MHD Free Convection Flow and Heat Transfer of Ferrofluids over a Vertical Flat Plate with Aligned and Transverse Magnetic Field

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Abstract

Objective: This paper aims to examine the Magnetohydrodynamics (MHD) ferrofluid’s steady natural convection boundary layer flow over a vertical flat plate. Method: Two types of base fluids (kerosene and water) were selected to formulate a problem that incorporates embedded magnetite ferroparticle. The governing nonlinear partial differential equations are modified with similarity transformations to a system of nonlinear ordinary differential equations. Then the Keller Box method is employed to numerically solve transform equations. The different values of governing parameters features and the characteristics of heat transfer and flow are analysed and discussed. Results: The results of the experiment are illustrated graphically to show the effects of the aligned and transverse magnetic field parameter, the particle volume fraction parameter and free convection parameter on heat transfer, skin friction and velocity and temperature profiles. The results were then compared with the published papers, which were found to agree well. Conclusion: At the plate surface with Fe$_3$O$_4$-kerosene ferrofluid, the heat transfer rate was higher than in the case of Fe$_3$O$_4$-water. The heat transfer rate at the plate surface increased with the rise in ferroparticle volume fraction, the angle of magnetic field, local Grashof number and magnetic field parameter.

Keywords: Aligned Magnetic Field, Boundary Layer, Ferrofluid, Natural Convection, Vertical Flat Plate

1. Introduction

The size of magnetic nanofluids, also known as ferrofluids, ranges from 5 to 15 nm. Ferrofluids are primarily employed to control the rate of heat transfer and fluid flow. They find applications in the field of aerospace, industrial engineering, medical, aeronautical, and science and technology. In the presence of Brownian motion, discussed the intermediate scale structure analysis of ferrofluids with magnetic nanoparticles. In employed computational fluid dynamics technique to illustrate the heat transfer analysis in ferrofluids. Recent studies reveal that the presence of an external magnetic field resulted in significant increase in the thermal conductivity of ferrofluids. Noted enhancement of thermal conductivity for Fe$_3$O$_4$-iron oxide ferrofluid by 300% and 200%, respectively. The formation of chainlike structures in ferrofluids imparts improved thermal conductivity, which increases with the intensity of the magnetic field.

Many researchers are now putting interest in the laminar boundary layer flows of heat and mass transfer of nanofluids on a flat surface due to the wide application, especially in different areas of science and engineering. Such flows are prominent in the cooling systems’ design for electronic devices, especially in the field of heat exchangers, geothermal reservoirs and cooling of a nuclear reactor. Many technological and industrial applications, such as micromixing of physiological samples, MHD pumps, drug delivery and biological transportation, experience the frequent MHD flow past a flat surface, which is of special technical significance. An external magnetic field is introduced to control the flow...
and heat transfer in any electrically conducting fluid flow system. The electrical conductivity property of nanofluids is improved due to the presence of nanoparticles, which also makes them more susceptible to the magnetic field as compared with conventional base fluids.

Recent studies that include the work of show the effects of chemical reaction and radiation on MHD thermostolutal nanofluid flow over a vertical plate in a porous medium. In studied the MHD boundary layer flow of nanofluids with gyrotactic microorganisms past a vertical plate and navier slip. In analyzed transfer rate of ferrofluids flow and heat over a flat plate with uniform heat flux. In examined the MHD slip flow of alumina water nanofluids over a flat plate. In elucidated the flow of a steady MHD boundary-layer in water-based nanofluids over a moving permeable flat plate.

It is to be noted that in the above discussion, studies were focussed mostly on free convection flows in nanofluids and not in ferrofluids. Therefore, this study investigates the free convection flow of ferrofluids over a flat plate, keeping the temperature constant and maintaining no slip condition. The study evaluates the effects of the transverse magnetic field, aligned magnetic field, Grashof number and volume fraction of ferroparticles on the velocity, skin friction, dimensionless temperature and heat transfer rate.

