A Novel Solution Methodology for the Optimization of Thermal Analysis in the Solar Parabolic Trough Collector

Y. K. Nayak1*, S. P. Sharma2, U. K. Sinha1, P. Kumar2 and N. Kumar1

1 Department of Electrical and Electronics Engineering, National Institute of Technology, Jamshedpur, Jharkhand – 831014, India; ee512035@nitjsr.ac.in
2 Department of Mechanical Engineering, National Institute of Technology, Jamshedpur, Jharkhand – 831014, India

Abstract

Objectives: The maximized nano-fluid Nusselt number and minimized pressure drops are the most effective options for obtaining the enhanced thermal frontiers in solar parabolic trough collector. Methods/Analysis: In view of this, numerous researches had proposed hybrid algorithms for the optimization of the thermal analysis. Obtaining Pareto optimal solution, tending to local optimum point and the time consumption are the main drawbacks of the previous algorithms. Hence, in order to overcome the above difficulties, present work proposes a new innovative approach for optimization of thermal analysis in SPTC. Particle Swarm Optimization (PSO) based solution methodology is proposed to gain the benefits of the global optimum solution and overcome the difficulties of the previous approaches. Findings: In this multi objective non-linear optimization problem, the effect of Nusselt number and pressure drops are considered as the main objectives to obtain the most beneficial values of the design variables. Inlet velocities, concentration ratio of nano-particles and absorber tube diameter are considered as the most preferable design variables in the proposed optimization problem. Application/Improvement: Five case studies based on different temperature levels are considered to check the suitability of the proposed solution methodology. Results explore the effectiveness of the proposed approach in the optimization of thermal analysis in SPTC.

Keywords: Heat Transfer, Nano-particle, Parabolic Trough Collector, Particle Swarm Optimization

1. Introduction

Day by day the percentage of the energy consumption increases due to growing world electricity demand. Thermo-economic forms of renewable energies are seen as the possible solution of growing energy challenges. Several renewable energies are highlighted due to the limited amount of existing fossil fuel, oil, gas, global warming, and contamination of the environment. Many scientists, researchers and engineers are exploring the possibilities of harnessing energy from various renewable energies like biomass, tidal, hydrogen, geothermal and solar energy.

Solar energy is the richest form of renewable energies and hence, can be seen in various applications throughout the worldwide. Solar tower, central receiver system, parabolic trough, linear Fresnel and the dish-stirling engine are the available technologies of concentrating solar collector1-6. Parabolic Trough Collector (PTC) is well known, cost effective and familiar solar heat collector element in solar thermal power system7,8, utilized to produce heat up to 400°C.

Optimal design of trough solar thermal power system has become key problem in the field of solar energy due to limited resources and expensive costs of the present conventional power plants9,10. In paper11, detailed parameter
study of the complete characteristics along with photothermal performance of a PTC system has been presented. Also, the effects of the parameters on the characteristics and performance have been numerically studied. Several investigations in heat transfer enhancement in PTCs have been carried out and their potential for improving the collector heat transfer performance has been presented in papers\textsuperscript{11-15}. Thermal conductivity and convective heat transfer coefficient are enhanced by implementing the nano-fluids, as the heat transfer fluid instead of base fluids\textsuperscript{16-17}. About 13 nm in diameter of $\text{Al}_2\text{O}_3$ particles at 4.3% volume fraction increased the thermal conductivity of water about 1.5% \textsuperscript{14}.

Heat transfer processes are widely used in numerous areas like: heat exchanger, cooling, heating and chemical processes. The poor heat transfer properties of the common fluids are the primary obstacles in the effectiveness of the heat process. Employing large particles in the fluid may tend to clogging. Hence, fluids with nanometre particles (nano-fluid) are chosen as the alternatives in obtaining the enhanced thermal conductivity, long term stability and low pressure drop\textsuperscript{18}. Nano-fluids are simply engineered, diluted, colloidal suspensions of particles with sizes in the nano-scale range (less than 100 nm) in a base fluid.

Nano-fluid have received utmost attention in enhancement of convective heat transfer. They have been proved superior over the current working fluids in increasing the performance of heat transfer devices and exchangers. The uses of nano-fluids have recently received significant attention for the enhancement of convective heat transfer. Nano-fluids are thought to have an enormous potential to increase the heat transfer performance of heat transfer devices and heat exchangers beyond the capabilities of current working fluids\textsuperscript{20}.

