A Study on BFO Algorithm based PID Controller Design for MIMO Process using Various Cost Functions

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Abstract

Background/Objectives: The objective of this paper is to design a decentralised PID controller for MIMO process model using Bacterial Foraging Optimization (BFO) algorithm using three different cost functions. Methods/Statistical analysis: In this work, each decentralised PID controller is separately designed for the top product loop and bottom product loop. The proposed method is tested on Wood and Berry (WB) model and Wardle and Wood (WW) model. A comparative study is presented on the considered cost function using the quality measure values, such as ITSE, ITAE, number of iteration and CPU time taken for the simulation. Findings: The result confirms that, all the cost function based approaches offers similar result. Application/Improvements: In future, this method can be tested using some most successful heuristic algorithms existing in the literature.

Keywords: Bacterial Foraging Optimization, Distillation Column, ITSE, ITAE, Multi-Input-Multi-Output System, PID Control

1. Introduction

In chemical process industries, tuning of controllers to stabilize the process loops are really a challenging job and is more difficult for Multi Input and Multi Output (MIMO) systems. In process control literature, Proportional + Integral + Derivative controller is widely considered because of their structural unfussiness, reputation and trouble-free execution, in spite of the considerable progress in advanced process control schemes existing in the literature1-3. A range of model based studies on fine tuning the PID controllers have provided insight for better understanding of the controller performance for a variety of process models4-7.

The conventional centralised and decentralised PID tuning techniques considered by most of the researchers for MIMO processes are purely based on an approximated state space or transfer function models. The tuning procedure employed for one MIMO system will not offer better response for other process models. Hence, in recent years, heuristic algorithm based controller design is widely adopted by the researchers8-11.

Heuristic algorithm based methods are user friendly procedures, widely adopted to obtain better result for the chosen problems. Due to its efficiency, in recent years, heuristic methods are very famous in the field of system identification and model based controller design. In heuristic algorithm based approach, the algorithm arbitrarily searches a D-dimensional search space until it identifies optimal solution for the chosen problem. In this approach, the accuracy of the identified value depends mainly on the cost function, which guides the optimization search in order to get the D-number of values.

The choice of a particular heuristic algorithm for a problem depends mainly on: 1. Reputation, 2. Search dimension, 3. Number of initial algorithm parameters to be assigned, 4. Adaptability, 5. Real time implementation, etc.

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In this paper, we attempted a decentralised multi-loop PID controller design for distillation column model (2 X 2) using the Bacterial Foraging Optimization (BFO) algorithm\textsuperscript{12-16}. In this paper, we proposed an assessment between various cost functions existing in the literature\textsuperscript{10,15,16}. During the optimization search, the BFO based approach is considered for the $PID_T$ (controller in top product loop) and $PID_B$ (controller in bottom product loop) using a chosen cost function. This process is repeated for 10 times individually and the mean value is chosen as the final controller values.

The performance of the controller design process with various cost functions are compared using the process quality measures, such as Integral Time Squared Error (ITSE), Integral Time Absolute Error (ITAE), average iteration time taken by for BFO based search and average CPU time for the search.

The remaining portion of the paper is arranged as follows: Section 2 presents the summary of the BFO algorithm, Section 3 outlines the various cost functions considered in this work, the simulated results of this work is discussed in section 4 and the conclusion of the work is presented in section 5.

### 2. Bacterial Foraging Optimization

Bacterial Foraging Optimization (BFO) algorithm is one of the nature inspired heuristic method proposed by Passino in 2002 to design the controller for a tank liquid level control problem\textsuperscript{14}. It is one of the most flourishing heuristic practices, developed based on the mathematical model of the foraging activities in Escherichia coli (E.coli) bacteria.

In this method, a group of artificial bacteria work together to find the potential solution in the D-dimensional search space during the optimization exploration. The earlier research has reported the dominance of the BFO based search in a variety of engineering optimization problems\textsuperscript{8-13}. In the literature, a considerable number of works are available to justify the dominance of BFO algorithm in the field of controller design\textsuperscript{14-16}. Implementation of the BFO has following steps:

**Step 1:** Initialization: assign population size $N$, dimension of search $D$ and stopping criteria ($J_{\text{min}}$).

**Step 2:** (i) Start of the swimming or tumbling operation, (ii). Start of the elimination-dispersal loop, (iii). Star of the reproduction loop, (iv). Start of the chemotaxis loop.

