Design and Analysis of Lyapunov Function based Controller for DC-DC Boost Converter

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Abstract

Objectives: To design a control system for DC-DC boost converter based on lyapunov stability theorem that fulfills all control purposes. Methods/Statistical Analysis: A DC-DC boost converter is controlled using switching function extraction method based on lyapunov stability theorem. Switching function extraction is a proper method to control switching devices such as DC-DC converters. At first state-equations of the system are obtained from large signal averaged model of the system. After that, fundamentals of lyapunov theorem are applied to state equations that lead to the calculation of switching function by which the converter is controlled. Findings: Using the presented method in this paper based on lyapunov theorem makes the converter system remain stable in all operational conditions. Also, it is shown that using proposed system for controlling DC-DC converter; state variable tracks its reference with an acceptable response. The current controller is also compared with conventional PI controller and it is observed that lyapunov based controller operates better in many ways. By using this controller stability of the system is guaranteed and when there is a sudden change in the reference of state variable, the state variable tracks down and corresponds with the new value. Application/Improvements: This controller can be used in all kinds of DC-DC, DC-AC and other kinds of converters. It is also applicable to all switching devices such as switching power sources.

Keywords: Converter, Control, DC-DC, Lyapunov, Stability

1. Introduction

Due to energy conversion characteristics and power transfer capabilities of power converters, they have an important role in electrical power circuits. In recent years, with renewable energy sources introduced into the industry, renewable energy conversion technology finds an important role, and therefore application of DC-DC converters becomes more important and a lot of works have been carried out on the control of these converters.

Different control methods have been used in previous works for DC-DC converters. For example PI controllers have been applied in some papers or some papers have used sliding mode controllers to control power converters. Furthermore, model predictive controllers, adaptive controllers, robust controllers and digital controllers have been used.

In this paper a DC-DC boost converter is used in order maintain an output voltage with a constant value. The control of this converter is based on lyapunov stability function. The advantage of lyapunov method is not only fast transient responses to sudden changes, but also it guarantees that system stays stable in all operating conditions.

The paper is organized as follows: In section 2, DC-DC boost converter schematic model is presented and its large signal averaged model is obtained. In section 3, circuit equations are calculated using averaged model and then state-space equations are generated, that indicate the behavior of the system under different conditions. After that, switching functions are determined based on lyapunov stability theorem, by which control of the system is carried out. Switching functions extraction is an appropriate method to control the switching devices. For this purpose, switching functions are extracted based on circuit equations such as state-space equations. Also, energy management system is proposed in order to prevent the battery from overcharging. In section 4,
simulation of a DC-DC boost converter with lyapunov function based control is carried out and results are presented.

2. Power Electronic and Averaged Model of the System

Figure 1(a) shows the electrical power circuit presented in this paper. The circuit consists of a DC-DC buck converter along with a voltage source as input and battery as output. The main purpose of the system is to control the battery's charging current and prevent the battery from overcharging.

The large signal averaged model of this circuit is shown in Figure 1(b). The averaged model is a common and proper method for analysis power electronic converter circuits which leads to a simpler equivalent circuit and also reduction of circuit complexities. The $u$ parameter is duty cycle of the converter and output of a PWM circuit, which is originally calculated in Equation (1), as shown in Figure 1. Changes in $u$ determine the voltages and currents of the system.

$$u = \frac{t_{on}}{t_{off} + t_{on}} \quad (1)$$

Whenever $u$ is 1 or a value close to 1, switch is on and current flows through it, but state 0 of $u$ will make the switch off.

3. Control System Strategy

Controller design is based on two steps. The first is to generate the state-space equations of the system based on circuit's equations. After that, switching functions are extracted using principals of lyapunov stability function, by which system's switching is controlled.

3.1 State-Space Equations of Boost Converter

To obtain the equations of the system, KVL and KCL are applied to the circuit shown in Figure 1. In this case, output voltage ($V_C$) and inductor's current ($I_L$) are state variables of the system. Using KVL law in left mesh of the circuit, the first equation is obtained as:

$$-V_{in} + L \frac{dI_L}{dt} + (1-u)V_C = 0 \quad (2)$$

Equation (3) shows the time derivation of Equation (2) based on state variable after mathematical manipulation:

$$\frac{dI_L}{dt} = -\frac{V_C}{L} + \frac{V_{in}}{L} \quad (3)$$

Also using KCL law in right node of the circuit, the second equation is obtained as:

$$(1-u)I_L - C \frac{dV_C}{dt} - \frac{V_C}{R} = 0 \quad (4)$$

This leads to Equation (5) after mathematical manipulation:

$$\frac{dV_C}{dt} = \frac{(1-u)I_L}{C} = \frac{V_C}{RC} \quad (5)$$

From Equation (3) and (5), state variables matrix can be defined as:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} I_L - I_L^* \\ V_C - V_C^* \end{bmatrix} \quad (6)$$