A past research reveals, the key parameters to enhance the thermal conductivity of nano-fluids are particle concentration, temperature, dispersion, stability and its size\textsuperscript{14}. The titanium nano-particles with a diameter of 21 nm are additionally employed in the receiver of the parabolic trough collector to investigate the heat transfer\textsuperscript{22}. In paper\textsuperscript{23}, heat transfer improvement is observed for a PTC tube employing synthetic oil – $\text{Al}_2\text{O}_3$ as nano-fluid for the particle concentration less than 5% at operating temperatures of 300 K, 400 K and 500 K. It is revealed that the heat transfer performance is directly proportional to the volumetric concentrations of nano-particles and inversely proportional to the operating temperatures.

The thermodynamic performances of the PTC employing syltherm 800 - $\text{Al}_2\text{O}_3$ nano-fluid are evaluated using an entropy generation minimization method\textsuperscript{24}.

Additionally, many of the researchers obtained optimal design parameters of the solar collector for enhancing the performances\textsuperscript{24}. In view of this, Genetic Algorithm is applied to obtain the optimal parameters of the PTC\textsuperscript{25}.

In the past, the performance of solar collector systems is enhanced by using the optimal parameters obtained by the various optimization techniques. The optimization techniques like LSSVM (least squares support vector machine)\textsuperscript{26}, GA (Genetic algorithm) and hybrid algorithms\textsuperscript{27} are the existing methodologies for obtaining the optimal parameters of the PTC. Though the GA methods have been employed successfully to solve complex optimization problems, recent research has identified some deficiencies in GA performance. The efficiency of the evolutionary process is reduced while solving the objective functions involving highly correlated design variables\textsuperscript{28}. Moreover, the premature convergence, tending to Pareto optimal or local minima solution of GA degrades its performance and reduces its search capability\textsuperscript{28}.

On the other hand, PSO is a useful technique in finding the global optimum values within less number of iterations. PSO has been implemented in wide variety of problems consisting of high dimensional, non-linear and non-convex. Recently, it is implemented to classify the preterm birth with a set of minimized rules\textsuperscript{29}. In paper\textsuperscript{30}, PSO is implemented to design a concrete gravity dam optimally. In this, the production cost is minimized subjected to various constraints like dam seismic forces, overturning, slip and cracks caused by fatigue in normal and earthquake conditions. Similarly, PSO is integrated with many other techniques in order to have superior characteristics. In view of this, it is adjoined with Apriori\textsuperscript{31}, Fuzzy Neural Networks (FNN)\textsuperscript{32} and Fuzzy C-Means clustering\textsuperscript{33} algorithms to improve energy in wireless sensor networks, for weather forecasting and for software cost estimation respectively.

Hence, to overcome the above difficulties and to gain the benefits like global optimal solution, PSO is proposed as one of the modern heuristic algorithm. It was organized by the simulation of a social system, and well implemented in solving continuous nonlinear optimization problems\textsuperscript{35-37}. It provides expert solution with better convergence characteristics and calculation times than the other stochastic methods\textsuperscript{38}. PSO has the advantage of controlling between the global and local exploration of
the search space which enhances the search capability of the evolutionary process.\textsuperscript{13}

Optimal design of PTC plays an important role and an enriched area of interest to engineers and researchers in the present competitive field. In past, researches have been utilized to solve complex problems of solar systems using various optimization algorithms.\textsuperscript{40,41} The algorithms employed are of either deterministic or stochastic solution methodologies, to achieve various objectives. A review of various optimization studies on solar thermal applications is elaborated in\textsuperscript{42,43}.

1.1 Contributions of the Present Work

1. Particle Swarm Optimization based solution methodology is proposed for optimization of heat transfer characteristics in SPTC.
2. Objectives like maximization of Nusselt number and minimization of pressure drops are considered simultaneously in the process of optimization.
3. Five different cases based on temperature levels are considered to check the suitability of the proposed approach in reaching the optimal value of the design variables.
4. The results obtained are presented and compared partially with the similar works done in the past.

The time line of the paper is as follows, after a brief introduction in section 1, the problem formulation and SPTC modelling are presented in section 2. Solution methodology is presented in section 3. The flow chart describing the proposed approach in reaching the optimal value of the design variables. Results and related discussions are presented in section 5. Finally, the findings of the research work are presented as conclusions in section 6.

2. Problem Formulation

This section provides the mathematical problem formulation for obtaining optimal design values for enhanced heat transfer characteristics of the SPTC.