**Step 3:** Evaluate the cost function: The objective function values of the bacteria’s are evaluated using the performance criteria for the convergence.

**Step 4:** Update the bacteria: The bacterial swarm is updated using the elimination-dispersal process. Update all the bacteria and sort based on the health.

**Step 5:** Stopping criteria: If the stopping criteria are met, stop the search and display the values. Else, repeat step 1 to step 4 until the specified iteration is completed.

**Step 6:** Display the optimal controller values, iteration number and CPU time.

In this work, the enhanced BFO algorithm discussed in\textsuperscript{11} is adopted.

The initial BFO parameters are assigned as follows:

\[
N = 30; \ N_c = \frac{N}{2}; \ N_s = N_{re} = \frac{N}{3}; \ N_{ed} = \frac{N}{4}; \ N_r = \frac{N}{2};
\]

\[
P_{ed} = \left( \frac{N_{ed}}{N + N_r} \right); \ d_{\text{attract}} = W_{\text{attract}} = \frac{N_s}{N};
\]

\[
h_{\text{repell}} = W_{\text{repell}} = \frac{N_r}{N}
\]

Where, $N$- number of E.Coli bacteria, $N_c$ – number of chemotactic steps, $N_s$-Swim length during the search, $N_{ed}$- number of elimination-dispersal events, $N_r$-number of bacterial reproduction, $P_{ed}$- probability of the bacterial elimination, $d_{\text{attract}}$= $W_{\text{attract}}$-width and depth of attraction, $h_{\text{repell}}$= $W_{\text{repell}}$- height and width of repellent signal.

Initially, all the artificial E.Coli bacteria are randomly placed in the $D$-dimensional search space and the bacteria continuously travel around the search space to get the optimal solution based on the chosen cost function. Figure 1 shows the convergence of bacteria in a two dimensional search space.
3. Implementation

The main aim of this controller design problem is to find optimal values of \( K_p \), \( K_i \) and \( K_d \) for the top loop and the bottom loop using the proposed approach. Figure 2 presents the BFO based tuning procedure implemented for the top loop. Similar procedure is followed for the bottom loop.

The parallel form of PID structure considered in this work is shown in Equation (2).

\[
\text{PID} = K_p + \frac{K_i}{s} + K_ds
\]

The quality measures considered in this paper to tune the PID and to analyze the process response are presented below:

\[
\text{IAE} = \int_0^T |e(t)|dt = \int_0^T |r(t) - y(t)|dt
\]

\[
\text{ISE} = \int_0^T t|e(t)|dt = \int_0^T t|r(t) - y(t)|^2 dt
\]

\[
\text{ITAE} = \int_0^T t|e(t)|dt = \int_0^T t|r(t) - y(t)|dt
\]

\[
\text{ITSE} = \int_0^T t|e(t)|dt = \int_0^T t[r(t) - y(t)]^2 dt
\]

where \( e(t) \) = error, \( r(t) \) = reference input (set point), \( y(t) \) = process output, \( \text{IAE} \) = integral of absolute error, \( \text{ISE} \) = integral square error, \( \text{ITAE} \) = integral of time multiplied absolute value of error and \( \text{ITSE} \) = integral of time multiplied square error criterion.

In this paper, a comparative analyze is presented among various cost functions considered in the literature to design the PID controller for more complex process models.

Equation (7) is the cost function considered to design the setpoint weighted PID controller for a class of unstable process models. In this work, the required shape of the process output is mathematically assigned using various time domain specifications\(^{10}\). This cost function offered better result for the complex unstable system models. Equation (8) and (9) presents the cost functions widely adopted in the literature to design the controller for Single Input Single Output (SISO) systems.

\[
J_{\text{min1}} = CF_1 = W_1 \int_{T_1}^{T_2} Y + W_2 \int_{T_2}^{T_3} Y_1 + W_3 \int_{T_3}^{T_4} Y_2 + W_4 \int_{T_4}^{T_5} Y_4 + W_5 \int_{T_5}^{T_\text{max}} Y_5
\]

where \( T_1 \) – delay time, \( T_2 \) – rise time, \( T_3 \) – peak time, \( T_4 \) – time for undershoot, \( T_5 \) – settling time (t), \( T_{\text{max}} \) – simulation period, \( Y \) – desired process output, \( Y_1 \) – peak overshoot \( (M_p) \), \( Y_2 \) – peak undershoot, \( Y_3 \)– tolerable error, \( Y_4 \) – maximum limit of tolerable \( Y_5 \) and \( W_1 \) to \( W_5 \) – weighting factors for performance measures.