2. Mathematical Model

The ferroparticle is characterised by a steady two-dimensional laminar natural convection boundary layer flow with heat transfer over a vertical semi-infinite flat plate in kerosene and water-based ferrofluids with magnetite (Fe₃O₄). Thermal equilibrium is maintained for base fluid (kerosene and water) and the ferroparticle and ensuring that no slip occurs between them. We consider the shape of the ferroparticle to be spherical and ferroparticle’s volume fraction is taken into account. α is the aligned magnetic field introduced to the flow. It is considered to be a function of the distance from the origin, which is expressed as \( B(x) = B_0 / \sqrt{x} \) with \( B_0 \neq 0 \). Here, \( B_0 \) is the magnetic field strength and \( x \) is the coordinate along the plate. The ferroparticle moments quickly get oriented along the magnetic field lines when a magnetic field is introduced. However, the particle moments get quickly randomised on removing the magnetic field. Based on the above assumption coupled with the assumptions of Boussinesq and boundary layer approximations, the basic equations governing the steady MHD-free convection boundary layer flow of a ferrofluid near a vertical flat plate can be expressed as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g(T - T_0) - \sigma B^2(x) \sin^2 \alpha (u - U_\infty)
\]

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_{nf}}{\rho_{nf} C_{pf}} \frac{\partial^2 T}{\partial y^2}
\]

And, with the boundary conditions:

\[
u(x, 0) = 0, \quad v(x, 0) = 0, \quad T(x, 0) = T_w \quad (1)
\]

\[
u(x, \infty) = U_{nf}, \quad v(x, \infty) = 0, \quad T(x, \infty) = T_0 \quad (2)
\]

Here, \( u \) is the fluid velocity and \( v \) is the normal velocity components along the \( x \)- and \( y \)-axes. \( \alpha \) is the inclined angle, \( T_w \) is the constant wall temperature, \( T \) is the temperature of the ferrofluid, \( T_0 \) is the free stream temperature, \( g \) is the gravity acceleration, \( U_{nf} \) is the free stream velocity, \( \rho_{nf} \) is the effective density, \( \sigma \) is the electrical conductivity, \( (\rho\beta)_{nf} \) is the thermal expansion coefficient, \( \mu_{nf} \) is the effective dynamic viscosity and \( \alpha_{nf} \) is the thermal diffusivity of the ferrofluid, which are given as follows:

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}
\]

\[
(\rho\beta)_{nf} = (1 - \phi) (\rho\beta)_f + \phi (\rho\beta)_s, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}
\]

Here, \( \mu_f \) is the dynamic viscosity of the base fluid, \( \phi \) is the solid volume fraction, \( \rho_f \) and \( \rho_s \) are the densities of pure fluid and nanoparticles, respectively, \( (\rho C_p)_{nf} \) is the heat capacity of the ferrofluid and \( k_{nf} \) is the thermal conductivity of the ferrofluid, which are given as follows:

\[
(\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_s
\]

\[
k_{nf} = k_s + 2k_f - 2\phi (k_f - k_s)
\]

\[
\frac{k_f}{k_s} = \frac{k_f}{k_s} + 2\phi (k_f - k_s)
\]
Here, $k_f$ and $k_s$ are thermal conductivities of the ferrofluid and ferroparticles and $(\rho C_p)_f$ and $(\rho C_p)_s$ are the specific heat parameters of the base fluid and nanoparticles, respectively. As given by\textsuperscript{11}, the viscosity of the ferrofluid $\mu_{nf}$ can be approximated as the viscosity of a base fluid $\mu_f$ with dilute suspension of fine spherical particles. The Maxwell-Garnett’s model is used to approximate effective thermal conductivity of the ferrofluid $k_{nf}$, which is used for studying heat transfer enhancement that employ ferrofluids. The continuity Equation (1) is automatically satisfied by the stream function $\psi$.