The derogation of pressure drop and escalation of Nusselt number in the turbulent flow regime are the key performance indices of heat transfer in SPTC. The optimization of the heat transfer characteristics, keeping the minimum pressure drop is necessary for upgrading the existing heating or cooling systems. Incorporation of nano-particles increases pressure drop and involves extra pumping power which is improper in terms of cost. Hence, in this paper, two objectives mainly, maximization of Nusselt number along with minimization of pressure drop are considered simultaneously to obtain the optimal heat transfer characteristics. The combined objective function is expressed as below:

Maximize

\[
Z = \left[ Nu_{nf} \frac{\partial P}{\rho_{nf} \times V_i^2} \right]
\]

Subjected to:

\[
Re_{min} \leq Re \leq Re_{max}
\]

\[
Ri_{min} \leq Ri \leq Ri_{max}
\]

Where,

- \( Nu_{nf} \): Nusselt number
- \( \partial P \): Pressure drop \((N/m^2)\)
- \( \rho_{nf} \): Density of nano-fluid \((kg/m^3)\)
- \( V_i \): Fluid inlet velocity \((m/s)\)
- \( Re \): Reynolds number
- \( Re_{min} \): Minimum value of Reynolds number
- \( Re_{max} \): Maximum value of Reynolds number
- \( Ri \): Richardson number
- \( Ri_{min} \): Minimum value of Richardson number
- \( Ri_{max} \): Maximum value of Richardson number

In the above problem formulation (1) represents the optimization problem, (2) and (3) represents the lower and upper bound values of the Reynolds number and Richardson number respectively. The objective function is transferred into the form consisting of the design variables like, fluid inlet velocity \( V_i \), concentration of nano-particles \( \theta \), and inner diameter of receiver tube \( D_i \). Hence, the objective function will be transferred to the form containing design variables with the help of following equations (4) and (5). The Nusselt number is formed with the help of Re and Prandtl number \((Pr)\).

\[
Nu_{nf} = 0.023(Re)^{4/5}(Pr)^{0.4}
\]
Similarly, the pressure drop can be expressed as:

$$\hat{P} = \frac{fL\rho_{nf}V_i^2}{2D_i}$$  (5)

Where,
- \( f \) Friction factor
- \( L \) Length of absorber tube \((m)\)

In the present work, Re and Pr are evaluated based on SPTC physical model.

### 2.1 SPTC Physical Model

In the present section the modelling of SPTC is emphasized and the effects of nano-particle concentration on the mixed convection heat transfer rate of the nano-fluid in a fully-developed turbulent flow are studied. A non-uniform heat flux is considered on the outer surface of the inner absorber tube. In this work Al\(_2\)O\(_3\) (nano-particles) – synthetic oil (base fluid) is regarded as the working fluid and the density, viscosity, thermal conductivity, and specific heat are varied with respect to the operational temperatures. The parameters like absorber tube length \((L)\), diameter \((D_i)\) and size of nano-particle are selected based on insight given by related case study in paper\(^{45}\) and are considered as 7.8 m, 0.07 m and 10 nm respectively. As a case study, Al\(_2\)O\(_3\)/ synthetic oil was used as HTF in the collector field and nano-fluid flow was considered single phase flow and mass flow rate was taken as 0.9 kg/s.

The characteristics of the nano-fluid are dependent on the thermo-physical properties of the synthetic oil, concentration ratio of Al\(_2\)O\(_3\) and Temperature (T). To reveal the importance of nano-particles concentration in the heat transfer of SPTC, a plot between heat transfer coefficient and concentration on nano-particles for different operational temperatures is shown in Figure 1.

It can be observed from the figure that an increase in concentration of nano-particles results a similar increase in heat transfer coefficient. It is also observed that zero concentration of nano-particles (only base fluid) at different temperatures have lighter heat transfer coefficient than the nano-fluid.

