An inventory Model for seasonal Goods with time Dependent Seasonal Revenue and Profit Loss in Fuzzy Sense

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Abstract

Background/Objective: To develop fuzzy set theory and the fuzziness in the inventory problem, the purpose is to find the optimal order quantity corresponds to the total cost and the associated cost also fuzzy. Methods/Statistical Analysis: This approach is to find the quantity which has the minimum cost with maximum profit. When the profit gained from selling one unit of the item without deterioration and other shortage cost must be avoided. Findings: Mathematical model has been developed in two steps. (1) The Fuzzy economic order quantity and with the fuzzy total cost, (2) The Fuzzy revenue and the related with fuzzy net profit, (3)Fuzzy economic order quantity and the total cost values with seasonal/time dependent revenue corresponding to the profits. Applications/Improvements: Our aim is to find total cost and profit with fuzzy sense. Numerical examples are given and sensitivity analysis is carried out to conclude the result.

Keywords: Economic Order Quantity, Fuzzy Profit, Mid-Season and Off Seasons, On Season, Revenue, Total Cost, Trapezoidal Fuzzy Numbers

1. Introduction

Seasons are given by god as well as weather conditions heavy rain fall no rainfall will affects the seasons some times. Seasonal goods are vegetables, root vegetables, fruits, spices, grains and all eatables etc. In the season time all vendors will come into the business with attractive prices, quality of the product may vary, but availability of the product in high level with less price. In this paper we divide the seasons into three on season, midseason and off season. In the off season goods are in very expensive price some vendors are will away from the trade by the time demand will be moderate depend the affordability of the customers, in midseason goods are in different price depends the necessity of the customers.

1.2. Representation of Seasons.
The fuzzy optimization was written very first by1. The process through certain situations over time related to deterioration and customers demand, observed that when a product is introduced in the market consumers buying behavior and categorize them according to their affordability to purchase. Fuzzy models constitute a
wide range of useful tools in assessing the business promotions\textsuperscript{4}. Various researchers made their writings in developing inventory models using fuzzy concept are as follows. The inventory problem with unexpected time\textsuperscript{5}. The economic order quantity formula by wilson's corresponds to the fuzziness with the additional costs inthe fuzzy set environment\textsuperscript{6}. Fuzzy models for the newsboy problem\textsuperscript{7}. The model with trapezoidal fuzzy number\textsuperscript{8}. A method to find the fuzzy profit when the demand with seasons and selling cost are represented as fuzzy numbers\textsuperscript{9}. An economic order quantity model with fuzzy numbers depends on time, demand and cost coefficients\textsuperscript{10}. An Economic order quantity model with reliability and demand-dependent unit cost\textsuperscript{11}. The case of revenue and seasonal variable quantity in consumer products\textsuperscript{12}. Inventory model for decaying items keeping selling price fuzzy sense and developing the model for total relevant cost also the triangular fuzzy numbers\textsuperscript{13}. Optimization of fuzzy production inventory model\textsuperscript{14}. Model for multiple items with profit finding, to maintain appropriate methods used to handle the shortages\textsuperscript{15}. The mathematical model for buyer and seller also developed the weibull deterioration used to find the realistic solutions\textsuperscript{16}. Multilevel inventory techniques used for reduce the relevant cost\textsuperscript{17}. Two parameter weibull distribution used to reduce the deterioration\textsuperscript{18} Two stage vendor scheduling to minimize relevant cost in different times with triangular fuzzynumbers\textsuperscript{19}. The standard of manufacturing technique is to obtained optimal fuzzy rule\textsuperscript{20}, \textsuperscript{21}analysed fuzzy trapezoidal rule for average mean value method. Fuzzy optimization plays the main role in data mining and share market tradings\textsuperscript{22},\textsuperscript{23}. The objective is to determine the total cost in maximizing the resultant profit by considering the fuzzy demand, ordering, holding, selling Price. The solution procedure is presented using the Left and Right functions of the fuzzy Operations with respect to their corresponding Seasonal Price values.

2. Notation and Assumptions

- H is the Holding cost per unit cycle.
- O is the setup cost per order.
- \( L_{Sn} \) is the Length of the plan in on season.
- \( L_{Sm} \) is the Length of the plan in midseason.
- \( L_{Sf} \) is the Length of the plan in off season.
- L is the length of the plan.
- r is the order quantity per cycle.
- D is the total demand.
- \( D_{Sn} \) is the Demand in on season.
- \( D_{Sm} \) is the Demand in midseason.
- \( D_{Sf} \) is the Demand in off season.
- \( SP_{Sn} \) is the selling price in on season.
- \( SP_{Sm} \) is the selling price in midseason.
- \( SP_{Sf} \) is the selling price in off season.
- \( r^* \) is the Economic order quantity.
- \( \tilde{T}_c \) is the Fuzzy total cost.
- \( \tilde{H} \) is the Fuzzy Holding cost.
- \( \tilde{O} \) is the Fuzzy Ordering cost.
- \( \tilde{R}_e \) is the Fuzzy total Revenue.
- \( \tilde{D}_q \) is the Fuzzy demand quantity.
- \( \tilde{S}_p \) is the selling price.
- \( \tilde{N}_p \) is the Fuzzy Net Profit.