\[
J_{\text{min2}} = CF_2 = W_1 M_p + W_2 t_s + W_3 \text{ITSE} + W_4 \text{ITAE}
\]

\[
J_{\text{min3}} = CF_3 = W_1 M_p + W_2 t_s + W_3 \text{ITSE} + W_4 \text{ITAE}
\]

Where \( W_1 \) to \( W_6 \) are weighting values. In all the cases, \( W_1 = W_2 = 10 \) and \( W_3 \) to \( W_6 = 5 \).

During the heuristic search, following values are assigned: dimension of search is chosen as three \( (K_p, K_i, K_d) \), number of bacteria is assigned as 20, the simulation time is chosen as 500 (for WB model) and 5000 (for WW model) and stopping criteria is \( J_{\text{min}} \). The heuristic search is repeated 10 times individually for the top and bottom loop and the mean value is recorded as the optimal controller parameter.

4. Results and Discussions

This section presents the performance of BFO based PID controller on a class of MIMO benchmark process models\(^{17-24}\). The simulation work is executed using Matlab 2010.

Initially, the PID design procedure is implemented on the top product (TP) loop of WB model (Equation 10) with unity step input for the top product loop and zero input for Bottom Product (BP) loop and vice versa. The
mean of the optimal controller values are presented in Table 1.

\[
G(s) = \begin{bmatrix}
12.8 e^{-s} & -18.9 e^{-3s} \\
16.7s + 1 & 21s + 1 \\
6.6 e^{-7s} & -19.4 e^{-3s} \\
10.9s + 1 & 14.4s + 1
\end{bmatrix}
\]

(10)

Figure 3 shows the reference tracking response for the TP and bottom product loops. Figure 3(a) and (b) presents the corresponding reference tracking for the bottom product. Similar procedure is once again repeated with the above process by assigning unity step signal as the setpoint for the bottom product. In both the cases, the simulation time is assigned as 300 sec. The performance measure values are tabulated in Table 2.

This procedure is then implemented in WW process model presented in Equation 11. Above said procedure is repeated for this model and the optimized controller values are recorded in Table 1 and the corresponding results are presented in Table 3.

\[
G(s) = \begin{bmatrix}
0.126 e^{6s} & -0.101 e^{12s} \\
60s + 1 & (48s + 1)(45s + 1) \\
0.094 e^{8s} & -0.12 e^{8s} \\
38s + 1 & 35s + 1
\end{bmatrix}
\]

(11)

### Table 1. Optimal controller values for the WB and WW model

<table>
<thead>
<tr>
<th>Process</th>
<th>Method</th>
<th>PID&lt;sub&gt;Τ&lt;/sub&gt;</th>
<th>PID&lt;sub&gt;Β&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K&lt;sub&gt;Τ&lt;/sub&gt;</td>
<td>K&lt;sub&gt;Τ&lt;/sub&gt;</td>
<td>K&lt;sub&gt;ΡΤ&lt;/sub&gt;</td>
</tr>
<tr>
<td>WB</td>
<td>CF1</td>
<td>0.6418</td>
<td>0.0528</td>
</tr>
<tr>
<td></td>
<td>CF2</td>
<td>0.5716</td>
<td>0.0535</td>
</tr>
<tr>
<td></td>
<td>CF3</td>
<td>0.6028</td>
<td>0.0474</td>
</tr>
<tr>
<td>WW</td>
<td>CF1</td>
<td>21.8446</td>
<td>0.5938</td>
</tr>
<tr>
<td></td>
<td>CF2</td>
<td>22.6500</td>
<td>0.5538</td>
</tr>
<tr>
<td></td>
<td>CF3</td>
<td>23.0775</td>
<td>0.6082</td>
</tr>
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</table>