The physical properties of different temperature are very important for engineering calculations. The density of nanofluid is based on the physical principle of the mixture rule. The density of nano-fluid is determined by the following equation.

$$\rho_{nf} = (1-\varphi)\rho_{bf} + \varphi\rho_{np}$$  (6)

Where,
- \( \rho_{bf} \) Density of base fluid
- \( \rho_{np} \) Density of nano-particle
- \( \varphi \) Concentration ratio of nano-particles

The specific heat of nano-fluid can be determined by assuming the thermal equilibrium between the nano-particles and the base fluid phase. Some authors\(^{40-43}\), prefer to use a simpler expression given as.

$$C_{p_{nf}} = (1-\varphi)C_{p_{bf}} + \varphi C_{p_{np}}$$  (7)

Where,
- \( C_{p_{nf}} \) Specific heat of nano-fluid
- \( C_{p_{bf}} \) Specific heat of base fluid
- \( C_{p_{np}} \) Specific heat of nano-particle

A wide range of experimental and theoretical studies was conducted by the researchers for thermal conductivity of nano-fluid\(^{46}\). In the present study the thermal conductivity is taken from\(^{45}\) which is as follows:

$$K_{nf} = \frac{k_{np} + 2k_{bf} - 2(1+\beta)^3\varphi(k_{bf} - k_{np})}{k_{np} + 2k_{bf} + (1+\beta)^3\varphi(k_{bf} - k_{np})}$$  (8)

Where,
- \( K_{nf} \) Thermal conductivity of nano-fluid
- \( K_{bf} \) Thermal conductivity of base fluid
- \( K_{np} \) Thermal conductivity of nano-particles

![Figure 1. Heat transfer coefficient v/s concentration of Nano-particle with different temperature.](image-url)
\[ \beta \text{ Ratio of nano-layer thickness to the nano-particles diameter} \]

A few theoretical formulas can be used to estimate the suspension, which is derived from the work of Einstein\(^46\).

\[ \mu_{nf} = (1 + 2.5\beta) \mu_{bf} \tag{9} \]

Where,

\[ \mu_{nf} \text{ Viscosity of nano-fluid} \]
\[ \mu_{bf} \text{ Viscosity of base fluid} \]

3. Solution Methodology

The PSO algorithm is introduced as an solution methodology to the optimization problem by Eberhart and Kennedy in the year 1995. It simulates on the mechanism of bird flocking in which a group of birds randomly look for food in an area\(^35\). It is commenced with a random population as solutions of optimization problem similar to many other algorithms. This involves updating of random population using the velocity factor. PSO is utilized in various fields such as training of neural networks, image recognition, fuzzy system control, optimization of electric power distribution networks, system identification in biomechanics, optimal shape and sizing design, topology optimization of structural optimization problems and process biochemistry.

Like other techniques, PSO consists of a number of swarm particles, sharpening their knowledge towards to the potential solution to an optimization problem in a given multidimensional search space. Each particle in the search space is appended with a position and velocity reflecting a candidate solution. The particles update their positions in a multidimensional search space to explore better fitness positions. Each particle has a memory function, and adjusts its path according to two pieces of information, the best position that it has so far achieved, and the global best position achieved by the whole swarm. The best position attained so far is called the local best and the best position among all individual local best positions is called the global best. Like in other evolutionary computation, the population is updated by applying some kind of operators, according to the fitness information, so that the particles can be expected to move towards a more appropriate solution. The mathematical formulations of the PSO algorithm are as follows.

Let the position vector of the \( j \text{th} \) particle in a \( D \)-dimensional space is represented as \( x_j = (x_{j,1}, x_{j,2}, \ldots, x_{j,D}) \). Similarly the velocity vector in a \( D \)-dimensional space is represented as \( v_j = (v_{j,1}, v_{j,2}, \ldots, v_{j,D}) \). The best previous position of the \( j \text{th} \) particle is recorded and represented as \( P_{best,j} = (P_{best,j,1}, P_{best,j,2}, \ldots, P_{best,j,D}) \).

The index of the best particle among all of the particles in the group is represented by the \( g\text{best}_d \). The updated position and velocity of the each particle can be evaluated using the current velocity and the distance from \( P_{best,j,d} \) to \( g\text{best}_d \) as shown in the following equations:

\[ v_{j,d}^{(t+1)} = w x_{j,d}^{(t)} + c_1 r_1 (P_{best,j,d} - x_{j,d}^{(t)}) + c_2 r_2 (g\text{best}_d - x_{j,d}^{(t)}) \tag{10} \]
\[ x_{j,d}^{(t+1)} = x_{j,d}^{(t)} + v_{j,d}^{(t+1)} \tag{11} \]

Where,

\[ c_1, c_2 \text{ Non negative acceleration constant} \]
\[ r_1, r_2 \text{ Random number between 0 and 1} \]
\[ w \text{ Inertia weight factor} \]