2.1 Fuzzy Operators,

- is the Fuzzy addition.
- is the Fuzzy Subtraction.
- is the Fuzzy Multiplication.
- is the fuzzy division.

2.2 Assumptions

- Seasons are fixed time Horizon.
- Selling cost is fixed.
- Shortages and Deteriorations avoided.
- Ordering and holding cost are constant.
- \( Sn \) is the on season is fixed for 3 months.
- \( Sf \) is the off season is fixed for 3 months.
- \( Sm_{(1,2)} \) is the midseason is fixed for

3. Formulation of the Inventory Model

3.1 Economic Order Quantity

By the Wilson EOQ formula,

\[
EOQ = r^* = \sqrt{\frac{2OD}{HL}}
\]  

3.2. For Seasonal Goods

Now we are applying fuzzy variables for Economic order quantity formula:
\[ \text{[EOQ]}_{\text{Sn}} = (r^*)_{\text{Sn}} = \sqrt{\frac{2OD}{H(Sp)_{\text{Sn}}}} \]  
\[ \text{[EOQ]}_{\text{Sm}} = (r^*)_{\text{Sm}} = \sqrt{\frac{2OD}{H(Sp)_{\text{Sm}}}} \quad \text{Here, } Sm = (Sm_1, Sm_2) \]  
\[ \text{[EOQ]}_{\text{Sy}} = (r^*)_{\text{Sy}} = \sqrt{\frac{2OD}{H(Sp)_{\text{Sy}}}} \]  

### 3.2.1 Total Cost

Total cost is addition of ordering and holding cost respectively,
\[ \tilde{T}_c = \frac{HL_t}{2} + \frac{OD}{r} \quad \text{and } r = r^* \]  

### 3.3 Fuzzy Inventory Model

Let \( \bar{H} = (H_1, H_2, H_3, H_4) \) and \( \bar{O} = (O_1, O_2, O_3, O_4) \) are trapezoidal fuzzy numbers and \( \bar{r} \) are fuzzy variables for the total cost formula with LR form we get,
\[ Lr^* = \sqrt{2D(O_1 + O_2 + O_3 + O_4)} \left( \frac{H_1 + H_2 + H_3 + H_4}{L} \right) \]  

From (5), \( \tilde{T}_c = \left( \frac{H}{2} + \frac{OD}{r} \right) \odot \left( \frac{H}{2} + \frac{OD}{r} \right) \)
\[ \tilde{T}_c = \left( \frac{H_1L_1}{2} + \frac{H_2L_2}{2} + \frac{H_3L_3}{2} + \frac{H_4L_4}{2} \right) \odot \left( \frac{O_1D}{r} + \frac{O_2D}{r} + \frac{O_3D}{r} + \frac{O_4D}{r} \right) \]  
\[ \tilde{T}_c = \left( \frac{H_1L_1}{2} + \frac{H_2L_2}{2} + \frac{H_3L_3}{2} + \frac{H_4L_4}{2} \right) \odot \left( \frac{O_1D}{r} + \frac{O_2D}{r} + \frac{O_3D}{r} + \frac{O_4D}{r} \right) \]  
\[ \tilde{T}_c = (\Omega_1, \Omega_2, \Omega_3, \Omega_4) \text{(say)} \]  

In the Left and Right form,
\[ A_L(\alpha) = \Omega_1 + (\Omega_2 - \Omega_1)\alpha \]  
\[ A_R(\alpha) = \Omega_4 - (\Omega_4 - \Omega_3)\alpha \]

By using signed distance method,
\[ \tilde{T}_c = \frac{1}{2} \int [A_L(\alpha) + A_R(\alpha)]d\alpha \]

### 3.4 To find Revenue with Profit with Fuzzy Sense

Let
\[ D = (\bar{D}_{\text{Sn}}, \bar{D}_{\text{Sm}}, \bar{D}_{\text{Sy}}, \bar{D}_{\text{Sp}}), \quad \bar{Sp} = (\bar{Sp}_{\text{Sn}}, \bar{Sp}_{\text{Sm}}, \bar{Sp}_{\text{Sy}} , \bar{Sp}_{\text{Sp}}) \]  
and \( z(r^*) = \left[ \tilde{Z}(r^*)_{\text{Sn}}, \tilde{Z}(r^*)_{\text{Sm}}, \tilde{Z}(r^*)_{\text{Sy}}, \tilde{Z}(r^*)_{\text{Sp}} \right] \)

