Abstract

Background/Objectives: The article considers the necessity to form stochastic culture in upper-formers as their integral quality, the premise for effective mental activity in stochastics, a composite index of stochastic competence. Methods: Philosophical and cultural approaches have been used to construct a structural-functional model of stochastic culture of the pupil’s personality. Axiological and epistemological factors form the basis for the model development. The upper-formers’ stochastic culture has been assessed by parameterization in quantitative or qualitative terms. Findings: The authors have proposed a procedure to diagnose and calculate the levels of upper-formers’ stochastic culture well-formedness, distinguishing a critical, admissible, advanced and optimal levels. The upper-formers’ stochastic culture will be formed according to the suggested conceptual model, in the framework of which four sections are distinguished: methodological, informatory, procedural-evaluative and diagnostic. Each section is correlated with its implementation plan. The results of experimental work confirmed the hypothesis about the effectiveness of the developed model of upper-formers’ stochastic culture formation (the model was introduced in more than 70 secondary schools in the Lipetsk region, Russia). Teaching stochastics, conducted according to the developed technique, contributes to the enhancement of the upper-formers’ stochastic level of competence, educational interest to mathematics, which ultimately indicates a high level of well-formedness of upper-formers’ stochastic culture. Application: The calculated structural-functional model of stochastic culture of the pupils’ personality is required for the teachers in their teaching activities, which will help arrange properly the educational process.

Keywords: Stochastic Competence, Upper-Formers’ Stochastic Culture

1. Introduction

The modern period of the life of society is characterized by the rapid growth of uncertainty in on-going events, phenomena and processes. It is impossible to give a definite answer, and even to suggest the future result of accomplishment that happened (or not yet happened). For this reason, it becomes urgent to consider elements of stochastic dimension, largely explaining the laws of all that surrounds us in the school course of mathematics. In addition, stochastic dimension introduces the pupil to the dynamics of everything accidental.

Surrounding reality currently has begun to be considered and examined from the perspective of cultural analysis. In this regard, the basic trends of studying culture, which are interconnected, are defined. One of the trends explores society culture, and the other – the person culture. Self-preservation and development of the society, as well as educational processes observed in this society are represented by researchers of the first approach: V.Ye. Davidovich, M.S. Kagan, E.S. Markaryan. Researchers representing the second approach, V.S. Bibler, I.A. Iliyaeva, V.M. Mezhuev, consider as basic the personal aspects of culture, creativity and self-perfection of a person as the subject of activity and communication.

The person culture and the society culture are marked by objects, synchronously developing, creating and enriching each other. Due to the above mentioned, the level of the person culture development is dependent on the society culture, which is formed by culture of individuals. The society culture generates separate system-wide interests and values, traits and qualities that are inherent to all individuals included in it. However, the meaning, the strength and the power of these characteristics is acquired.
only in the absence of suppression of unique features. Characteristics should act as a means of development of representatives of this culture. The interdependence between the society culture and the person culture is a mechanism of the society self-movement.

Thus, stochastic culture is a part of the person culture in a logically subordinated series: common culture – the person culture – mathematical culture – stochastic culture. A consistent refinement of the above mentioned concepts will lead to the definition of a stochastic culture.

We will select the most significant, in our view, definitions of culture among numerous ones.

Culture is a purposeful activity of the subject, represented by either a person or a social group, or humanity in general. Culture is a kind of a complex unity. Culture is the highest degree of manifestation of human education and professional competence.

In turn, the person culture is part of common culture represented by the individual that is a complex system and is manifested in a person's education and competence.

In the system of the person culture the special role is played by the mathematical knowledge of the quantitative relations and spatial forms of reality, so the degree of scientific, technological and social progress is set by the level of current mathematical culture. The presence of the mathematical formalism and mathematical characteristics of reality is a necessary condition for the existence of modern society. Mathematical culture is a complex, dynamic quality of the personality that characterizes willingness and ability of the pupil to acquire, use and develop mathematical knowledge and skills in the educational activity.

Mathematical culture disobjectification results in cultural and personal qualities of the pupil in the investigated field of mathematics. Having become a prerequisite for the creative activity deployment, mathematical culture suggests the pupil's ability to possess and safely operate with the relevant competencies.

