAE is integral absolute error. The weights are assigned as \( W_1 = W_2 = 5 \) and \( W_3 = W_4 = 10 \).

The problem of assigning the optimal parameters for multi-loop PI controllers which stabilise TITO system is addressed in the paper. A centralised PI controller design procedure is implemented on some well known benchmark systems, such as WW and VL model. A comparative study among the heuristic algorithms, such as PSO, BFO and FA are presented.

The heuristic algorithm based search continuously explores the four dimensional search universe until the cost function is minimized. For all the heuristic algorithms, the population size is assigned as 30 and maximum iteration number is chosen as 500 and the stopping criterion is chosen as \( J_{\text{min}} \).

Initially the proposed work is tested on the WW distillation column model. In this work, the optimization exploration is repeated 10 times with each considered heuristic algorithm and the mean value among the trial is chosen as the optimal controller parameter.

Table 1 presents the controller values obtained with the heuristic algorithms for WW and VL processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>Method</th>
<th>( K_p_1 )</th>
<th>( K_i_1 )</th>
<th>( K_p_2 )</th>
<th>( K_i_2 )</th>
<th>Average Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>WW</td>
<td>PSO</td>
<td>-1.0345</td>
<td>-0.4149</td>
<td>2.3552</td>
<td>0.3067</td>
<td>268</td>
</tr>
<tr>
<td></td>
<td>BFO</td>
<td>-0.9566</td>
<td>-0.5088</td>
<td>1.9903</td>
<td>0.4028</td>
<td>302</td>
</tr>
<tr>
<td></td>
<td>FA</td>
<td>-0.8879</td>
<td>-0.5024</td>
<td>2.1887</td>
<td>0.4022</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td>TLBO</td>
<td>-1.0041</td>
<td>-0.4353</td>
<td>2.0881</td>
<td>0.4159</td>
<td>274</td>
</tr>
<tr>
<td>VL</td>
<td>PSO</td>
<td>-1.2270</td>
<td>-0.0882</td>
<td>2.7112</td>
<td>0.3975</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>BFO</td>
<td>-1.5033</td>
<td>-0.1162</td>
<td>2.3938</td>
<td>0.4174</td>
<td>337</td>
</tr>
<tr>
<td></td>
<td>FA</td>
<td>-1.1977</td>
<td>-0.1225</td>
<td>2.9003</td>
<td>0.7034</td>
<td>258</td>
</tr>
<tr>
<td></td>
<td>TLBO</td>
<td>-1.0106</td>
<td>-0.0958</td>
<td>2.8113</td>
<td>0.5470</td>
<td>252</td>
</tr>
</tbody>
</table>

Initially a unity setpoint is applied for the top product \((r_1)\) by keeping the other input \((r_2)\) as zero. A disturbance of 0.3 (30% of set point) is then applied at 100 sec. From Figure 3(a) and 3(b), it can be observed that, the proposed method outperforms the PSO and BFO algorithms.

Then, similar procedure is repeated on the VL model with unity setpoint on the bottom product \((r_1)\) by keeping the other input \((r_2)\) as zero. A disturbance of 0.3 (30% of set point) is then applied at 150 sec. From Figure 4(a) and 4(b), it can be observed that, the proposed method outperforms the PSO and BFO algorithms.
Table 2. Performance measures obtained for disturbance rejection

<table>
<thead>
<tr>
<th>Process</th>
<th>Method</th>
<th>Top product</th>
<th></th>
<th></th>
<th></th>
<th>Bottom product</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$M_p$</td>
<td>$t_e$</td>
<td>ISE</td>
<td>IAE</td>
<td>$M_p$</td>
<td>$t_e$</td>
<td>ISE</td>
</tr>
<tr>
<td>WW</td>
<td>PSO</td>
<td>0.127</td>
<td>23.74</td>
<td>1.544</td>
<td>1.242</td>
<td>0.330</td>
<td>30.72</td>
<td>0.113</td>
</tr>
<tr>
<td></td>
<td>BFO</td>
<td>0.160</td>
<td>20.81</td>
<td>1.026</td>
<td>1.013</td>
<td>0.379</td>
<td>27.04</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>FA</td>
<td>0.186</td>
<td>20.06</td>
<td>1.053</td>
<td>1.025</td>
<td>0.345</td>
<td>26.88</td>
<td>0.066</td>
</tr>
<tr>
<td></td>
<td>TLBO</td>
<td>0.127</td>
<td>23.74</td>
<td>1.402</td>
<td>1.184</td>
<td>0.350</td>
<td>25.18</td>
<td>0.061</td>
</tr>
<tr>
<td>VL</td>
<td>PSO</td>
<td>0.000</td>
<td>51.27</td>
<td>34.163</td>
<td>5.845</td>
<td>0.351</td>
<td>22.57</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>BFO</td>
<td>0.000</td>
<td>53.34</td>
<td>19.682</td>
<td>4.436</td>
<td>0.269</td>
<td>34.02</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>BA</td>
<td>0.000</td>
<td>56.02</td>
<td>17.715</td>
<td>4.208</td>
<td>0.255</td>
<td>12.84</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>TLBO</td>
<td>0.000</td>
<td>58.71</td>
<td>28.951</td>
<td>5.381</td>
<td>0.226</td>
<td>21.79</td>
<td>0.036</td>
</tr>
</tbody>
</table>

5. Conclusion

The problem of finding the best possible controller parameters for TITO system using TLBO algorithm is discussed in this paper. Proposed centralised PI controller design procedure is tested using some well known benchmark distillation column, such as WW and VL model. This study shows that, the proposed PI controller design procedure offers better performance measure values for the reference tracking and disturbance rejection operations compared with the alternatives considered in this work.

6. References