Where $I_L^*$ is the reference output current and $V_C^*$ is the reference of capacitor's voltage. Combining Equation (3), (5) and (6), state-space equations are calculated as:

$$\begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} 0 & \frac{1-u}{L} \\ \frac{1-u}{C} & \frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} \quad (7)$$

Equation (7) shows the state-space matrix of the system, originally shown in Equation (8):
\[
\begin{bmatrix}
\dot{x}
\end{bmatrix} = [A][x] + [B]
\]  

Where matrices A and B are state space and input matrices respectively. According to Equation (6) and (7), matrix B is defined as:

\[
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} = 
\begin{bmatrix}
0 & \frac{1-u}{L} & \frac{V_m - \frac{dI_L}{dt} \ast}{L} \\
\frac{1-u}{C} & \frac{1}{RC} & \frac{dV_c}{dt} \ast
\end{bmatrix}
\]  

According to above equations, DC-DC boost converter is a non-linear and time-variant system. Switching function \(u\) is able to control the output current and capacitor's voltage. The switching function \(u\) can be obtained by classic control as well as modern methods. In this paper \(u\) is generated using lyapunov stability function.

### 4. Switching Function Extraction using Lyapunov Stability Function

Control strategy in this paper is based on the lyapunov stability function. According to lyapunov theorem, a non-linear time-variant system is globally and uniformly stable if satisfies the following conditions:

\[
\begin{align*}
V(0) &= 0 \\
\alpha(||x||) &< V(x) < \beta(||x||) \\
\dot{V}(x) &< -\gamma(||x||) \\
\lim_{t \to \infty} \|x\| &\to \infty
\end{align*}
\]  

Where, \(x\) is state variable and \(V\) is lyapunov function.

In this paper lyapunov function is defined as the summation of the state variables squares:

\[
V = \frac{1}{2}Lx_1^2 + \frac{1}{2}Cx_2^2
\]  

\(V\) can also be written as:

\[
V = X^T PX
\]  

Therefore the following condition is stated:

\[
\lambda_{\min}(P) \|x\| \leq V \leq \lambda_{\max}(P) \|x\|
\]  

In which \(\lambda_{\min}\) and \(\lambda_{\max}\) are minimum and maximum eigen values of \(P\) matrix respectively. Merging Equation (12) and inequality (13):

\[
X^T PX \leq \lambda_{\max}(P) \|x\|
\]  

Multiplying both sides of (21) with \(-\frac{1}{\lambda_{\max}}\):

\[
-\|x\| \leq -\frac{1}{\lambda_{\max}} X^T PX
\]  

To analyze the condition of (10) for \(\dot{V}\) term, the function \(-\gamma(||x||)\) is defined as:

\[
-\gamma(||x||) = -k \|x\|
\]  

In which \(k\) is control matrix and must be defined in a way to satisfy \(\dot{V}\) condition. Furthermore control parameters are defined as:

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix}
\frac{1}{2\lambda_{\max}} \\
\frac{Lk_1}{Ck_2}
\end{bmatrix}
\]  

Equation (18) satisfies the first, second and fourth conditions of potential function of (10). According to (10), (13) and (15), the DC-DC boost converter is globally and uniformly stable if the derivative of function \(V\) satisfies following equation:

\[
\dot{V} = Lx_1x_1 + Cx_2x_2 < \alpha x_1^2 + \beta x_2^2
\]  

Substituting Equation (7) and (9) into Equation (18), lyapunov function's derivative form is simplified as:

\[
\begin{align*}
\dot{V} &= Lx_1(\frac{1-u}{L} x_2 - \frac{1-u}{L} V_c \ast + \frac{V_m}{L}) \\
&+ Cx_2(\frac{1-u}{C} x_1 - \frac{1}{RC} x_2 + \frac{1-u}{C} I_L \ast) \\
&- \frac{1}{RC} V_c \ast < \alpha x_1^2 + \beta x_2^2
\end{align*}
\]  

Since the values of \(\frac{dI_L}{dt}\ast\) and \(\frac{dV_c}{dt}\ast\) (in Equation (9)) are zero in steady state, they are neglected in Equation (19):

\[
\begin{align*}
\dot{V} &= -(1-u)x_1x_1 - (1-u)x V_c \ast + V_m x_1 \\
&+(1-u)x_2x_2 - \frac{1}{R} x_2 x_2 + (1-u)x_2 I_L \ast \\
&- \frac{1}{R} x_2 V_c \ast < \alpha x_1^2 + \beta x_2^2
\end{align*}
\]  

