Dynamic Modelling and Control Design for a Vehicle in its Longitudinal Motion

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Abstract

Background/Objectives: The purpose of developing a model based control study is to understand the longitudinal dynamics in detail. This paper develops mathematical model of longitudinal dynamics of the vehicle and develops a suitable control methodology to have a reliable safe system. Methods/Statistical Analysis: Here the longitudinal dynamics of the system is modeled using white box modeling and suitable PID control algorithm is developed to have a stable dynamics of the systems. To develop the required model Physical laws of equations are used. Findings: For a tuned PID controller, the system stayed within the set point of acceleration and the change in throttle has not affected the system's velocity and acceleration. Applications/Improvements: This model can be imparted in the driverless car models for complete analysis. Modern control methodologies can be developed for the developed model to have a better performance.

Keywords: Longitudinal Dynamics, Longitudinal Motion, Mathematical Model, Physical Laws of Equations, PID Control, White Box Modeling

1. Introduction

Research in the field of Advanced Vehicle Control System (AVCS) has increased in large numbers due to the development of Intelligent Transportation Systems (ITS) in last decade. Integration of modern model-based automatic control systems is an important research area. Model based automatic control system describes the dynamics behavior of an automobile and accesses the unknown parameters. Vehicles with close spacing require a high performance based longitudinal controller. Safety of the passengers is the prime motive in developing any longitudinal controller. As safety is the important concern for designing the controller, uniform spacing should be ensured during the whole duration of driving. In this paper we are focusing on a single vehicle using the conventional control algorithm. Jullierme Emiliano presents the longitudinal model identification which benefits to design the longitudinal vehicle model and control the velocity using PI controller. In general, studies on the design of autonomous vehicle involves two types of control,

- The steering control (lane keeping)
- The longitudinal control

This paper exclusively deals with the longitudinal control of the vehicle. The proposed technique will reduce the space between vehicles and also avoids accidents due to frontal collision. In this paper we use the conventional controller (PID) to control the velocity of the vehicle in order to avoid collision and accidents.

This paper is categorized as follows. In section 2 we describe the approach towards the modeling of the required vehicle model. After analyzing the vehicle model, in section 3 we design the control algorithm and test in the developed vehicle model. We discuss the experimental results in section 4 and finally section 5 concludes the paper with suggestions to be considered for future research.
2. Modeling

The proposed system is designed in order to regulate the vehicle speed so as to avoid collision and accident. The system is used to follow the command based on the feedback signal. The free body diagram of the Car is shown in Figure1. The equation from this free body diagram is represented in equation 1. The vehicle has different forces such as Aerodynamics force, Braking force, Viscous Force or frictional force and engine force. Except the engine dynamic force all the forces opposes the movement of the vehicle that means all other forces are opposite in direction to that of the engine force. So in order to move the vehicle the engine force should be much higher in comparison to all other forces shown in free body diagram.

As we are concern about the design of the controller so the modeling of the vehicle will not take principal role. Our aim is to design the controller in order to fulfill the required task, so by considering the system from and adapting certain derivation, we can realize the simple linear transfer function model for designing the controller. Frank describes Longitudinal Control for Automated Automobiles and Trucks Operating on a Cooperative Highway, presents adaptive control for interconnected nonlinear dynamics system. documents a smooth Intelligent Cruise Control (ICC) structure. Reader’s wants a depth study on the model can refer to which deals with the different types and methods of modeling technique. The longitudinal vehicle model by using different physical equations. The longitudinal dynamics of the vehicle model using Simulink platform solves the problem in automated highway by designing the vehicle model and the controller.

Figure 1. Various forces acting on a vehicle.

The longitudinal dynamics of a vehicle is designed by referring the Newton's law of motion. As per the consideration of Newton's law of motion the force being exerted to the vehicle is the summation of different individual forces such as Viscous force \( F_{visc} \), Aerodynamic force \( F_{aero} \), Braking force \( F_{brake} \) and Climbing force or the Downgrade force \( F_{grav} \). This force is also called as gravitational force. These forces plays a role of deciding factor for calculating the force exerted in the vehicle to move the vehicle or for the motion of the vehicle. The mathematical equation to explain the law of motion is given in equation 1.

\[
\dot{z}(mV) = F_{visc} + F_{aero} + F_{brake} + F_{grav} + F_{engine} \tag{1}
\]

From equation 1 'M' represents the mass of the vehicle and 'V' represents the longitudinal velocity. The force equation is posturized in the free body diagram in Figure 1. Aerodynamics force, viscous force and braking force as always act on the opposite direction of the movement of the vehicle. To design the required system assumption has to be made for a flat and leveled surface and the brake condition has been eliminated. For the above condition the force related to gravitational, brake, and aerodynamics are neglected. Now the engine force \( F_{engine} \) is the only force used to run the car. Which affects the motion of the vehicle. The transmission system consists of gear box, clutch, speed and torque conversions etc. The transmission system of a vehicle is a very complicated system. For driving the wheels the transmission system plays an important role by relating the output. So it is a matter of challenge to design the mathematical model of the transmission system.