In the present work the value of non-negative acceleration constant is taken as 2 according to past experiences\(^35\). The balance between the global and local exploration capabilities of the particles is obtained by the introduction of the inertia weight factors. As originally developed, \( w \) often decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weight \( w \) is set according to the following equation

\[ w = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter \tag{12} \]

Where,

\[ iter_{max} \text{ Maximum iteration count} \]
\[ w_{max} \text{ 0.9} \]
\[ w_{min} \text{ 0.4} \]

The termination criterion for iterations is determined according to whether the maximum generations or a
designed value of the fitness is reached. In this present work the maximization problem is converted to a minimization problem for obtaining the optimum values of the design variables. In general the PSO algorithm is used for minimization of the fitness function. Hence the function $Z$ is changed to $J$ as shown below:

$$J = \frac{K}{1+Z}$$  \hspace{1cm} (12)

Where,

$K$ A positive integer valued 100

4. Flow Chart for the Proposed Methodology

The proposed method aims to obtain a near optimal solution for a problem comprising of maximization of Nusselt number and minimization of pressure drop using Particle Swarm Optimization. The optimum values of design variables are obtained through the proposed flow chart as shown in Figure 2.

5. Result

In the present work an alternative approach is proposed for the optimization of thermal analysis in solar parabolic trough collector. The thermal analysis includes the optimization of the ratio of the Nusselt number to the pressure drop. As, it is obvious that lower the pressure drop and higher the Nusselt number results in higher heat transfer characteristics of the SPTC.

Particle Swarm Optimization based solution methodology is proposed for obtaining the optimum values of the design variables for the optimization problem defined in equation (1). It is known that, simple nature in concept, easy implementation and computational efficiencies of PSO results for great potential to solve the optimization problems. Hence, in this work PSO is chosen as the other alternative to obtain the solution of the optimization problem. To check the suitability of the proposed approach for the thermal analysis in the SPTC the optimization problem is solved for five case studies depending on the temperature levels. These are emphasized as below:

- **Case A**: Optimization problem for $T=300$ K
- **Case B**: Optimization problem for $T=350$ K
- **Case C**: Optimization problem for $T=400$ K
- **Case D**: Optimization problem for $T=450$ K
- **Case E**: Optimization problem for $T=500$ K

![Figure 2. Flow chart for obtaining the optimum value of design variable.](image-url)
Table 1. Lower and upper limits of the design variables

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Unit</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid input velocity</td>
<td>m/s</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Concentration ratio of nanoparticles</td>
<td>-</td>
<td>0.001</td>
<td>0.06</td>
</tr>
<tr>
<td>Diameter of the inner receiver tube</td>
<td>m</td>
<td>0.05</td>
<td>0.15</td>
</tr>
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</table>

The results of the proposed approach for the case A, case B, case C, case D and case E are shown in Table 2. The heat transfer coefficient is also evaluated for the clear comparison purposes of the present work with the previous works at different operational temperatures. The plot representing the heat transfer coefficient for all the case studies are shown in Figure 3. It is clear from the figure that the heat transfer coefficient is increased in the present study as compared to the work preferred in paper[25]. The convergence plots of the proposed approach for the five case studies are shown in Figure 4. Hence the results obtained in the present research work are more effective and efficient when compared to the past researches.

The above figures reveal the various convergence characteristics of the thermal analysis, optimization problem in SPTC. It is also revealed that the proposed approach of optimization is well suited for the thermal analysis in SPTC.

6. Conclusion
In the present work, a novel solution methodology is proposed for the optimization of thermal analysis in the solar parabolic trough collector. Five design variables are considered for the optimization problem in order to enhance the heat transfer characteristics. To check the suitability, the optimization problem is solved for various case studies classified on different temperature levels. The results conveyed that the proposed work has superior characteristics in solving the issues related to the thermal analysis in STPC.

Table 2. Results of the proposed PSO approach for the optimization problem at different case studies

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Optimum values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case A T=300K</td>
</tr>
<tr>
<td>Fluid input velocity</td>
<td>0.803</td>
</tr>
<tr>
<td>Concentration ratio of nanoparticles</td>
<td>0.0505</td>
</tr>
<tr>
<td>Diameter of the inner receiver tube</td>
<td>0.1365</td>
</tr>
</tbody>
</table>

Figure 3. Heat transfer coefficient for different temperatures

Figure 4. Convergence curve for objective function
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7. References

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