\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \nu = -\frac{\partial \psi}{\partial x} \quad (7)
\]

The variable $\eta$ and a dependent variable $f$ in term of the stream function $\psi$ are defined to obtain a similarity solution of Equations (1)–(3) along with boundary conditions (4):

\[
\eta = y \sqrt{\frac{U_x}{v_{f,x}}} = V_f \sqrt{Re_x} \eta, \quad \psi = v_f \sqrt{Re_x} \psi(\eta), \quad \theta = \frac{T - T_w}{T_\infty - T_w} \quad (8)
\]

Here, $Re_x = \frac{U_x x}{v_f}$ is the Reynolds number.

Similarity variables (7) and (8), (1)–(3) are used to reduce the following nonlinear system of ordinary differential Equations:

\[
f'' + (1 - \phi)^2 \left[ 1 - \phi + \phi (\rho_{s} / \rho_{f}) \right] \frac{1}{2} f' + (1 - \phi)^2 \left[ 1 - \phi + \phi \left( \frac{\rho C_p}{\rho C_p} \right) \right] \frac{\left( \rho \beta \right)_f}{\left( \rho \beta \right)_s} \frac{Gr_x}{\eta} \theta
+ (1 - \phi)^2 \left( \frac{\mu_{nf}}{\mu_{nf}} \right) \frac{Pr}{2} \left[ 1 - \phi + \phi \left( \frac{\rho C_p}{\rho C_p} \right) \right] f \theta' = 0 \quad (9)
\]

When subjected to the boundary conditions (5), the Equations become:

\[
f'(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1
\]

\[
f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0, \quad \text{as} \quad \eta \rightarrow \infty \quad (11)
\]

Here, primes represent the differentiation with respect to $\eta$, $M = \sigma B_0^2 / \rho U_\infty$ is the magnetic parameter, $Pr = \left( \mu C_p \right)_f / \mu_f$ is the Prandtl number and $Gr_x = g \beta_f \left( T_w - T_\infty \right) x / U_\infty^2$ is the local Grashof number. The parameter $Gr_x$ must be constant to achieve a true similarity solution. This condition is satisfied if the thermal expansion coefficient $\beta_f$ is proportional to $x^{-1}$. Hence, on assuming\textsuperscript{12},

\[
\beta_f = ax^{-1} \quad (12)
\]

Where $a$ is constant with appropriate dimension. Substituting (12) into the parameter $Gr_x$ gives us:

\[
Gr = \frac{a g \left( T_w - T_\infty \right) x}{U_\infty^2} \quad (13)
\]

In this study, quantities of practical interest are the skin-friction coefficient, $C_f$, at the surface of the plate and the local Nusselt number, $Nu_x$ is defined as:

\[
C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{q_w}{k_f \left( T_w - T_\infty \right)} \quad (14)
\]

where $\tau_w$ represents the shear stress or wall skin friction at the plate and $q_w$ represents the plate’s heat flux, which is given by:

\[
\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad (15)
\]

Substituting (8) into (15) and using (14) gives us:

\[
\frac{C_f}{\left( Re_x \right)^{2.5}} = \frac{1}{(1 - \phi)^2.5} f'(0), \quad \frac{Nu_x}{(Re_x)^{2}} = \frac{k_{nf}}{k_f} \theta'(0) \quad (16)
\]

### 3. Results and Discussion

The Keller Box method was used to solve the above set of Equations (9)–(10) numerically exposed to the boundary conditions (11). To compute the local Nusselt number and the skin friction coefficient in (16), both velocity and temperature profiles were taken and utilized. We have considered two types of base fluids with ferroparticle of $Fe_3O_4$. Table 1 gives the thermophysical properties of kerosene, water and $Fe_3O_4$.

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Water</th>
<th>Kerosene</th>
<th>$Fe_3O_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho \left( kg / m^3 \right)$</td>
<td>997.1</td>
<td>780</td>
<td>5200</td>
</tr>
<tr>
<td>$C \left( J \ kgK \right)$</td>
<td>4179</td>
<td>2090</td>
<td>670</td>
</tr>
</tbody>
</table>

Table 1. Thermophysical properties of base fluids and ferroparticle\textsuperscript{15,16}
MHD Free Convection Flow and Heat Transfer of Ferrofluids over a Vertical Flat Plate with Aligned and Transverse Magnetic Field

For the base fluids, the values of Prandtl number of water and kerosene were taken to be 6.2 and 21, respectively. The impact of the solid ferroparticle volume fraction is studied in the range of $0 \leq \phi \leq 0.2$. Here, $\phi = 0$ indicates the pure fluid kerosene or water. Our investigation's results were found to concur with the studies available in the literature (Table 2).