In this paper simplification proposed by Jullierme Emiliano and Khaireuddin Osman is adapted by taking in to account to the difficulties faced in transmission system. Jullierme tells the use of the first order linear system. The system consists of a time constant 'T', static gain 'K', and a time delay '\\( \tau \)'. The input of the system also has been limited to a certain range. The throttle signal which correlate with the model's input has been restricted to the sluggish speed of the vehicle and to the maximum value which is most appropriate for the safety purpose. Analyzing equation 1 in frequency domain the longitudinal dynamics of the car has been presented in the block diagram in Figure 1. Now simplifying the block diagram shown in Figure2, the transfer function of the system is presented in equation 2.

\[
G(s) = \frac{v(s)}{u(s)} = \frac{k e^{-\tau s}}{mT} \frac{1}{(s + \frac{1}{m})(s + B \frac{1}{T})} \tag{2}
\]
where \(v(s)\) and \(u(s)\) are the output and the input of the system respectively. \(V(s)\) is the velocity of the vehicle and \(u(s)\) is the throttle input to the system. Again considering the symbols represent in the study that is ‘\(m\)’ shows the mass of the vehicle and the passengers. The above transfer function is the final transfer function of the system. Different parameter values are adopted from the research done by frank. These parameters constant values are displayed in Table.1. After getting the simplified model the controller design has been started. The vehicle longitudinal dynamics obtained by Khairuddin Osman is shown in Figure 2. Now assume all the initial condition to be zero. Applying the initial conditions that are zero wind gusts and no grading the model is converted to only the forward path gain and the feedback as shown in Figure 2. Now the output state equations are represented in equation 3 and equation 4.

\[
\frac{d}{dt}v' = \frac{1}{m}(F_d - c_g v^2) \tag{3}
\]

\[
F_d = \frac{1}{T}(c_i u(t - T) - F_d) \tag{4}
\]

\[Y = v \tag{5}\]

Due to the square term present in equation 3 nonlinearity arises, to make this nonlinear equation into linear, we have to differentiate each terms. After differentiating the equations it becomes as represented in equation 6 and equation 7.

\[
\frac{d}{dt}v' = \frac{1}{m}(-2c_g v' v + \delta F_d) \tag{6}
\]

\[
\frac{d}{dt}F_d = \frac{1}{T}(c_g \delta u(t - T) - F_d) \tag{7}
\]

And the output equation becomes

\[Y = \delta v \tag{8}\]

Now the transfer functions we get by solving these equations are

\[
\frac{v(s)}{u(s)} = \frac{C_1 e^{-s}}{mT} \frac{e^{-s}}{(s + \frac{2 c_g v}{M})(s + \frac{1}{T})} \tag{9}
\]

The time delay also has been simplified by using the power series expansion.

\[e^{-s} = \frac{1}{(1 + rs)} \tag{10}\]

Substituting above equations in the transfer function and by substituting the adapted constant values we got the transfer function as

\[
G(s) = \frac{v(s)}{u(s)} = \frac{2.4767}{(s + 0.0476)(s + 1)(s + 5)} \tag{11}
\]

From the above transfer function it shows a third order type 0 system. Due to the time delay approximation, the important fact in realizing the system is that the system has been linearized, as the system has been linearized so the further calculation will become much simpler as compared to the nonlinear system.

**III. Control Schemes**

In this research the velocity controller has been proposed using conventional control techniques. The focus of this
paper is to control the velocity of the vehicle as per the compared values between the set point and the feed-back velocity. The set point limit has been obtained from the literature survey which gives the different comfort limit. The model has been analyzed in the previous section which is used as the process for the controller. As per the controller we need to control the input variables to the process and so from the previous section. The control algorithm uses vehicle throttle position and brake as two control parameter for the control of the velocity of the vehicle. The controller used in this technique is the PID controller which behaves linearly. This chosen PID controller is adaptable in structure, flexible and tuning compatible. The block diagram of the proposed model has been shown in Figure 3. The controller schemes has been classified in conventional, intelligent category. PID controller is one of the conventional control schemes used most commonly. The Invoke of throttle setting and the brake setting has been controlled by a switching logic shown in Figure 4. It decides the required controller to switch in to the said parameters for the motion of the car. The development of dynamic and robust traffic management system based on fuzzy logic approach. Basic methods for computer formulation and solution of the equations of motion.

\[ u(t) = k(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{d}{dt}(e(t)) \] ------ (12)

From the Figure5 ‘y’ is the measured process variable, ‘r’ is the reference variable, ‘u’ is the control signal and ‘e’ is the error. The control signal generated from the controller has the combination of 3 terms that is P-term, I-term and the D-term.