According to A.V. Khutorskoy, competence is a set of interrelated personal qualities (ways of life and KSAs (knowledge, skills, acquirements), set by the connection to a specific range of objects or processes of productive action in their relation. And competency is the person's skillful possession of a suitable competence, which includes his personal attitude to the subject of his activities.

I.A. Zimnyaya says about competence as integral personal formations including intellectual, moral and emotional components.

Competence is the level of skills of a person which is able to reflect the degree of conformity of a particular competency, and allows acting constructively in changing social conditions.

From the point of view of T.M. Churekova competence is a category, referring to the sphere of relations between practices and knowledge that integrates knowledge and skills, learned ways of activity and abilities in specific circumstances, as well as the readiness to implement all kinds of activity.

Competence in practical and operational application of knowledge and valuable relation to that sphere of activity in which the individual is realized determines the knowledge of how to act.

In the study of mathematics stochastic competence – the basic component of the mathematical competence – becomes fundamental for a pupil. Stochastic competence is the manifested readiness for activity, enabling to master the basic concepts of statistics, combinatorics and probability theory; the ability to apply these concepts in practice.

Stochastic competence largely characterizes the phenomenon of the pupil's stochastic culture. Based on this provision, we formulate a definition of upper-formers' stochastic culture. Stochastic culture of upper-formers is the integral quality of the individual, the premise and the condition for effective mental activity in the field of stochastic culture, a composite index of stochastic competence.

The conceptual meaning of stochastic culture is formed by four main elements:

1) the world view of random processes and phenomena;
2) probabilistic thinking;
3) methods of stochastics;
4) language of stochastics.

We disclose briefly the content of these components.

What is the world view of random processes and phenomena? First of all, these are many episodes of the reality around us in various spheres of human activity. For example, they are: construction of a new telephone exchange, organization of ambulance points, etc. In the first case, no one knows the number of subscribers who want to use the telephone services, and how often calls will be made. In the second case – the number of medical teams and cars for the timely assistance, as well as how many people will turn to the reception point. The content of these problems itself indicates the presence of random events, and probability-theoretic methods are required to solve the problems.
Stochastics (probability theory, combinatorics, and mathematical statistics) is one of the fundamental dimensions of school mathematics. The study of probability theory contributes to the pupil’s probabilistic thinking formation, enabling to apply methods of strictly logical thinking in a very uncertain situation, with specification of the concepts and terminology clearness.

From the standpoint of philosophical views, probability theory extends the idea about natural connections that exist in the outside world. The concept of probabilistic determinism contributed to the discovery of methods of quantitative research of the laws of probabilities, which find their fundamental applications in knowledge of natural phenomena and thinking.

Application of probability theory also consists in the construction of the languages that will be suitable for rational as well as objective non-deductive conclusions. In the formation of a language the probability theory acts as syntax, which has some differences in its interpretation, in connection with which among neither mathematicians, nor statisticians and philosophers have complete agreement in relation to how such stochastic language should look like.

We would like to note that the study of the theory of probability enables to better establish the relationship between reality and mathematics, as well as to create mathematical models of reality. In case if in the course of mathematics the “Statistics, Combinatorics, Probability Theory” topic is totally ignored, then the pupils have a misconception about the true nature of mathematics and its applications. Those pupils, who were unfamiliar with probability theory, have a totally incorrect point of view, consisted in the fact that mathematical methods are only needed for simple and precise relationships between precisely measurable and computable values. To prevent this, math teachers should concentrate their attention on the study of the stochastic approach of school course, not only giving deep theoretical knowledge to pupils, but also to form their stochastic culture.

### 2. Method


To construct the upper-formers’ stochastic culture model we can use diverse approaches. We consider philosophical and cultural approach, in the framework of which we construct a structural-functional model of stochastic culture of the pupil’s personality.

Factors of axiological and epistemological nature are the basis in the development of the model.

Axiological foundations of the pupils’ personality culture are the basis of value perception of the world around, reflecting pupils’ value and motivational orientations. Epistemological foundations of the pupils’ personality culture are orientation in the course of educational activities presented by cognitive and competency-based orientations, directly practical skills and reflective-evaluative abilities. Ontological foundations of the pupils’ personality culture contain axiological and epistemological foundations and represent the totality of upper-formers’ achievements, which are received in educational institutions and used in other areas.