Table 2. A comparison of the skin friction coefficient for different values of volume fraction of ferroparticle

<table>
<thead>
<tr>
<th>Volume Fraction</th>
<th>Khan et al. [9]</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Fe}_3\text{O}_4$-water</td>
<td>0.01</td>
<td>0.34324</td>
</tr>
<tr>
<td>0.1</td>
<td>0.45131</td>
<td>0.451324</td>
</tr>
<tr>
<td>0.2</td>
<td>0.59517</td>
<td>0.595192</td>
</tr>
<tr>
<td>$\text{Fe}_3\text{O}_4$-kerosene</td>
<td>0.01</td>
<td>0.34557</td>
</tr>
<tr>
<td>0.1</td>
<td>0.47336</td>
<td>0.473360</td>
</tr>
<tr>
<td>0.2</td>
<td>0.63950</td>
<td>0.639499</td>
</tr>
</tbody>
</table>

Figures 1 and 2 display the impacts of inclined magnetic field on temperature and velocity profiles. It was found that increasing the aligned angle improved the velocity profiles; however, the temperature profiles for both ferrofluids were found to decrease. When $\alpha = \pi/2$, the aligned magnetic field behaved like transverse magnetic field and the magnetic field attracted ferroparticles. Figures 3 and 4 present the decreasing on the momentum and thermal boundary layer thickness by increasing the magnetic field strength. In general, the increase in magnetic field leads to automatic arranging of ferroparticles in a defined order.

Figures 5 and 6 present the effect of volume fraction of ferroparticles on temperature and velocity profiles for both ferrofluids, i.e. $\text{Fe}_3\text{O}_4$-kerosene and $\text{Fe}_3\text{O}_4$-water. Momentum boundary layer thickness increases with an increase in the volume fraction of ferroparticles, while an increase in the volume fraction led to an increase in the plate surface temperature and thermal boundary layer.

Figures 7 and 8 illustrate the boundary layer velocity and temperature variation with Grashof number ($Gr_x$). Increase in Grashof number ($Gr_x$) parameter increased the velocity and consequently decreased the boundary layer’s thickness due to buoyancy forces. Grashof number ($Gr_x$) increment value resulted in decay of ferrofluids’ thermal boundary layer thickness.

Tables 3 and 4 show the effect of non-dimensional governing parameters on local Nusselt number and local skin friction for $\text{Fe}_3\text{O}_4$-kerosene and $\text{Fe}_3\text{O}_4$-water ferrofluids respectively. It is to be noted that the increase in local skin friction coupled with local Nusselt number at the plate surface increased the magnetic field parameter ($M$), aligned angle ($\alpha$), Grashof Number ($Gr_x$) and volume fraction of ferroparticle.

$\begin{array}{|c|c|c|}
\hline
k (W/ mK) & 0.613 & 0.149 & 6 \\
\hline
\beta (K^{-1}) & 21 & 99 & 1.3 \\
\hline
\end{array}$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Effect of inclined angle, $\alpha$ on the velocity profiles for $M = 1, \phi = 0.1, Gr_x = 0.1$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Effect of inclined angle, $\alpha$ on the temperature profiles for $M = 1, \phi = 0.1, Gr_x = 0.1$.}
\end{figure}
Figure 3. Effect of magnetic field parameter, $M$ on the velocity profiles for $\alpha = \pi / 2, \phi = 0.1, Gr_x = 0.1$.

Figure 4. Effect of magnetic field parameter, $M$ on the temperature profiles for $\alpha = \pi / 2, \phi = 0.1, Gr_x = 0.1$.