It leads to:

\[
\dot{V} = (1-u)(-x V_c \ast + x_2 I_L \ast) \\
+ V_m x_1 - \frac{1}{R} x_2 - \frac{1}{R} x_2 V_c \ast \\
- \alpha x_1^2 - \beta x_2^2 < 0
\]
Since the term is $-\frac{1}{R}x_2^2$ always negative, it can be neglected. Furthermore, $u$ must be calculated in such a way that left side of Equation (28) stays negative and system remains stable:

$$u = 1 - \frac{-V_i x_1 + \frac{1}{R} x_2 V_c^* + \alpha x_1^2 + \beta x_2^2}{-x_2 V_c^* + x_2 I_L^*}$$ (22)

Equation (22) is system's switching function in which $\alpha$ and $\beta$ are control parameters and calculated in Equation (17). The values of $k$ in Equation (17) define the controller's performance. Figure 2 shows the block diagram of the complete system.

Figure 2. Block diagram of proposed system.

5. Simulation

The performances of the developed nonlinear controller will now be illustrated using numerical simulations in Matlab/Simulink. System's parameters are presented in Table 1:

<table>
<thead>
<tr>
<th>Parameter's Value</th>
<th>Parameter's Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{in}=15$ V</td>
<td>Input Voltage</td>
</tr>
<tr>
<td>$L=500$ mH</td>
<td>Inductor</td>
</tr>
<tr>
<td>$C=47$ uF</td>
<td>Capacitor</td>
</tr>
<tr>
<td>$R=10$ kΩ</td>
<td>Output Resistance</td>
</tr>
<tr>
<td>$F_{sw}=30$ kHz</td>
<td>Switching Frequency</td>
</tr>
</tbody>
</table>

As it was mentioned, the main purpose of the proposed control system is to maintain the output voltage in desired value that is 50 V in here. The speed of this voltage to reach the reference value depends on control parameters and output current reference. Figure 3 shows output voltage and current for references 50 V and 0.1 A respectively in no-load condition. It can be seen that output voltage reaches the preset reference in about 10 ms and is maintained constantly.

Figure 3. (a) Output voltage and (b) Output current in non-load condition.

Figure 4 shows the simulation results when there is a resistive load in the system. The output voltage reaches the reference value and then is consumed in the resistor and drops down. In this case, controller is required to compensate consumed voltage and bring the output voltage to its reference again. From Figure 4 A is the moment in which load consumes the output voltage and B is the moment in which controller compensates the consumed voltage.

Since the method used to control the converter is based on lyapunov stability function, controller must be able to guarantee the system's stability under variation in input voltage. Figure 5 shows the condition in which $V_{in}$ suddenly changes from 15 V to 25 V. As observed, in this period the variation in output voltage is limited to 0.3 V that proves the system has an acceptable response over the
input variations and instabilities. It can also be seen that when the input voltage returns to its initial value, output voltage is set on reference value with a small difference.

Extracted parameters from the proposed controller are seen in Table 2. As observed, these values are acceptable in general for a DC-DC converter and it can be concluded that all the control purposes are achieved.

Table 2. Parameters of proposed control system

<table>
<thead>
<tr>
<th>Parameter's value</th>
<th>Parameter's name</th>
</tr>
</thead>
<tbody>
<tr>
<td>85 ms</td>
<td>Rise Time</td>
</tr>
<tr>
<td>110 ms</td>
<td>Settling Time</td>
</tr>
<tr>
<td>0.505 %</td>
<td>Overshoot</td>
</tr>
<tr>
<td>2.2 %</td>
<td>Undershoot</td>
</tr>
<tr>
<td>0.248 V</td>
<td>Steady State Error</td>
</tr>
</tbody>
</table>

In order to analyze the controller's performance more precisely, it is compared with a conventional PI controller as observed in Figure 6. It can be seen that proposed controller has a better steady state error, meaning that proposed controller works better under conditions in which system must work for a longer period. Output voltage in proposed system also reaches the reference value faster than PI controller and proposed controller has a better overshoot compared to PI controller. This implies that proposed system has a faster response in comparison with PI controllers.

Figure 4. (a) Output voltage and (b) Output voltage in loaded condition.

Figure 5. (a) Output current and (b) Output voltage when there is a sudden change in the input voltage.

Figure 6. Comparison of proposed controller with PI controller.

6. Conclusion

According to the simulation results, it can be concluded that the proposed control system is a proper method for DC-DC boost converters. Using lyapunov stability theorem for extracting switching functions makes sure that state variables track down their references in
a short time with an appropriate speed. In addition, using lyapunov function ensures that system's stability is maintained in all operating conditions. The proposed controller works better in comparison with conventional PI controllers and can achieve an acceptable response when there is variation in the input voltage.

7. References