Structural and functional model of upper-formers’ stochastic culture is represented in axiological and epistemological sections.

In axiological cross-section we select a motivational and value component, in epistemological cross-section — cognitive and competency-based, effective and practical, reflexive and evaluative components.

A motivational and value component forms motivational orientations and guidelines of activity with the help of which a pupil has a desire to engage in intellectual activity classes when solving combinatorial and probabilistic problems, performing statistical analysis of data. Component foundations help in identifying and developing orientation of intellectual honesty and creativity, on the realization that the truth is ambiguous. Presented component acts as an aesthetic perception of intellectual practices in solving stochastic problems in different ways and their results. The presence of this component makes it possible to use information and computer technologies as a tool in the learning process.

A cognitive and competency-based component generates pupils’ stochastic competence, raises the level of mathematical literacy.

An effective and practical component covers all practical activities of the pupil: it determines the level of self-reliance, awareness in incremental steps in finding the correct solution to problems.

A reflective-evaluative component carries out reflection in relation to the mathematical activity in the study of combinatorics, statistics and probability theory.

These components of pupils’ stochastic culture are available for parameterization, but in different degrees.
Table 1. Structural and functional model of upper-formers’ stochastic culture

<table>
<thead>
<tr>
<th>Axiological foundation</th>
<th>Epistemological foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Motivational and value component</strong></td>
<td>Cognitive and competency-based component</td>
</tr>
<tr>
<td><strong>Formation of motivational orientation systems of activity, initiation of intellectual activity in the field of stochastics</strong></td>
<td>Formation of knowledge and skills in the field of stochastics</td>
</tr>
<tr>
<td><strong>Development of orientations on intellectual honesty and creativity, on ambiguity of the truth</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Use of information and computer technologies, formation of computer knowledge image</strong></td>
<td>Formation of a stochastic competence;</td>
</tr>
<tr>
<td><strong>Formation of aesthetic perception of reality</strong></td>
<td>Formation of probabilistic thinking;</td>
</tr>
<tr>
<td><strong>Awareness of the value of stochastic knowledge and skills, algorithmization of one’s own activity</strong></td>
<td>Development of extraordinary perception of the world of random phenomena and processes;</td>
</tr>
<tr>
<td><strong>Creation of a stochastic knowledge complex</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Encouraging the development of innovative abilities and thinking criticism of upper-formers’ personality in the stochastic dimension</strong></td>
<td>Formation of a stochastic competence;</td>
</tr>
<tr>
<td><strong>Development of skills that contribute to the perception of the diverse manifestations of the world harmony</strong></td>
<td>Formation of probabilistic thinking;</td>
</tr>
<tr>
<td><strong>Ontological foundations</strong></td>
<td></td>
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<tr>
<td>Totality of upper-formers’ achievements, received in educational institution and used by him in various spheres of activity</td>
<td></td>
</tr>
</tbody>
</table>

Thus, stochastic competence and mathematical literacy of cognitive and competency-based components are parameterized in assessing of KSA. Other components are measured by specially developed systems of psychological tests. Result of parameterization is a quantitative or qualitative assessment of the relevant component of upper-formers’ stochastic culture.

Each structural component of the pupil’s stochastic culture model performs its distinctive functions. We move on to the functions of stochastic culture, clarifying the definition of “function”.