### Table 2. Performance measure values for WB model

<table>
<thead>
<tr>
<th>Simulation sequence</th>
<th>Method</th>
<th>ITSE&lt;sub&gt;Τ&lt;/sub&gt;</th>
<th>ITAE&lt;sub&gt;Τ&lt;/sub&gt;</th>
<th>ITSE&lt;sub&gt;Β&lt;/sub&gt;</th>
<th>ITAE&lt;sub&gt;Β&lt;/sub&gt;</th>
<th>Avg. Iteration</th>
<th>Avg. CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit step given for TP loop</td>
<td>CF1</td>
<td>2.833</td>
<td>36.56</td>
<td>34.23</td>
<td>121.4</td>
<td>214</td>
<td>54.38</td>
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<tr>
<td></td>
<td>CF2</td>
<td>3.241</td>
<td>38.16</td>
<td>35.58</td>
<td>123.4</td>
<td>225</td>
<td><strong>41.06</strong></td>
</tr>
<tr>
<td></td>
<td>CF3</td>
<td>3.562</td>
<td>39.77</td>
<td><strong>32.96</strong></td>
<td>113.9</td>
<td>201</td>
<td>55.16</td>
</tr>
<tr>
<td>Unit step given for BP loop</td>
<td>CF1</td>
<td><strong>221.9</strong></td>
<td>428.8</td>
<td>744.2</td>
<td>806.1</td>
<td>296</td>
<td>72.18</td>
</tr>
<tr>
<td></td>
<td>CF2</td>
<td>231.8</td>
<td><strong>428.3</strong></td>
<td>661.5</td>
<td>740.1</td>
<td><strong>288</strong></td>
<td><strong>68.19</strong></td>
</tr>
<tr>
<td></td>
<td>CF3</td>
<td>266.6</td>
<td>467.4</td>
<td>544.8</td>
<td>614.7</td>
<td>336</td>
<td>84.10</td>
</tr>
</tbody>
</table>
5. Conclusion

The problem of assigning the best possible controller parameters for MIMO system using BFO algorithm is addressed in the paper using three different cost functions. Proposed decentralised PID controller design procedure is implemented on some well known bench mark systems, such as WB and WW model. This study confirms that, all the cost function offers similar performance measure value, whereas CF1 offers better CPU time. In future, the proposed method can be implemented for the centralised PI/PID controller design for MIMO system.

Table 3. Performance measure values for WW model

<table>
<thead>
<tr>
<th>Simulation sequence</th>
<th>Method</th>
<th>ITSE&lt;sub&gt;T&lt;/sub&gt;</th>
<th>ITAE&lt;sub&gt;T&lt;/sub&gt;</th>
<th>ITSE&lt;sub&gt;B&lt;/sub&gt;</th>
<th>ITAE&lt;sub&gt;B&lt;/sub&gt;</th>
<th>Avg. Iteration</th>
<th>Avg. CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit step given for TP loop</td>
<td>CF1</td>
<td>3.504</td>
<td>57.19</td>
<td>27.13</td>
<td>98.13</td>
<td>170</td>
<td>37.63</td>
</tr>
<tr>
<td></td>
<td>CF2</td>
<td>3.735</td>
<td>50.38</td>
<td>24.63</td>
<td>85.87</td>
<td>237</td>
<td>46.86</td>
</tr>
<tr>
<td></td>
<td>CF3</td>
<td>4.445</td>
<td>61.25</td>
<td>21.65</td>
<td>67.68</td>
<td>173</td>
<td>38.35</td>
</tr>
<tr>
<td>Unit step given for BP loop</td>
<td>CF1</td>
<td>288.2</td>
<td>621.5</td>
<td>102.4</td>
<td>1.3e4</td>
<td>236</td>
<td>58.06</td>
</tr>
<tr>
<td></td>
<td>CF2</td>
<td>302.8</td>
<td>622.6</td>
<td>899.6</td>
<td>1.04e4</td>
<td>252</td>
<td>68.55</td>
</tr>
<tr>
<td></td>
<td>CF3</td>
<td>355.2</td>
<td>665.6</td>
<td>857.3</td>
<td>689.1</td>
<td>230</td>
<td>61.10</td>
</tr>
</tbody>
</table>

Figure 4. Reference tracking outcome of WW model, (a) TP response 1 (b) BP response 1 (c) TP response 2 (d) BP response 2.

Figure 4 shows the reference tracking response for the TP loop and BP loop for the unity reference signal. In this case, the simulation time is chosen as 2500 sec. From Table 2 and Table 3, one can observe that, all the considered cost functions offers similar performance measure values, whereas CF1 offers better average CPU time.

6. References