Figure 5. Effect of volume fraction, $f$ on the velocity profiles for $\alpha = \pi / 2, M = 1, Gr_x = 0.1$.

Figure 6. Effect of volume fraction, $f$ on the temperature profiles for $\alpha = \pi / 2, M = 1, Gr_x = 0.1$.

Figure 7. Effect of Grashof number, $Gr_x$ on the velocity profiles for $\alpha = \pi / 2, M = 1, \phi = 0.1$.

Figure 8. Effect of Grashof number, $Gr_x$ on the temperature profiles for $\alpha = \pi / 2, M = 1, \phi = 0.1$. 
Table 3. Variation in skin friction coefficient and Nusselt number for Fe$_3$O$_4$-water ferrofluids at different non-dimensional parameters

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\phi$</th>
<th>M</th>
<th>$Gr_x$</th>
<th>$C_f\left(Re_x\right)^{\frac{1}{2}}$</th>
<th>$Nu_x\left(Re_x\right)^{-\frac{1}{2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
<td>0.554242</td>
<td>0.752440</td>
</tr>
<tr>
<td>45°</td>
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<td>1</td>
<td>0.1</td>
<td>0.982291</td>
<td>0.868373</td>
</tr>
<tr>
<td>70°</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
<td>1.212317</td>
<td>0.914855</td>
</tr>
<tr>
<td>90°</td>
<td>0.01</td>
<td>1</td>
<td>0.1</td>
<td>1.105335</td>
<td>0.842419</td>
</tr>
<tr>
<td>90°</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
<td>1.274275</td>
<td>0.926087</td>
</tr>
<tr>
<td>90°</td>
<td>0.2</td>
<td>1</td>
<td>0.1</td>
<td>1.517503</td>
<td>1.018429</td>
</tr>
<tr>
<td>90°</td>
<td>0.1</td>
<td>1</td>
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<td>1.274275</td>
<td>0.926087</td>
</tr>
<tr>
<td>90°</td>
<td>0.1</td>
<td>2</td>
<td>0.1</td>
<td>1.714156</td>
<td>0.994113</td>
</tr>
<tr>
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<td>3</td>
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<tr>
<td>90°</td>
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<td>0.5</td>
<td>1</td>
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</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1.793629</td>
<td>1.005371</td>
</tr>
</tbody>
</table>

Table 4. Variation in skin friction coefficient and Nusselt number at different non-dimensional parameters for Fe$_3$O$_4$-kerosene ferrofluids

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\phi$</th>
<th>M</th>
<th>$Gr_x$</th>
<th>$C_f\left(Re_x\right)^{\frac{1}{2}}$</th>
<th>$Nu_x\left(Re_x\right)^{-\frac{1}{2}}$</th>
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</thead>
<tbody>
<tr>
<td>0°</td>
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<td>1</td>
<td>0.1</td>
<td>0.541970</td>
<td>1.225903</td>
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<tr>
<td>70°</td>
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</tr>
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</tr>
<tr>
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</tr>
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<td>0.5</td>
<td>1.432538</td>
<td>1.595729</td>
</tr>
<tr>
<td>90°</td>
<td>0.1</td>
<td>1</td>
<td>1</td>
<td>1.634822</td>
<td>1.646262</td>
</tr>
</tbody>
</table>

4. Conclusion

The paper was based on the numerical study of natural boundary layer flow of MHD ferrofluids over a flat plate with aligned magnetic field. A similarity transformation was employed for the governing partial differential equations, which were modified into ordinary differential equations and then the Keller box method was used to solve numerically. Fe$_3$O$_4$-water and Fe$_3$O$_4$-kerosene are the two types of water-based ferrofluids considered. The heat transfer rate at the plate surface was found to increase with increasing angle of magnetic field, magnetic field parameter, ferroparticle volume fraction and local Grashof number. In addition, the heat transfer rate at the plate surface was found to be higher with Fe$_3$O$_4$-kerosene ferrofluids than with Fe$_3$O$_4$-water.

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6. References