**Figure 6.** Overview analysis of the system.

Different control parameters are proportional gain (K), Integral gain (T_i), and derivative time (T_d). These three parameters belong to proportional, integral and derivative part respectively which deals with past, present and future. The controller block diagram is shown in Figure 5 with T_i=1, T_d=1 and K=1. The visualization of adaptive cruise control has been presented in Figure 6. The speed of the vehicle has been set to a particular speed after detecting a vehicle in front of the host Vehicle.

By designing the controller with the mathematical equations will not fulfill the required task. Some parameters have been considered in order to achieve the required control action. These parameters are

- Noise filtering and high frequency roll-off
- Set point weighting
- Wind up
- Tuning
- Computer implementation

The PID controller transfer function can be described by equation 13.

\[ G(s) = k\left(1 + \frac{1}{sT_i} + sT_d\right) \] ------ (13)

The equation 13 describes the non-interacting controller. Sometimes it is claimed that the interacting controller is easier to tune manually. If a different controller changed the PID controller then the controller also has to change.

Another representation of the PID controller is shown in equation 14.
\[ G'(s) = K + \frac{k_t}{s} + sK_d \]  \hspace{1cm} (14)

Figure 7. Speed of vehicle with respect to Throttle setting.

The equation 14 is similar that of the equation 13. Only the parameters used are different. This equation is basically used for analytical calculation because of the linear appearance. Proportional, Integral or derivative action can be prevail from the above given equation by varying the parameter values. Using integral action in a controller will lead to increasing error, which results the integral term will become very large. Wind up technique is used to reduce the integral action error, results due to the above conditions based on the Figure 5. Throttle and breaking action has been selected by the switching logic technique. The switching criterion used in this work considers the required acceleration to achieve the velocity. The throttle loop is choosen when the required velocity is high and the brake is selected when the required velocity is very low. Between certain ranges the hysteresis loops comes in to play. The error between the set point velocity and the actual velocity is calculated and then processed as the variable to the PID controller with \( K_p = 1 \), \( Ti = 1 \), \( Td = 1 \) respectively. These are used because of the limit comfort to the passenger. Filter is required because the vehicle doesn’t permit the high frequency noise.

3. Result Analysis

The whole system is modeled and controlled using the graphical programming language LabVIEW. The vehicle model is the first part of the paper. The vehicle modeled has been designed by two conventional methods. The first method is by implementing the traditional Newton’s law of motion and the Second method is to find the transfer function from\(^1\). This model depends on friction, slip, torque, aerodynamics. As our target of this paper is to design the controller so these Parameters are neglected .we basically focus on the designing of the controller which is the key goal of the paper. In order to design the controller, the first experiment was to validate the model. After getting the vehicle model we implement the PID controller in order to control the vehicle speed. The diagram shows in Figure 3 defines the block diagram of the controller and the vehicle mathematical model. The controller has been designed in order to assure safety and comfort to the driver and the passengers. The output has been obtained by running the simulation. The switching logic is used to switch between the throttle and brake controller. Different values of \( K_p \), \( T_i \) and \( T_d \) are used in order to get the settling time quickly. In this research the input is the throttle percentage that is the accelerator to increase the speed and the brake applied is used to control the speed of the vehicle. The analysis has two different modes. One is manual mode another is cruise control mode. In the manual mode the vehicle is controlled by the driver. In automatic mode if certain condition arises like the driver is tired and wants to put the vehicle speed in automatic condition, then cruise control mode is activated. In cruise control mode the driver will not able to control the speed of the vehicle until he applies the brake. When the driver applies the brake the cruise control mode will automatically switched to manual mode. This situation is well known as Ideal condition. Figure 7 shows the throttle opening and closing and corresponding vehicle speed. The road profile has been designed in order to develop the longitudinal controller. Figure 8 shows the different modes of the system. The simulation have been verified for certain range of speed.

Table 1. Parameter Estimation

<table>
<thead>
<tr>
<th>Constant parameters</th>
<th>Assumed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1500 Kg/</td>
</tr>
<tr>
<td>Ca</td>
<td>1.19N/(m/s)^2</td>
</tr>
</tbody>
</table>
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### 4. Conclusion

The vehicle model has been developed by using the physical law of equations like Newton's law of motion. The developed model is controlled using the designed conventional PID controller, which is used to control the speed of the vehicle. From the result analysis, we could find the developed controller has better control over the longitudinal dynamics in terms of speed than that of the PI based controller. The vehicle model used in this paper doesn't involve external forces like aerodynamics forces, viscous forces etc. So the future work may be focused on developing an appropriate model considering the all above said parameters for better representation of the actual considered vehicle. The simulation has been verified using the rapid prototyping software called LabVIEW. The results are analyzed in section 5. Research can be done to improve the speed profile of the vehicle by using intelligent controller such as Advanced Control Systems which may result in assured safety, stability and improved performance.

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### 6. References