are the trapezoidal fuzzy numbers, then,
With the use of Operations on trapezoidal multiplication we can get,
\[ \bar{R}e = (\bar{Sp}_{\text{Sn}} \odot \bar{D}_{\text{Sn}}, \bar{Sp}_{\text{Sm}} \odot \bar{D}_{\text{Sm}}, \bar{Sp}_{\text{Sy}} \odot \bar{D}_{\text{Sy}}, \bar{Sp}_{\text{Sp}} \odot \bar{D}_{\text{Sp}}) \]

\[ \bar{R}e = (\bar{Sp}_{\text{Sn}} \odot \bar{D}_{\text{Sn}}, \bar{Sp}_{\text{Sm}} \odot \bar{D}_{\text{Sm}}, \bar{Sp}_{\text{Sy}} \odot \bar{D}_{\text{Sy}}, \bar{Sp}_{\text{Sp}} \odot \bar{D}_{\text{Sp}}) \]

\[ \text{Profit (or) Loss} = 1 \odot Lr \odot [\bar{R}_e \ominus \tilde{Z}(r^*)] \]
An inventory Model for seasonal Goods with time Dependent Seasonal Revenue and Profit Loss in Fuzzy Sense

\[
\text{Profit/loss} = 1 \odot L_{in} \otimes [\mathcal{R}_s \oplus \mathcal{Z}(r*)_{sm}] \oplus 1 \odot L_{sm} \otimes [\mathcal{R}_s \oplus \mathcal{Z}(r*)_{sm}]
\]

\[
\text{Profit/loss} = 1 \odot L_{in} \otimes [\mathcal{S}_p \oplus \mathcal{D}_r \oplus \mathcal{Z}(r*)_{sm}] \oplus 1 \odot L_{sm} \otimes [(\mathcal{S}_p) \oplus \mathcal{D}_r \oplus \mathcal{Z}(r*)_{sm}]
\]

We can use the above mathematical model for finding the profit through the total cost along with economic order quantity. The following examples are derived from the Equations (6), (13) and (14), Simple Mathematical calculations used for finding the values of Total cost, Revenue, Profit as well as corresponding graphical images has been drawn.

4. Problem Calculations

4.1 Example: 1
Let us consider an optimization problem with the following data:
\[
\hat{O} = (40,25,25,10) \quad \hat{H} = (36,22.5,22.5,9) \quad \mathcal{S}_p = (5,12.5,12.5,20) \quad \mathcal{D}_r = (400,250,3250,100)
\]
Find the Profit.

4.2 Example2
Let us consider an inventory system with the following data:
\[
\hat{O} = (150,120,90,60) \quad \hat{H} = (160,120,80,40) \quad \mathcal{S}_p = (10,20,30,40)
\]
Find the Profit/Loss.

4.2.1 Sensitivity Analysis Table (2)

<table>
<thead>
<tr>
<th></th>
<th>(D)</th>
<th>(\hat{O})</th>
<th>(\hat{H})</th>
<th>(r^*)</th>
<th>(\mathcal{Z}(r^*))</th>
<th>(\mathcal{R}_e)</th>
<th>(N_p)</th>
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<tbody>
<tr>
<td>On season</td>
<td>400</td>
<td>40</td>
<td>36</td>
<td>13.333327</td>
<td>900.00029</td>
<td>8000</td>
<td>2366.66657</td>
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<tr>
<td>Midseason1</td>
<td>250</td>
<td>25</td>
<td>22.5</td>
<td>6.666666</td>
<td>1012.50008</td>
<td>3125</td>
<td>704.16664</td>
</tr>
<tr>
<td>Midseason2</td>
<td>250</td>
<td>25</td>
<td>22.5</td>
<td>6.666666</td>
<td>1012.50008</td>
<td>3125</td>
<td>704.16664</td>
</tr>
<tr>
<td>Off season</td>
<td>100</td>
<td>10</td>
<td>9</td>
<td>3.333333</td>
<td>787.500071</td>
<td>500</td>
<td>-95.833357</td>
</tr>
</tbody>
</table>

Graph 1. Graphical Representation.

Graph 2. Graphical Representation.

5 Conclusion and Future Research

From the Tables (1) and (2), it can be observed that,
- Demand quantity is increases when in the on season.
- Demand quantity is moderate when in the midseason.
- Demand quantity is decreases when in offseason.
- Selling Price is increases when offseason.
- Selling Price is decreases when in on season.
- Revenue is increases when demand increase.
- Order and holding costs are same in both a tables but Profit percentage is varies due to demand, affordability of the customers and seasons.
A fuzzy inventory model for demand with constant selling price, ordering cost and holding cost has been developed with fuzzy sense. Trapezoidal fuzzy models are found for profit/loss. Without shortages and deterioration has been found. A numerical example is also given in support the theory. A future research is to extend the model under deterioration with shortages.

6. References