Comparison of the Performance of Hexagonal Grid and Half-cuboctahedron Grid Tensegrity Systems in Roof Structures

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Abstract

Objective: To study and compare the behaviour of half-cuboctahedron grid and hexagonal grid tensegrity system. Methods/Statistical Analysis: Ever since the discovery of tensegrity, most works have been concentrated on the classification and form finding techniques but very less on the mechanics of these structures. This study focuses on the behaviour of two basic tensegrity modules, the half-cuboctahedron and hexagonal. Findings: Later, the study has been extended to tensegrity grids of spans 2x2, 4x4 and 8x8 of both the configurations (hexagonal modules and half-cuboctahedron modules) which were developed using an FEM based software. The grids were compared on the basis of nodal displacements and member forces, resulting in the half-cuboctahedron grid to be a more feasible configuration for large span roof structures. Applications/Improvements: The application of the tensegrity structure is to applied in the large span roof structures.

Keywords: Form Finding, Half-cuboctahedron, Hexagonal, Member Forces, Nodal Displacements, Tensegrity

1. Introduction

“Islands of compression in an ocean of tension” is how R.B. Fuller describes tensegrity systems. Tensegrity structures can be defined as a pattern that results when the ‘push’ provided by struts and the pull provided by tendons achieve a win-win relationship with each other.

Topology of the tensegrity structure provided by connectivity matrix as initial parameters. This paper illustrates the model of ‘S’ type load cell before optimization and finally obtained optimized model. The configurations generate maximum directive beam with reduced side lobe level. Two optimization techniques were used, particle swarm optimization and genetic algorithm. Both the arrays are evaluated in terms of efficient null placing and side lobe levels reduction using the algorithms taken into consideration. Pull is continuous whereas push is discontinuous. The continuous pull is balanced by the discontinuous push, producing the integrity of tension and compression. Two tensegrity are easily recognizable in the systems of the human body. The muscular-skeletal system is a tensegrity of muscles and bones, the muscles provide continuous pull, the bones discontinuous push. This forms the basis for all human physical mobility. The central nervous system can also be seen as using the analogy of tensegrity where motor neurons and sensor neurons, complement each other in a balance.

Tensegrity structures are perfect candidates for deployable structures as they can change their shape and size from a compact state to the prestressed service state. This work concentrates on the behaviour of deployable tensegrity grids. The struts in a tensegrity structure are attached to each other through ball joints and not through the joints that impart torque. Very few works have been reported on the mechanics of these structures. Then conducted a study on the double layered tensegrity grids consisting of triangular prisms. And also conducted experimental investigations under static load on a three-unit span DLTG consisting of seven triangular prismatic units. Some researcher conducted studies on

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n-strut tensegrity\textsuperscript{2}. And found the design equations of deployable n-strut tensegrity systems\textsuperscript{10} and conducted the study on the development of deployable structures, based on the tensegrity concept, for applications in space\textsuperscript{11} conducted a detailed study on general methods for creating tensegrity structures\textsuperscript{12} conducted the experimental and analytical study on half cuboctahedron module DLTG\textsuperscript{13}, introduced a new technique for development of tensegrity structures and their deployment in field\textsuperscript{14} and some researchers conducted a detailed study on the investigation on a multistable tensegrity structure.

The behaviour of two basic deployable tensegrity configurations, the hexagonal and the half-cuboctahedron has been concentrated in this work. Both the modules were developed and analysed using an FEM based software based on the nodal displacements and the member forces. The study was then extended to tensegrity grids for roof structures of spans 2x2, 4x4 and 8x8 for both the configurations.

2. Description of the Modules

The two basic tensegrity modules, the hexagonal configuration and the half-cuboctahedron modules have both been designed using the design equations developed by Ian Stern\textsuperscript{9}. As per the design, the half-cuboctahedron module consists of 12 cables and 4 struts. This 4 strut tensegrity module attains a stable shape when the top platform is rotated by π/2 radians with respect to the bottom layer. At this state the pre stressing forces in the members can be calculated using the equations,

\[ F_s = \frac{\sqrt{2} L_s}{a} F_b \]
\[ F_i = \frac{\sqrt{2} L_i}{a} F_b \]
\[ F_a = \frac{b}{a} F_b \]

The length of the respective members (cables and struts) can also be calculated from the following design equations,

\[ L_s = \frac{\sqrt{2}}{2} \sqrt{b^2 - \sqrt{2}ab + a^2 + 2h^2} \]

where, \( F_s \) is the force in the top cables, \( F_b \) force in the bottom cables, \( F_i \) force in the leg ties and \( F_a \) force in the struts. \( a \) is the length of top cables, \( b \) the length of the bottom cables, \( L_s \) the length of leg ties and \( L_i \) the length of struts respectively.

Similarly, the hexagonal module consists of 6 struts and 18 cables (6 on the top platform, 6 on the bottom platform and 6 as leg ties). The 6-strut tensegrity module also attains its stable position when the top platform is rotated by π/2 radians with respect to the bottom platform. The pre stressing forces in the members and the lengths of the respective members in the module at the stable position can be calculated from the following design equations,

\[ F_s = \frac{L_s}{a} F_b \]
\[ F_i = \frac{L_i}{a} F_b \]
\[ F_a = \frac{b}{a} F_b \]
\[ L_s = \sqrt{b^2 + ab + a^2 + h^2} \]
\[ L_i = \sqrt{b^2 - ab + a^2 + h^2} \]

where, \( F_s \) is the force in the top cables, \( F_b \) force in the bottom cables, \( F_i \) force in the leg ties and \( F_a \) force in the struts. \( a \) is the length of top cables, \( b \) the length of the bottom cables, \( L_s \) the length of leg ties and \( L_i \) the length of struts respectively. The prototype configuration of the half-cuboctahedron and hexagonal tensegrity modules are shown in Figures 1 and 2 respectively.