The concept of “function” is ambiguous and finds its application in humanities, natural and mathematical sciences. Thus, in the humanities, function acts as characteristic or sign of any systemic effects. Sciences, exploring the human activities from the socio-pedagogical point of view, speak about the function as a qualitative characteristic aimed at the development, maintenance and
preservation of system. In mathematics, function is the
dependence of any kind between two or more variables.
In the system of culture, except structural components,
functional components related to them are highlighted.
Missing connections of any of components with oth-
ers displace them from the established system. Analysis
of culture by functional method is not a novelty in the
humanities. One of the central cultural issues is the prob-
lem of functions of culture, which was considered in the
works by A.I. Arnoldov17, Ye.M. Babosova18, M.S. Kagan1,
E.V. Sokolov19. Functions of the personality’s mathematical
culture are reflected in the works by V.M. Galynskiy16, A.S.
Garkun16, N.K. Kisel16, Yu.V. Poznyak16, V.V. Samokhval16,
G.G. Shvarkova16. Any theoretical or practical investiga-
tions regarding the stochastic culture functions have not
been conducted.
Adhering to the model representation of stochastic
culture, we define the list of functions that it implements.
Functions of axiological dimension:
Functions of the motivational-value component:
• formation of motivational attitudes and orientations
  of activity, initiation of intellectual activity in the field
  of stochastics;
• development of orientations on intellectual honesty
  and creativity, ambiguity of the truth;
• use of information and computer technologies, forma-
tion of the computer image of knowledge;
• formation of aesthetic perception of reality;
• awareness of the value of stochastic knowledge and
  skills, algorithmization of one’s own activity;
• creation of the stochastic knowledge complex;
• encouraging the development of innovative abilities
  and thinking criticism of upper-former personality in
  the stochastic dimension;
• development of skills that contribute to the perception
  of the diverse manifestations of the world harmony;
Functions of the epistemological dimension.
Functions of the cognitive and competency-based
component:
• formation of knowledge and skills in the field of sto-
  chastics;
• formation of the stochastic competence;
• formation of probabilistic thinking;
• development of extraordinary perception of the world
  of random phenomena and processes;
• algorithmization of probabilistic schemes of activity;
• Functions of the effective and practical component:
• formation of skills to use stochastic knowledge in
  practice;
• formation of skills to distinguish between deter-
  ministic (predefined) and random phenomena and
  processes;
Features of the reflective and evaluative component:
• mastering the style of probabilistic thinking;
• laying the foundations of notions on content dynamics
  of stochastic knowledge;
• formation of skills to demonstrate the reflection
  regarding the process of activity in the study of sto-
  chastics;
• formation of skills to show reflection regarding the
  activity results in the study of stochastics.
Thus, the unity of all the components and all the functions
is integrated in the structural-functional model of upper-
formers’ stochastic culture. Using the proposed model
will offer new opportunities to research teachers’ practi-
cal activity and also will help identify new approaches to
the study of the stochastic dimension in the school course
of mathematics.

3. Results
The formation of upper-formers’ stochastic culture will
be carried out according to the conceptual model sug-
gested by us, in the framework of which we distinguish
four sections: methodological, informatory, procedural
and evaluative and diagnostic. Each section is correlated
with its implementation plan.
The methodological section there includes the fol-
lowing conceptual idea – the design of conditions for
the formation of upper-formers’ stochastic culture will
be achieved on the basis of implementation of the
structural-functional model of pupils’ stochastic culture;
the technology of the pupils’ stochastic culture formed-
ness using info-communication tools. Methodological
approaches are proposed and principles of training are
proved.
The informatory section of the model involves the for-
amation of components of structural-functional model
of upper-formers’ stochastic culture: motivation and
evaluative, cognitive and competency-based, effective
and practical, reflexive evaluative.
Table 2. Diagnosis of the level of well-formedness of upper-formers’ stochastic culture

<table>
<thead>
<tr>
<th>Components of mathematical culture and their weighted coefficients</th>
<th>Diagnosis methods</th>
<th>Formulas for determining the value of mathematical culture components formation level</th>
</tr>
</thead>
</table>
| Value and assessment component \( a = 0.25 \) | Observation, conversation, techniques, “Unfinished thesis”, “Story” | \[
K_1 = \sum_{i=1}^{a} \frac{n_i}{3 \cdot a}
\] |
| Cognitive and information component: 1) mathematical knowledge, math language \( \beta_1 = 0.2 \) | Tests, conversation, observation | \[
K_{2.1} = \sum_{i=1}^{b_1} \frac{n_i}{3 \cdot \beta_1}
\] \( b_1 \) – the total number of mathematical concepts included in the content of mathematics education; \( n \) – coefficient characterizing the level of well-formedness of mathematical concept \( (n = 0, 1, 2, 3) \); \( n = 0 \) – value orientation is not formed, \( n = 1 \) – partially formed, \( n = 2 \) – well enough formed, \( n = 3 \) – formed at a high level for this age).  
| 2) logical ways of thinking (analysis, synthesis, comparison, classification, generalization, seriation) \( \beta_2 = 0.2 \) | | \[
K_{2.2} = \sum_{i=1}^{b_2} \frac{n_i}{3 \cdot \beta_2}
\] \( b_2 \) – the total number of formed logical methods of thinking; \( n \) – coefficient characterizing the level of well-formedness of logical thinking techniques \( (n = 0, 1, 2, 3) \) |
| Effective and practical component \( \gamma = 0.2 \) | Tests, conversation, | \[
K_3 = \sum_{i=1}^{c} \frac{n_i}{3 \cdot \gamma}
\] \( c \) – total number of jobs offered; \( n \) – coefficient characterizing the level of independence, accuracy, awareness \( (n = 0, 1, 2, 3) \) |
| Reflexive evaluative component \( \delta = 0.15 \) | Conversation, observation | \[
K_4 = \sum_{i=1}^{d} \frac{n_i}{3 \cdot \delta}
\] \( d \) – the total number of indicators of this component formation; \( n \) – coefficient characterizing the ability to self-evaluation, self-control, activity in the performance of tasks \( (n = 0, 1, 2, 3) \) |
| Calculation of the value of the person’s mathematical culture formedness level in general is carried out by the formula: \( K = (a \cdot K_1 + \beta_1 \cdot K_{2.1} + \beta_2 \cdot K_{2.2} + \gamma \cdot K_3 + \delta \cdot K_4) \) | |

The **procedural section** distinguishes levels of upper-formers’ stochastic culture formedness:

- the critical level is characterized by inconclusively formed idea of an upper-former on the world view of random phenomena and processes;
- the permissible level is characterized by a well-developed probabilistic thinking;
- at the advanced level upper-formers skillfully own stochastic methods;
- the optimal level suggests the acquisition of stochastic language.

The **evaluative and diagnostic section** is a control and diagnostics of levels of upper-formers’ stochastic culture formation. The criteria include a number of value orientations, a number of stochastic notions, the total number of proposed tasks, the total number of indicators of reflection and evaluation formation.

4. **Discussion and Conclusion**

As the studies showed, until recently, it was very problematic to diagnose the level of pupils’ stochastic culture formation\(^{20}\). All attempts to define the well-formedness pointed only at individual characteristics, without which there is no place for stochastic culture itself. Among these were:

- motivation of acquiring stochastic knowledge in teaching and cognitive process;
• understanding of stochastic material and the ability to apply it in different situations;
• identification, investigation and solution of stochastic problems in various conditions, establishment of inter-subjective, interdisciplinary, inter-science relations;
• development of summarizing, systematization, abstraction and planning skills, etc.;
• construction and application of the simplest mathematical models in practice;
• assessment of stochastics beauty, demonstration of interest to the etymology of stochastic concepts, their application in practice.

After L.V. Voronina and L.V. Moiseyeva\(^ {\text{11}} \) had proposed a method of determining the level of well-formedness of the personality’s mathematical culture, it became possible to carry out the diagnosis of the level of upper-formers’ stochastic culture formation.

According to the proposed diagnostics, all four components of structural-functional model of upper-formers’ stochastic culture must pass the assessment levels: motivation and evaluative, cognitive and competency-based, effective and practical, reflexive evaluative. Observation, interviews, tests, etc. are the ways by which this methodology is implemented.

The level of well-formedness of upper-formers’ stochastic culture will be calculated as follows:

\[
K = (a \cdot K_2 + \beta_1 + K_2 + \beta_2 + K_2 + \gamma \cdot K_2 + \delta \cdot K_2) \cdot \%^{15}
\]

where the values of the constant coefficients equal to: \( \alpha = 0.25, \beta_1 = 0.20, \beta_2 = 0.20, \gamma = 0.20, \delta = 0.15 \) (specified and revealed by L.V. Voronina and L.V. Moiseyeva using a peer review method).

Subsequently, the level of well-formedness of upper-formers’ stochastic culture is critical if its value is less than 50%, admissible, if its value is 50–70%, advanced, if it is 71–90%, and optimal, if it is more than 90%.

Thus, the calculated level of upper-formers’ stochastic culture formation will help the teacher in his teaching activity. A teacher will be able not only to more deeply explore the phenomenon of stochastic culture, but also to “redirect” the educational process in the right direction.

5. Acknowledgements

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