![Half-cuboctahedron module](image1)

Figure 1. Half-cuboctahedron module.
2. Analysis

The basic tensegrity modules, the half-cuboctahedron and hexagonal configurations has been developed and analysed using a finite element package. Then study has been then extended to tensegrity grids developed by agglomerations of the basic tensegrity modules. In this study the compression members (struts) have been assigned the property of galvanised iron pipes of medium type conforming to the Indian Standard IS 1239-I (1990) and a 2.8 mm nominal diameter mild steel stranded wires of 6x19, confirming to IS 3459 (1977), were used as the tensile members.

2.1 Basic Tensegrity Modules

The tensegrity modules were designed as per Stern's design equations mentioned earlier. The basic tensegrity module consists of 4 bottom cables of 1m lengths and 4 top cables and 4 leg ties of lengths 0.707m lengths and 4 struts of lengths 1.224m. Similarly, the hexagonal configuration tensegrity module consists of 6 bottom cables of lengths 0.62m, 6 top cables of lengths 0.31m and 6 leg ties of lengths 0.733m. The 6 struts were designed for lengths of 0.96 m each. Assuming a pre stressing force of 1.5kN on the top cables the pre stressing forces in the other members were calculated respectively for both the configurations, using the relationship between the internal forces mentioned earlier. Figures 3 and 4 shows the plan view of the half-cuboctahedron and hexagonal configurations respectively.

2.2 Tensegrity Grids

The tensegrity grids were developed by agglomerations of the basic tensegrity modules in both the configurations. Additional cables were provided in the top layer of hexagonal configuration to ensure the stability of the structure. The grids were developed and analysed under the application of external forces varying from 250N to 2500N uniformly being applied on the top nodes. All the degrees of freedom has been locked for the bottom central nodes whereas, for the corner nodes all three degrees of translations has been locked. The larger span tensegrity grids of 4x4 and 8x8 grids were also developed and analysed based on the 2x2 grid. Figure 5(a, b) shows the 3D view of the 2x2 grids developed in SAP2000.
3. Results and Discussions

The single modules and tensegrity grids were analysed under different static loads to study the behaviour of the structures and the performances have been compared to suggest a feasible configuration for large span roof structures.

3.1 Basic Tensegrity Modules

The single module tensegrity systems have been analysed of loads varying from 250N to 2500N. Figure 6 shows a comparison of the nodal displacements of both the configurations. A larger displacement is observed on application of loads higher than 400N. Figures 7 and 8 shows the comparison of the cable forces and strut forces respectively. Not much variation is observed in the case of cable forces but the hexagonal module exhibits a lower value incase of the strut forces. Hence, a better performance is observed in case of the hexagonal tensegrity module as compared to the half-cuboctahedron tensegrity module.

3.2 Tensegrity Grids

The tensegrity grids of both configurations have been analysed under loads varying from 200N to 2000N. Figure 9 shows the comparison of the maximum nodal displacements observed in the 2x2 tensegrity grids. The nodal displacements of the hexagonal tensegrity grid shows a larger variation as compared to the half-cuboctahedron tensegrity grid. The permissible nodal displacement for a tensegrity grid is L/250, as per research values. Hence, the permissible displacement of 8mm incase of the 2x2 tensegrity grid is obtained at 750N in half-cuboctahedron tensegrity grid and 600N in hexagonal tensegrity grid respectively.

Figures 10 and 11 shows the comparison of the cable forces and the strut forces respectively. It can be observed from both the cases that the half-cuboctahedron tensegrity grid shows a much lower value as compared to the hexagonal tensegrity grid.

Figure 6. Comparison of the maximum nodal displacements.

Figure 7. Comparison of the maximum cable forces.

Figure 8. Comparison of the strut forces.

Figure 9. Comparison of the maximum nodal displacements.

Figure 10. Comparison of the maximum cable forces.

Figure 11. Comparison of the strut forces.
For larger span tensegrity grids of 4x4 and 8x8 grids, the hexagonal grid shows a lower nodal displacement up to 400N as compared to half-cuboctahedron tensegrity grids. But on further increase in the nodal loads a better performance is observed in case of half-cuboctahedron tensegrity grid. The permissible nodal displacement of 16mm (L/250) in half-cuboctahedron is observed at 1107N and in hexagonal grid is observed at 900N for 4x4 tensegrity grids. Whereas, for 8x8 tensegrity grids the permissible nodal displacement of 32mm is observed at a load of 1950N in half-cuboctahedron grids and 1900N in hexagonal grids. The member forces of the large span tensegrity grids shows a similar performance as that of the 2x2 tensegrity grids but both the configurations shows a slightly better performance in case of the 8x8 tensegrity grids hence, proving that the tensegrity grids are more feasible for large span structures. Figures 12 and 13 shows the comparison of 4x4 and 8x8 tensegrity grids.

Figure 12. Comparison of the maximum nodal displacements of 4x4 tensegrity grids.

Figure 13. Comparison of the maximum nodal displacements of 8x8 tensegrity grids.

4. Conclusion

This work mainly highlights on the comparison of the design and analysis of two tensegrity configurations i.e. the half-cuboctahedron and hexagonal tensegrity systems to suggest the best configuration for the chosen scenario. The performance of both the configurations have been analysed for single modules and 2x2, 4x4 and 8x8 tensegrity grids under external loads varying from 200N to 2500N. It could be observed that in case of a single module tensegrity system the hexagonal module shows a better performance but at the same time the half-cuboctahedron grid tensegrity system shows a better performance in case of tensegrity grid structures. It was also observed that the tensegrity grids are more feasible for large span structures. Wind analysis and modifications of joints are interests of further works.

5. References